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Collisional Transport in a Plasma with Steep Gradients

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abstract The validity is given to the newly proposed two weight δf method for neoclassical transport calculation, which can solve the drift kinetic equation taking into accoun effects of steep plasma gradients, large radial electric field, finite banana width, and an orbit topology near the axis. The new method is applied to the study of ion transport with steep plasma gradients. It is found that the ion thermal diffusivity decreases as the scale length of density gradient decreases, while the ion particle flux due to ion-ion self collisions increases with increasing gradient.

1. Introduction

The neoclassical theory[1] is constructed based on the assumption that $\rho_p \ll L_r$ and $M_p \ll 1$, where ρ_p is the poloidal Larmor radius, L_r is the radial gradient length of plasma parameters, and M_p is the poloidal Mach number. Recent fusion experiments are often operated in parameter regimes beyond this assumption. Therefore, the theory for neoclassical or collisional transports should be extended to include effects of finite orbit width dynamics, strong radial electric field, large radial gradients, and non-standard orbit topology near magnetic axis. It has been shown by Lin, Tang, and Lee [2] that the δf particle simulation, solving the drift kinetic equation, can be a powerful tool for neoclassical transport calculation in the new parameter regime.

Recently, the present authors have developed a new δf method to solve the drift kinetic equation [3], in which the collision scheme was much improved and the two weighting method was employed. Accurate implementation of Coulomb collisions is an important ingredient for neoclassical transport calculation. A linear like-particle collision scheme, almost perfectly conserving the particle number, momentum, and energy has been given in Ref. [3]. The benchmark calculation of neoclassical transport using this collision scheme demonstrated the increased accuracy in results, and the collision scheme in Ref.[3] seems to be most adequate for δf simulations. The nonlinear weighting scheme [4], which is used for gyrokinetic particle simulation, is difficult to be directly applied to the drift kinetic equation with diffusive motion due to Coulomb collisions. Chen and White [5] derived a rigorous collisional δf algorithm by treating the weight as a new dimension of particle motion. However, the question how to evaluate the marker density g for weight calculation still remained to be solved, although the precise estimation of g is essential for the δf method. The present autors presented a new scheme in which g is evaluated from its kinetic equation using the idea of δf method. The resultant weighting scheme consists of two weight equations, and is more effective and accurate to solve the drift kinetic equation. In the following, we give the validity of this two weighting method. Ion transports in a plasma with steep density gradients are studied by the δf simulation developed in Ref.[3].

2. Two weighting δf formulation

We start from the well-known drift kinetic equation [1] for a guiding center distribution function $f(\vec{v}, \vec{x}, t)$. Separating f into two parts $f = f_0 + f_1$ $(f_1 \ll f_0)$, we solve the equation for $f_1(\delta f)$ introducing the concept of weight. Chen and White [5] derived the weight equation treating the weight as a new dimension of the particle motion, in addition to the usual dimensions of $(\vec{x}, \vec{v})$ phase space. The weight equation for f_1 is given by [5]

$$\dot{w} = \frac{1}{g}\left[-\int wS_M dw - \vec{v_d}\cdot\nabla f_0 + C(f_0, f_1)\right] \tag{1}$$

$$\frac{D}{Dt}g = \frac{\partial g}{\partial t} + (\vec{v_\parallel} + \vec{v_d})\cdot\nabla g - C(g, f_0) = \int S_M dw \tag{2}$$

where $\vec{v_d}$ is the drift velocity, C is the collision operator, and S_M is the particle source. Note that equation (1) is derived under the assumption that $\dot{w}$ is independent of w. Instead of using $g = f$ in equation (1) as in the nonlinear weighting scheme, we now consider to solve g with $g(t = 0) = f_0$, using the idea of δf method. Set $g = g_0 + g_1$ $(g_1 \ll g_0)$ and $g_0 = f_0 = f_M$, where f_M is a Maxwellian distribution function. In the same way as in obtaining the weight w for f_1, we have the weight w_1 for g_1 as

$$\dot{w}_1 = \frac{1}{h^{(1)}}\left[-\int w_1\Omega_M^{(1)}dw_1 - \vec{v_d}\cdot\nabla f_0 + \int S_M dw\right] \tag{3}$$

$$\frac{D}{Dt}h^{(1)} = \int \Omega_M^{(1)}dw_1 \tag{4}$$

Now we have successively obtain equations, for $w_2, w_3, w_4, \cdots$,

$$\dot{w}_k = \frac{1}{h^{(k)}}\left[-\int w_k\Omega_M^{(k)}dw_k - \vec{v_d}\cdot\nabla f_0 + \int \Omega_M^{(k-1)}dw_{k-1}\right] \tag{5}$$

$$\frac{D}{Dt}h^{(k)} = \int \Omega_M^{(k)}dw_k \tag{6}$$

Thus, we have an infinite set of hierarchy equations. For $j-$th marker, we can write, for $k = 1, 2, \cdots$,

$$g_j = f_{0j} + w_{1j}h_j^{(1)}, \qquad h_j^{(k)} = f_{0j} + w_{k+1,j}h_j^{(k+1)} \tag{7}$$

Note that the source term S_M is chosen so as to satisfy physics requirements. On the other hand, $\Omega_M^{(k)}$ can be chosen arbitrarily. We choose $\Omega_M^{(k)}$ as, for $k = 1, 2, \cdots$,

$$\int w_k\Omega_M^{(k)}dw_k = 0, \qquad \int \Omega_M^{(k)}dw_k = (1 + \epsilon_k)\int S_M dw \tag{8}$$

Here, we will assume that $\epsilon_1 \neq \epsilon_2 \neq \cdots$ and $|\ \epsilon_k\ | \ll 1$ for all k. Since the dominant term of $h^{(k)}$ is f_0, we can approximate equation (5) to

$$\dot{w}_k \simeq \dot{w}_1 + \epsilon_k \frac{1}{f_0} \int S_M dw \tag{9}$$

From equation (7), g can be written as, omitting the index j,

$$g = (1 + w_1 + w_1 w_2 + w_1 w_2 w_3 + \cdots) f_0 + w_1 w_2 w_3 \cdots h^{(\infty)} \tag{10}$$

The second term on the right hand side vanishes because $|\ w_k\ | < 1$ for all k. If we take, for example, $\epsilon_k = \varepsilon(-1)^{k-1}/k$ with $|\ \varepsilon\ | \ll 1$, the series with f_0 can converge to yield

$$g = (1 + w_1 + w_1^2 + w_1^3 \cdots) f_0 + O(\varepsilon) = \frac{f_0}{1 - w_1} + O(\varepsilon) \tag{11}$$

Likewise,

$$h^{(k)} = \frac{f_0}{1 - w_1} + O(\varepsilon) \tag{12}$$

In the lowest order (we can choose ε as small as possible), replacing w_1 by ω, we recover our previous two weighting equations [3];

$$\dot{w} = \frac{1-\omega}{f_0} \left[-\int w S_M dw - \vec{v_d} \cdot \nabla f_0 + C(f_0, f_1) \right], \tag{13}$$

$$\dot{\omega} = \frac{1-\omega}{f_0} \left[-\vec{v_d} \cdot \nabla f_0 + \int S_M dw \right]. \tag{14}$$

Thus, the two weighting δf method for neoclassical study [3] has been validated and the equations (13) and (14) represent a general and accurate weighting scheme.

3. Ion transports with steep gradients

Particle simulations employing the δf method mentioned above were carried out to investigate ion neoclassical transports [3]. It was found that ion thermal transport and ion parallel flow near the axis are largely reduced due to the non-standard orbit topology near the axis.

In the present paper, we study ion collisional transports in a tokamak plasma with steep density gradients. The simulations are carried out for a simple equilibrium field with apsect ratio of 6 and the constant safety factor ($q = 1.1$), using 5,000,000 markers distributed in the whole poloidal cross section. The particle source is chosen as $S_M = \nu(t) s(\vec{x}) f_M \delta(w)$ and markers are added to the simulation domain in terms of the rate of $\nu(t)$ and spatial distribution $s(\vec{x})$. The radial particle flux Γ_i due to ion-ion self collisions, which vanishes in the conventional neoclassical theory, is calculated changing $\eta_b = \Delta_b / L_n$, where Δ_b is the banana width and L_n is the scale length of the density gradient. Γ_i vanishes for $\eta_b \leq 0.1$ and increases linearly as η_b increases. The ion thermal diffusivity χ_i is also examined. As η_b increases, χ_i decreases linaerly and $\chi_i \simeq \chi_i^{NC}$ for $\eta_b = 0.5$, where χ_i^{NC} is the value by the conventional neoclassical theory. From these results, it is expected that the δf method developed by the present authors would be useful for the study of collisional transports beyond the scope of the conventional neoclassical theoty.

4. Summary

We have validated the newly proposed two weighting method [3] by taking into account of the hierarchy equations for weights. Using this two weighting mathod in the δf simulation, we have studied ion collisional transport with a steep density gradient. It has been shown that the ion thermal diffusivity decreases as the density gradient scale decreases. It has also been shown that the particle flux due to ion-ion like collisions, which vanishes in the conventional neoclassical theory, increases with decreasing density gradient scale. The details of this investigation will be published elsewhere.

Collisional tranports in the presence of large electric field, steep gradients of temperatures, and reversed magnetic shear are under investigation.

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