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RADIO FREQUENCY FORCES, AFFECTING IONS IN CLOSED MAGNETIC TRAPS

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The radio frequency forces, affecting electrons, have been obtained on many occasions in plasma cylinders, axially-symmetric tokamaks and in closed magnetic traps[1]. Now, the theory of Alfvén and fast wave (AW and FW) forces, affecting electrons in toroidal devices, can be supposed to be well-elaborated and understood. The interest to radio frequency forces, affecting ions in toroidal devices, has appeared in the connection with the possibility of the transport barrier formation in tokamaks[2] by Alfvén waves[1]. Transport barriers were observed not only in tokamaks but also in stellarators. That is why it is in particular interesting to consider the problem of the transport barrier formation by Alfvén and fast waves in magnetic traps with closed magnetic surfaces. In this paper, general expressions for time- and surface-averaged radio frequency forces, affecting ions in closed toroidal devices, are obtained.

Summing hydrodynamic equations for electrons and ions, after the time averaging, we have

$$-\boldsymbol{\nabla}\sum_{\alpha} p_{\alpha 0} - \sum_{\alpha}\left(\boldsymbol{\nabla}\cdot\hat{\boldsymbol{\pi}}_{\alpha 0}\right)_0 + \frac{1}{c}\left[\mathbf{j}_0\times\mathbf{B}_0\right] + \mathbf{F}^h = 0, \qquad (1)$$

where $\mathbf{j} = \sum e_\alpha n_\alpha \mathbf{V}_\alpha$, $\omega_{p\alpha}^2 = 4\pi n_\alpha e_\alpha^2/M_\alpha$ is the particle Langmuir frequency,

$$\mathbf{F}^h = \mathbf{F}_n + \mathbf{F}_j + \mathbf{F}_R + \mathbf{F}_\pi, \quad \mathbf{F}_n = \left(\mathbf{E}_\omega \sum_\alpha e_\alpha n_{\alpha\omega}\right)_0, \quad \mathbf{F}_j = \frac{1}{c}\left(\left[\mathbf{j}_\omega\times\mathbf{B}_\omega\right]\right)_0, \qquad (2)$$

$$\mathbf{F}_R \approx -4\pi\sum_\alpha \nabla_i\left(\frac{j^i_{\alpha\omega}\mathbf{j}_{\alpha\omega}}{\omega_{p\alpha}^2}\right)_0 \qquad \mathbf{F}_\pi = \sum_\alpha\left(\boldsymbol{\nabla}\cdot\hat{\boldsymbol{\pi}}_\alpha\right)_0.$$

We employ coordinates r,θ,ζ, where r is any magnetic surface function, the angles θ and ζ are chosen so that magnetic field lines are straight in these coordinates $\mathbf{B}_0 = \left\{B_0^r;\ B_0^\theta;\ B_0^\zeta\right\} = \{0;\ \chi';\ \phi'\}/2\pi\sqrt{g}$, where χ and ϕ are perpendicular and parallel magnetic fluxes, respectively. We find the θ and ζ covariant components of Eq. (1)

$$-\frac{\partial\tilde{p}_0}{\partial\theta} - \left(\boldsymbol{\nabla}\cdot\hat{\boldsymbol{\pi}}_{i0}\right)_\theta - \frac{\sqrt{g}}{c}j_0^r B_0^\zeta + F_\theta^h = 0, \quad -\frac{\partial\tilde{p}_0}{\partial\zeta} - \left(\boldsymbol{\nabla}\cdot\hat{\boldsymbol{\pi}}_{i0}\right)_\zeta + \frac{\sqrt{g}}{c}j_0^r B_0^\theta + F_\zeta^h = 0. \qquad (3)$$

Using the ambipolarity condition $\langle j_0^r\rangle_g = \langle\sqrt{g}j_0^r\rangle/\langle\sqrt{g}\rangle = 0$, $\langle ...\rangle = \int_0^{2\pi}\int_0^{2\pi}(...)d\theta d\zeta/4\pi^2$, we obtain from Eq.(3)

$$-\langle\frac{\partial\tilde{p}_0}{\partial\theta}\rangle_g - \frac{3}{2}\langle\frac{\partial\ln B_0}{\partial\theta}\pi_{i0\parallel}\rangle_g + \frac{1}{2}\langle\frac{\partial\pi_{i0\parallel}}{\partial\theta}\rangle_g + \langle F_\theta^h\rangle_g = 0, \qquad (4)$$

$$-\langle\frac{\partial\tilde{p}_0}{\partial\zeta}\rangle_g - \frac{3}{2}\langle B_0^2\left(\frac{1}{q}\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\zeta}\right)\frac{\pi_{i0\parallel}}{B_0^2}\rangle_g - \frac{3}{2}\langle\pi_{i0\parallel}\frac{\partial\ln B_0}{\partial\zeta}\rangle_g + \frac{1}{2}\langle\frac{\partial\tilde{\pi}_{i0\parallel}}{\partial\zeta}\rangle_g + \langle F_\zeta^h\rangle_g = 0. \qquad (5)$$

In Eqs.(4) and (5) we used the conditions $B_{\theta p} \ll B_{\zeta p}$ $\beta_p = 8\pi p_0/B_0^2 < 1$, where $B_{\theta p}$ and $B_{\zeta p}$ are the "physical" poloidal and toroidal components of the equilibrium magnetic field, respectively, and $p_0 = p_{i0} + p_{e0}$ is the isotropic plasma pressure. Viscous forces should be calculated for the proper magnetic trap, using the time-averaged drift kinetic equation for ions.

In Eqs.(4) and (5) one should eliminate the terms containing the derivatives of the isotropic plasma pressure. For this purpose, we obtain from Eq.(1)

$$\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta}\right)\left(\tilde{p}_0 + \tilde{\pi}_{i0\|}\right) - \frac{3}{2}\left[\pi_{i0\|}\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta}\right)\ln B_0\right]^{\tilde{}} - \tilde{F}_\theta^h - q\tilde{F}_\zeta^h = 0, \qquad (6)$$

where $\tilde{A} = A - \langle A\rangle$.

Eq.(6) can easily be solved at least in two cases: 1) for conventional axially-symmetric tokamaks with arbitrary cross-section and spheromaks, and 2) for large aspect ratio devices with circular cross-section and with the curvature and torsion of magnetic axis depending on the angle ζ [some kinds of stellarators and the facility "Dracon" ("Dragon")]. In the first case, when $\partial/\partial\zeta = 0$, one obtains from Eq.(6)

$$\langle\frac{\partial\tilde{p}_0}{\partial\theta}\rangle_g - \frac{3}{2}\langle\left(\pi_{i0\|}\frac{\partial \ln B_0}{\partial\theta}\right)^{\tilde{}}\rangle_g + \langle\frac{\partial\tilde{\pi}_{i0\|}}{\partial\theta}\rangle_g - \langle\tilde{F}_\theta^h\rangle_g - q\langle\tilde{F}_\zeta^h\rangle_g = 0. \qquad (7)$$

Combining Eqs.(6) and (7), one obtains

$$\frac{3}{2}\left\{\langle\frac{\partial\tilde{\pi}_{i0\|}}{\partial\theta}\rangle_g - \langle\pi_{i0\|}\frac{\partial \ln B_0}{\partial\theta}\rangle_g - \langle\left(\pi_{i0\|}\frac{\partial \ln B_0}{\partial\theta}\right)^{\tilde{}}\rangle_g\right\} + \langle F_\theta^h\rangle_g - \langle\tilde{F}_\theta^h\rangle_g - q\langle\tilde{F}_\zeta^h\rangle_g = 0. \quad (8)$$

The terms in braces are the viscous forces affecting ions in axially symmetric tokamaks and spheromaks. The third term in braces in Eq.(8) can be substantial in spheromaks, and the terms $\langle\tilde{F}_\theta^h\rangle_g + q\langle\tilde{F}_\zeta^h\rangle_g$ can be important for the TAE investigation. In large aspect ratio tokamak, we have the well-known expression for the viscous forces.

In the second case above, the quantities which oscillate with the angles θ and ζ can be written in the form $\tilde{x} = (\tilde{x})_{(\theta)} + [\hat{x}\exp(i\theta) + c.c.]$, where $\tilde{x}$ is any oscillating function, $(\tilde{x})_{(\theta)} = \int_0^{2\pi}\tilde{x}d\theta/2\pi$. As a radial coordinate r, in the following we employ the parallel magnetic flux ϕ. In this case, the following relations hold,

$$\sqrt{g} = (R/2\pi B_0)\ \left\{1 - (\phi/\pi B_0)^{1/2}\left[K\exp(i\theta) + c.c.\right]\right\}.$$

Here, $K = k\exp(ih)$, h is a periodic function of ζ, which can be found from the equation $\partial h/\partial\zeta = -R(\kappa - \kappa_0)$, k and κ are the magnetic axis curvature and torsion, respectively, κ_0 is the averaged of κ over ζ, R is the magnetic axis length divided by 2π.

The magnetic field takes the form $B_0 = B_s\left\{1 + (1/2)(\phi/\pi B_0)^{1/2}\left[K\exp(i\theta) + c.c.\right]\right\}$, where B_s is the magnetic field on the magnetic trap axis. Introducing the quantities

$$X = K/B_0^{3/2}, \qquad Y = \hat{L}_\|^{-1}X, \qquad \hat{L}_\| = 1/q - i\partial/\partial\zeta, \qquad (9)$$

where the operator $\hat{L}_\|^{-1}$ is the inverse of operator $\hat{L}_\|$, one finds from Eq.(4)-(7)

$$-\frac{3}{4}\langle i\hat{\pi}_{i0\|}X^* + c.c.\rangle + \langle X^*\hat{L}_\|^{-1}\left[\frac{3}{2}\hat{\pi}_{i0\|}\frac{\partial \ln B_0}{\partial\zeta} + \frac{3i}{4q}\left(\frac{\phi}{\pi}\right)^{1/2} B_0 X\left(\pi_{i0\|}\right)_{(\theta)}\right] + c.c.\rangle - \quad (10)$$

$$-\left(\pi/\phi\right)^{1/2}\langle 1/B_0\left(F_\theta^h\right)_{(\theta)}\rangle+\langle X^*\left[\hat{L}_\parallel^{-1}\left(1/q\hat{F}_\theta^h+\hat{F}_\zeta^h\right)-\hat{F}_\theta^h\right]+c.c.\rangle=0,$$

$$\frac{3}{2}\langle\frac{1}{B_0}\rangle\langle\left(\pi_{i0\parallel}\right)_{(\theta)}\frac{\partial\ln B_0}{\partial\zeta}\rangle+\langle F_\zeta^h\rangle\langle\frac{1}{B_0}\rangle-\langle\frac{1}{B_0}\left(\tilde{F}_\theta^h\right)_{(\theta)}\rangle=0. \qquad (11)$$

Here "$*$" is the symbol of the complex conjugating. The terms in Eqs. (10),(11), containing the ion parallel viscosity $\pi_{i0\parallel}$, are the viscous forces affecting ions in toroidal devices with circular cross-sections.

Thus, to find the rf forces, affecting ions in magnetic traps, one should in particular calculate the quantities in Eq.(8)

$$F_{1\theta}^h=\langle F_\theta^h\rangle-q\left(\langle F_\zeta^h\rangle_g-\langle F_\zeta^h\rangle\right) \qquad (12)$$

in axially-symmetric tokamaks, and the terms

$$F_{2\theta}^h=\left(\frac{\pi}{\phi}\right)^{1/2}\langle\frac{1}{B_0}\left(F_\theta^h\right)_{(\theta)}\rangle+\langle X^*\left[\hat{L}_\parallel^{-1}\left(\frac{1}{q}\hat{F}_\theta^h+\hat{F}_\zeta^h\right)-\hat{F}_\theta^h\right]+c.c.\rangle, \qquad (13)$$

$$F_{2\zeta}^h=\langle F_\zeta^h\rangle\langle\frac{1}{B_0}\rangle-\frac{1}{q}\left[\langle\frac{1}{B_0}\left(F_\theta^h\right)_{(\theta)}\rangle-\langle F_\theta^h\rangle\langle\frac{1}{B_0}\rangle\right]. \qquad (14)$$

in toroidal devices with circular cross-sections. As in Eqs.(10) and (11) the viscous forces are present, we can include the time-averaged force $\mathbf{F}_\pi$ in these forces. Nevertheless, the input of the time-averaged force $\mathbf{F}_\pi$ into the viscous forces is usually small. Thus, we can calculate viscous forces without the rf terms. That is why we can only consider the time-averaged force

$$\mathbf{F}^h\approx\mathbf{F}_n+\mathbf{F}_j+\mathbf{F}_R, \qquad (15)$$

and, consequently, the forces

$$\langle F_\theta^h\rangle_g=\langle F_{n\theta}\rangle_g+\langle F_{j\theta}\rangle_g+\langle F_{R\theta}\rangle_g,\qquad\langle F_\zeta^h\rangle_g=\langle F_{n\zeta}\rangle_g+\langle F_{j\zeta}\rangle_g+\langle F_{R\zeta}\rangle_g. \qquad (16)$$

Using the covariant derivatives

$$\boldsymbol{\nabla}_k j_i=\frac{\partial j_i}{\partial x^k}-\Gamma_{ik}^m j_m, \qquad (17)$$

where Γ_{ik}^m are the Cristoffel's symbols, we simplify the rf forces

$$F_s^h=\frac{i}{4\omega}\sum_\alpha\left\{\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^k}\left(\sqrt{g}E_{\alpha s}^{ef}j_\alpha^{*k}\right)-j_\alpha^{*k}\frac{\partial}{\partial s}E_{\alpha k}^{ef}+\frac{2\pi i\omega}{\omega_{p\alpha}^2}\frac{\partial}{\partial s}\mid\mathbf{j}_\alpha\mid^2-c.c.\right\}, \qquad (18)$$

where

$$s=\theta,\zeta,\qquad E_{\alpha k}^{ef}=E_k+\frac{4\pi i\omega}{\omega_{p\alpha}^2}j_{\alpha k}. \qquad (19)$$

As it follows from Eqs.(13)and (14), we need to calculate the quantities

$$\langle F_s^h\rangle\approx\frac{i}{4\omega}\sum_\alpha\langle\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^k}\left(\sqrt{g}E_{\alpha s}^{ef}j_\alpha^{*k}\right)-j_\alpha^{*k}\frac{\partial}{\partial s}E_{\alpha k}^{ef}-c.c.\rangle, \qquad (20)$$

and

$$\langle F_s^h\rangle_g=\frac{i}{4\omega\langle\sqrt{g}\rangle}\sum_\alpha\left\{\frac{\partial}{\partial r}\langle\sqrt{g}E_{\alpha s}^{ef}j_\alpha^{*r}\rangle-\langle\sqrt{g}j_\alpha^{*k}\frac{\partial}{\partial s}E_{\alpha k}^{ef}\rangle+\right. \qquad (21)$$

$$+\frac{2\pi i\omega}{\omega_{p\alpha}^2}\langle\sqrt{g}\frac{\partial}{\partial s}\mid \mathbf{j}_\alpha \mid^2\rangle - c.c.\Big\}.$$

Note that Eqs.(13)-(21) are mainly supposed to be used to calculate AW forces, affecting ions, in toroidal devices with the complicated geometry of the magnetic field. In particular, these equations can be used to find AW forces connected with TAE. This is a very cumbersome and, therefore, separate problem. It is implied to be fulfilled later, and the proper analysis of AW forces will be done there.

To find radio frequency (rf) forces, affecting ions in magnetic traps with closed magnetic surfaces, it is convenient to use the magnetohydrodynamic description, when rf currents are given by the expressions

$$\mathbf{j}_{\alpha\omega} = \mathbf{j}_{\alpha\omega\perp} + j_{\alpha\omega\parallel}\mathbf{h}_0. \tag{22}$$

For the case $\omega \ll \omega_{ci}$ and $k_\perp\rho_i \ll 1$, where $k_\perp$ is the characteristic perpendicular wave number and $\rho_i = v_{Ti}/\omega_{ci}$ is the ion Larmour radius, we have

$$\mathbf{j}_{e\omega\perp} \approx \frac{\omega_{pe}^2}{4\pi\omega_{ce}}\left[\mathbf{E}_{\omega\perp}\times\mathbf{h}_0\right] + \frac{c}{B_0}\left[\mathbf{h}_0 \times \left\{\boldsymbol{\nabla}_\perp p_{e\omega\perp} + \left(p_{e\omega\parallel} - p_{e\omega\perp}\right)\boldsymbol{\nabla}_\perp \ln B_0\right\}\right], \tag{23}$$

$$\mathbf{j}_{i\omega\perp} = \frac{\omega_{pi}^2}{4\pi\left(\omega_{ci}^2 - \omega^2\right)}\left\{\omega_{ci}\left[\mathbf{E}_{\omega\perp}\times\mathbf{h}_0\right] - i\omega\mathbf{E}_{\omega\perp}\right\}. \tag{24}$$

In Eqs.(23) and (24), we neglect thermal effects, except the anisotropic electron pressure where one should take into account only parts containing the collisional or Landau damping, depending on collisional or collisionless plasmas is under consideration. The rf parallel ion current $j_{i\omega\parallel}$ is supposed to be equal to zero, and the rf parallel electron current and the electron anisotropic pressure should be found from the definitions

$$j_{e\omega\parallel} = e_e\int d\mathbf{v}v_\parallel f_{e\omega}, \qquad p_{e\omega\parallel} = M_e\int d\mathbf{v}w_\parallel^2 f_{e\omega}, \qquad p_{e\omega\perp} = \frac{1}{2}M_e\int d\mathbf{v}w_\perp^2 f_{e\omega}. \tag{25}$$

Thus, this is one of advantages of the magnetohydrodynamic approximation to the problem of the determination of AW and FW forces, affecting ions, that one should calculate only three scalar quantities $j_{e\omega\parallel}, p_{e\omega\parallel}, p_{e\omega\perp}$, to find the absorption of these waves in magnetic traps with closed magnetic surfaces. The perturbed electron distribution function can be found from the drift kinetic equation.

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