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Radiofrequency conductivity of Magnetized Plasmas for Low Aspect Ratio Tokamaks

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Introduction Recently, several experiments in spherical and low aspect ratio tokamaks (LART), such as the START tokamak[1], have produced efficient plasma confinement in high beta regimes, and nowadays the radio frequency (RF) plasma heating and current drive studies are very important parts of experimental and theoretical programs on these tokamaks. Analyses of these effects are based on the RF dissipation in plasmas.

Usually, the theoretical study of the RF field excitation and absorption in plasmas is based on the solution of the Vlasov-Maxwell set of the equations. On the first step the Vlasov equation should be solved and dielectric permeability tensor, which needs for solving of the Maxwell equations and colisionless absorption analysis. For simple toroidal plasmas with circular magnetic surfaces and small toroidicity parameter the analytical solution of Vlasov's equation was presented in Refs.[2, 3, 4].

Here, taking into account the low aspect ratio for circular magnetic surfaces, we present the analytical solution of Vlasov's equation. We develop the evaluation of the RF conductivity parallel to magnetic field and show the effect of the low aspect ratio parameter ($R_0/r = 1/\epsilon$) on the electron dissipation in LART.

The Vlasov equation We begin our discussion evaluating the electron distribution function that is governed by the Vlasov equation for LART geometry with the circular cross section of magnetic surfaces, using the quasitoroidal set of coordinates r, ζ, θ. All plasma parameters and equilibrium magnetic field, are assumed to be independent in time t and in ζ,which is the axially symmetric toroidal coordinate . Further, we are going to solve the linearized Vlasov's equation for the fluctuating component ($\sim \exp \mathrm{i}(n\zeta - \Omega t)$) of the electron distribution function in magnetized plasmas,

$$-\mathrm{i}\omega f_0 + k_0 v \cos\gamma \left(\frac{\partial f_0}{\partial \theta} + \frac{\mathrm{i}\, n q_t f_0}{1+\epsilon\cos\theta} \right) + \epsilon \frac{k_0 v \sin\theta \sin\gamma}{2(1+\epsilon\cos\theta)} \frac{\partial f_0}{\partial \gamma} = \frac{e\ \cos\gamma}{m_e} \frac{\partial F_M}{\partial v} E_3; \qquad (1)$$

where $q_t = r h_\zeta / R_0 h_\theta$ is the tokamak stability parameter, $e = |e|$, $k_0 = h_\theta / r$, and v, σ, γ are spherical coordinates in the velocity space with a polar axis along the magnetic field. The Krook's model used here to describe the collisions, $\hat{S}t[f] = -\nu_e f$, is represented through complex frequency, $\omega = \Omega - \mathrm{i}\nu_e$. The above equation, which is a zero order drift equation, is written only for Fourier amplitude f_0 of a fluctuating component expansion $f(t, \vec{r}, \vec{v}) = \sum_l f_l \exp(\mathrm{i}\, l\sigma)$ via the σ-coordinate, and l-harmonic coupling is neglected because of the approximation of the small Larmour radius (see Ref.[2]). The parallel component of the oscillating current j_3 contain only zero-harmonic term,

$$j_3 = -2\pi e \int_0^\infty v^3 \mathrm{d}v \int_0^\pi f_0 \sin\gamma \cos\gamma \mathrm{d}\gamma. \qquad (2)$$

Further, index 0 in the function f_0 is omitted. In Eq.(1), we are going to use the same solving method as in Refs.[2, 3, 4] but without assumption $\epsilon \ll 1$. As usually, the new variables κ and w for untrapped particles are introduced instead of γ and θ,

$$\kappa^2 = 2\epsilon/[1+\epsilon-\sin^2\gamma(1+\epsilon\cos\theta)], \quad w = \int_0^{\theta/2} \mathrm{d}\eta/\sqrt{1-\kappa^2\,\sin^2\eta}$$

and the variables for the trapped particles are defined by the equations:

$$\bar{\kappa}^2 = [1+\epsilon-\sin^2\gamma(1+\epsilon\cos\theta)]/(2\epsilon), \quad \bar{w} = \int_0^{\arcsin(\sin(\theta/2)/\bar{\kappa})} \mathrm{d}\eta/\sqrt{1-\bar{\kappa}^2\,\sin^2\eta},$$

Finally, we get the equation for the fluctuating component f_s^u of the untrapped particles:

$$\frac{\partial f_s^u}{\partial w} - \frac{2\mathrm{i}s\,\omega\,\kappa}{k_0\,v\kappa_0}\sqrt{1-\kappa_0^2\,\mathrm{sn}^2 w}\,f_s^u + \frac{\mathrm{i}\,2nq_t\,\mathrm{dn}w\,f_s^u}{(1+\epsilon)(1-\kappa_0^2\,\mathrm{sn}^2 w)} = -\frac{2e\,F_M}{k_0\,T_e}\sum_m E_m \mathrm{dn}w\exp 2\mathrm{i}m\,\mathrm{am}w \tag{3}$$

where $\kappa_0 = \sqrt{2\epsilon/(1+\epsilon)}$, and $s=1$ for $0\le\gamma\le\pi/2$, $s=-1$ for $\pi/2\le\gamma\le\pi$ with the boundary conditions, $f_s^u|_{w=K(\kappa)} = f_s^u|_{w=-K(\kappa)}$. For the trapped particles we have,

$$\begin{aligned}&\frac{\partial f_s^t}{\partial\bar{w}} - \frac{2\mathrm{i}s\,\omega}{k_0\,v\kappa_0}\sqrt{1-\kappa_0^2\bar{\kappa}^2\mathrm{sn}^2\bar{w}}\,f_s^t + \frac{2\mathrm{i}\,nq_t\bar{\kappa}\mathrm{cn}\bar{w}\,f_s^t}{(1+\epsilon)(1-\kappa_0^2\bar{\kappa}^2\mathrm{sn}^2\bar{w})}\\ &= -\frac{2e\,F_M}{k_0\,T_e}\sum_m E_m\bar{\kappa}\mathrm{cn}\bar{w}\exp\left(2\mathrm{i}m\bar{\kappa}\int_0^{\bar{w}}\mathrm{cn}u\mathrm{d}u\right)\end{aligned} \tag{4}$$

with the boundary conditions, $f_s^t|_{w=\pm K(\bar{\kappa})} = f_{-s}^t|_{w=\pm K(\bar{\kappa})}$. In this case, the equation for the current density $j_{3,p}^u$ of untrapped particles has the form:

$$j_{3,p}^u = -2e\kappa_0^2\sum_s s\int_0^\infty v^3\mathrm{d}v\int_{\kappa_0}^1\frac{\mathrm{d}\kappa}{\kappa^3}\int_{-K(\kappa)}^{K(\kappa)}\frac{f_s^u(\kappa,w)\mathrm{dn}w}{1-\kappa_0^2\,\mathrm{sn}^2\bar{w}}\exp\left(-2\mathrm{i}p\mathrm{am}w\right)\mathrm{d}w \tag{5}$$

The analogous equation for the current density $j_{3,p}^t$ of trapped particles is like this:

$$j_{3,p}^t = -2e\kappa_0^2\sum_s s\int_0^\infty v^3\mathrm{d}v\int_0^1\bar{\kappa}^2\mathrm{d}\bar{\kappa}\int_{-K(\bar{\kappa})}^{K(\bar{\kappa})}\frac{f_s^t(\bar{\kappa},\bar{w})\,\mathrm{cn}\bar{w}}{1-\kappa_0^2\bar{\kappa}^2\mathrm{sn}^2\bar{w}}\exp\left(-2\mathrm{i}p\bar{\kappa}\int_0^{\bar{w}}\mathrm{cn}\eta\mathrm{d}\eta\right)\mathrm{d}\bar{w} \tag{6}$$

Parallel rf conductivity for LART. To solve the Eqs.(3,4) we should present the coefficients in the left hand side as a sum of averaged and periodical parts,

$$\sqrt{1-\kappa_0^2\,\mathrm{sn}^2 w} = A^u(\kappa)+\phi^u(w,\kappa), \quad \frac{\mathrm{dn}w}{1-\kappa_0^2\,\mathrm{sn}^2 w} = \frac{\pi}{2K(\kappa)\sqrt{1-\kappa_0^2}}+\psi^u(w,\kappa) \tag{7}$$

analogously, in the case of the trapped particles,

$$\sqrt{1-\kappa_0^2\,\bar{\kappa}^2\mathrm{sn}^2\bar{w}} = A^t(\bar{\kappa})+\phi^t(\bar{w},\bar{\kappa}), \quad \bar{\kappa}\mathrm{cn}\bar{w}/(1-\kappa_0^2\,\bar{\kappa}^2\mathrm{sn}^2\bar{w}) = \psi^t(\bar{w},\bar{\kappa}), \tag{8}$$

where $K(\kappa)$ is the complete elliptic integral of the first kind. The functions $\phi^{u,t}(w,\kappa)$ and $\psi^{u,t}(w,\kappa)$ are even periodical functions and the function $\mathrm{am}w$ can be also presented as sum of periodical and non-periodical parts $\pi\,w/2K(\kappa)+\tilde{\mathrm{am}}w$. The expansions in Eqs.(3,4) are possible due to the properties of Jakobi functions [5]. Note that this solving scheme of Eqs.(3,4) was also demonstrated in Refs.[2, 3, 4]. Substitutinq f_s^u and f_s^t into

Eqs.(3,4) we find the parallel components of the rf conductivity tensor $\sigma^{p,m}_{33(u)}$ for the untrapped and trapped particles:

$$\frac{4\pi\mathrm{i}}{\Omega}\sigma^{p,m}_{33(u)} = \frac{\omega_{pe}^2\kappa_0^2}{\sqrt{2\pi}\Omega k_0 v_T}\sum_r \int_{\kappa_0}^1 \frac{\mathrm{d}\kappa}{\kappa^3}\int_{-\infty}^{\infty}\frac{\bar v^4 \exp\left(-\bar v^2/2\right) D^{u,r}_{p,1}(\bar v,\kappa)\, C^{u,r}_{m,1}(\bar v,\kappa)\,\mathrm{d}\bar v}{\bar v(r+nq_t/\sqrt{1-\epsilon^2})-\Omega/\omega_b^u-\mathrm{i}\nu_e/\omega_b^u}; \tag{9}$$

$$\frac{4\pi\mathrm{i}}{\Omega}\sigma^{p,m}_{33(t)} = \frac{\omega_{pe}^2\kappa_0^2}{\sqrt{2\pi}\Omega k_0 v_T}\sum_r \int_0^1 \bar\kappa\,\mathrm{d}\bar\kappa\int_{-\infty}^{\infty}\frac{\bar v^4 \exp\left(-\bar v^2/2\right) D^{t,r}_{p,1}(\bar v,\bar\kappa)\, C^{t,r}_{m,1}(\bar v,\bar\kappa)\,\mathrm{d}\bar v}{r\,\bar v-\Omega/\omega_b^t-\mathrm{i}\nu_e/\omega_b^t} \tag{10}$$

where the characteristic bounce frequencies are introduced

$$\omega_b^u = \pi\, k_0\, v_T\kappa_0/(2K(\kappa)\, A^u(\kappa)\,\kappa),\ \ \omega_b^t = 4^{-1}\pi\, k_0\, v_T\kappa_0 K^{-1}(\kappa)/A^t(\bar\kappa)$$

The coefficients $C^{u,r}_{m,s}(\bar v,\kappa)$, $C^{u,r}_{m,s}(\bar v,\kappa)$, $D^{t,r}_{m,s}(\bar v,\kappa)$ and $D^{u,r}_{m,s}(\bar v,\bar\kappa)$ are represented as,

$$\begin{aligned} C^{u,r}_{m,s}(\bar v,\kappa) &= \frac{1}{\pi}\int_{-K(\kappa)}^{K(\kappa)}\mathrm{d}w\mathrm{dn}w\ \exp\left[\mathrm{i}(m-r)\frac{\pi w}{K(\kappa)}+2\mathrm{i}\,\mathrm{a\tilde{m}}w+\mathrm{i}\Phi_s^u(v,w,\kappa)\right],\\ D^{u,r}_{p,s}(\bar v,\kappa) &= \frac{1}{\pi}\int_{-K(\kappa)}^{K(\kappa)}\frac{\mathrm{d}w\mathrm{dn}w}{1-\kappa_0^2\,\mathrm{sn}^2w}\exp\left[\mathrm{i}(r-p)\frac{\pi w}{K(\kappa)}-2\mathrm{i}p\,\mathrm{a\tilde{m}}w-\mathrm{i}\Phi_s^u(\bar v,w,\kappa)\right], \end{aligned} \tag{11}$$

$$\begin{aligned} C^{t,r}_{m,s}(\bar v,\bar\kappa) &= \frac{1}{\pi}\int_{-2K(\bar\kappa)}^{2K(\bar\kappa)}\mathrm{d}\bar w\ \bar\kappa\mathrm{cn}\bar w\exp\left[-\mathrm{i}\frac{r\pi\bar w}{2K(\bar\kappa)}+2\mathrm{i}m\bar\kappa\int_0^{\bar w}\mathrm{d}\eta\mathrm{cn}\eta+\mathrm{i}\Phi_s^t(v,\bar w,\bar\kappa)\right].\\ D^{t,r}_{p,s}(\bar v,\bar\kappa) &= \frac{1}{\pi}\int_{-K(\bar\kappa)}^{K(\bar\kappa)}\frac{\mathrm{d}\bar w\ \bar\kappa\mathrm{cn}\bar w}{1-\kappa_0^2\bar\kappa^2\mathrm{sn}^2\bar w}\exp\left[\mathrm{i}\frac{r\pi\bar w}{2K(\bar\kappa)}-2\mathrm{i}p\bar\kappa\int_0^{\bar w}\mathrm{d}\eta\mathrm{cn}\eta-\mathrm{i}\Phi_s^t(\bar v,\bar w,\bar\kappa)\right] \end{aligned} \tag{12}$$

where the dimensionless velocity $\bar v = v/v_T$ is introduced and the functions $\Phi_s^u(v,w,\kappa)$ and $\Phi_s^t(v,\bar w,\bar\kappa)$ are defined by integrals of the oscillating coefficient of Eqs.(7-8):

$$\begin{aligned} \Phi_s^u(v,w,\kappa) &= \int_0^w\left(\frac{2nq_t}{1+\epsilon}\,\psi^u(\eta,\kappa)-\frac{2s\omega\,\kappa}{k_0\,v\,\kappa_0}\,\phi^u(\eta,\kappa)\right)\mathrm{d}\eta\\ \Phi_s^t(v,\bar w,\bar\kappa) &= \int_0^{\bar w}\left(\frac{2nq_t}{1+\epsilon}\psi^t(\eta,\bar\kappa)-\frac{2s\omega}{k_0\,v\,\kappa_0}\,\phi^t(\eta,\bar\kappa)\right)\mathrm{d}\eta \end{aligned} \tag{13}$$

The boundary conditions for functions $f_s^u(\kappa,w)$ and $f_s^t(\bar\kappa,\bar w)$ are satisfied due to the periodicity of related functions (see Eqs.(7-8) and (13,12)). The real part of the parallel conductivity is very important for the analysis of wave dissipation in plasmas, because density of dissipated wave power is proportional to it. For the untrapped particles this value is given by the equation

$$\frac{4\pi}{\Omega}\mathrm{Re}\,\sigma^{p,m}_{33(u)} = \frac{\omega_{pe}^2\kappa_0^2}{\Omega k_0 v_T}\sqrt{\frac{\pi}{2}}\sum_r\int_{\kappa_0}^1\frac{\mathrm{d}\kappa v_{u,r}^4\exp\left(-v_{u,r}^2/2\right)D^{u,r}_{p,1}(v_{u,r},\kappa)\,C^{u,r}_{m,1}(v_{u,r},\kappa)}{\kappa^3|r+nq_t/\sqrt{1-\epsilon^2}|} \tag{14}$$

Analogous equations take place for the trapped particles:

$$\frac{4\pi}{\Omega}\mathrm{Re}\,\sigma^{p,m}_{33(t)} = \frac{\omega_{pe}^2\kappa_0^2}{\Omega k_0 v_T}\sqrt{\frac{\pi}{2}}\sum_r\int_0^1\frac{\bar\kappa\,\mathrm{d}\bar\kappa}{|r|}v_{t,r}^4\exp\left(-v_{t,r}^2/2\right)D^{t,r}_{p,1}(v_{t,r},\bar\kappa)\,C^{t,r}_{m,1}(v_{t,r},\bar\kappa) \tag{15}$$

where $\omega_{pe} = \sqrt{4\pi N_0 e^2/m_e}$ is the electron plasma frequency, $v_{u,r} = \Omega/\omega_b^u(r+nq_t/\sqrt{1-\epsilon^2})$ and $v_{t,r} = \Omega/\omega_b^t r$ are the bounce resonance values of the untrapped and trapped particle velocities A replacing of an integration limits was performed with an using of the relation

$\Phi^{u,}_{s,p}(v, w, \kappa) = \Phi^{u}_{-s,p}(-v, w, \kappa)$. It should be noted that the quantities $C^{u,r}_{m}(\kappa, x)$ and $D^{u,r}_{p}(\kappa, x)$ and the analogous values for the trapped particles are real.
Finally, we calculate numerically the parallel rf conductivity. In Fig.1, we show dependence of $4\pi \mathrm{Re}\, \sigma^{m,m}_{33(u)}(\epsilon)/\Omega = \mathrm{Im}\, \epsilon^{m,m}_{33(u)})$, which is proportional to the wave dissipation power, over the parameter $\omega/(k_{||} v_T)$, where $k_{||} = k_0(m + nq_t)$.

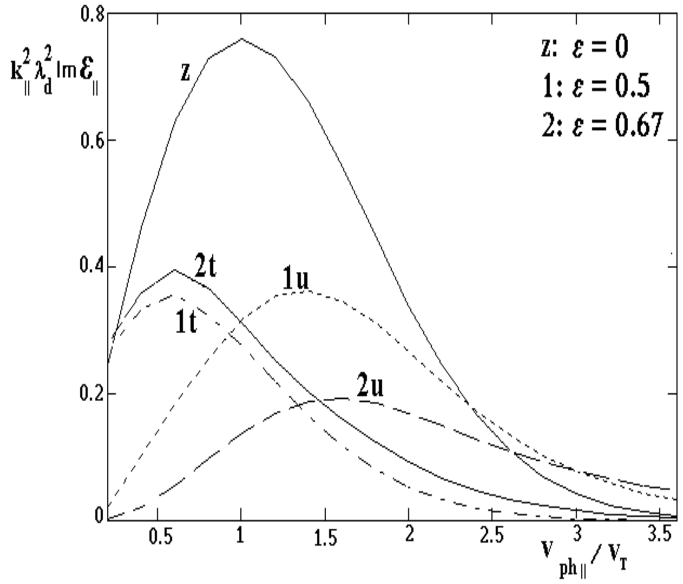

Fig.1. *Plot of the imaginary part of the parallel dielectric tensor component* $Im\varepsilon^{mm}_{33}$ *over normalized phase velocity* v_{ph}/v_{Te} *for different values toroidicity* $\epsilon = 0.0$, 1/2, 2/3.

In the conclusion, we can say that the wave dissipation is strongly enhanced for waves with phase velocity larger than thermal velocity because of strong modulation of parallel velocity of electrons. This phenomena can be effectively used for rf plasma heating and current drive in low aspect ratio tokamaks.

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