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Pressure-driven modes in an $L = 2/M = 10$ heliotron/torsatron system

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Introduction

From an analysis of finite-β MHD equilibria in an $L = 2/M = 10$ planar axis heliotron/torsatron system with an inherently large Shafranov shift (L and M are the polarity and toroidal field period of the helical coils, respectively), it has been shown:

1. The change of the local magnetic shear due to a large Shafranov shift is essentially axisymmetric, i.e., related to toroidicity. This change leads to the disappearance of the (integrated) local magnetic shear on the outer side of the torus, even in the region with a stellarator-like global magnetic shear.[1]

2. The local normal magnetic curvature consists of parts due to both toroidicity and helicity of the helical coils, which determines the three-dimensional properties of the high-mode-number ballooning modes.[2]

In three-dimensional MHD equilibria, the eigenvalues ω^2 for high-mode-number ballooning modes have the functional form $\omega^2 = \omega^2(\psi, \theta_k, \alpha)$ where ψ labels the flux surface, α labels the magnetic field line on it, and θ_k is the radial wave number. Since ω^2 has no α-dependence in axisymmetric systems, the stronger the α-dependence of ω^2 is, the more significant the three-dimensional properties of ω^2 are. The topological properties of the unstable eigenvalues ω^2 (< 0) in (ψ, θ_k, α) space, are the following [2]:

1. For Mercier-unstable equilibria, two types of topological level surfaces for ω^2 coexist. One is a tokamak-like cylindrical level surface whose axis is in α direction. The other is a spheroidal level surface inherent to three-dimensional systems, which is surrounded by the cylindrical level surface. Modes with spheroidal level surfaces for ω^2 have larger growth rates than those with cylindrical level surfaces.

2. In Mercier-stable equilibria, only a topologically spheroidal level surface exists.

From these results, it was previously conjectured [2] that the global structure of pressure-driven modes would have the following properties:

1. Global modes that correspond to modes in the local mode analysis with a topologically cylindrical level surface for ω^2 will be poloidally localized tokamak-like ballooning modes or interchange modes. Effects of the toroidal mode coupling on these modes are weak.

2. Global modes corresponding to modes in the local mode analysis with a topologically spheroidal level surface for ω^2 will be such ballooning modes inherent to three-dimensional systems with strong toroidal mode coupling that they have high poloidal and toroidal mode numbers and are localized in both the poloidal and toroidal directions.

3. For Mercier-unstable equilibria, where both topologically cylindrical and spheroidal level surfaces for ω^2 coexist, poloidally localized tokamak-like ballooning modes or interchange modes with weak toroidal mode coupling appear when their typical toroidal mode numbers are relatively low. As the typical toroidal mode numbers become higher, such ballooning modes inherent to three-dimensional systems appear with strong toroidal mode coupling that they have larger growth rates and are localized in both the poloidal and toroidal directions.

4. In Mercier-stable equilibria, where only a topologically spheroidal level surface for ω^2 exists, only such ballooning modes inherent to three-dimensional systems appear with strong toroidal mode coupling that they have high poloidal and toroidal mode numbers and are localized in both the poloidal and toroidal directions.

The purposes of the present work are to prove the above conjecture for both Mercier-unstable and Mercire-stable equilibria, and to clarify the inherent properties of pressure-driven modes, through a global mode analysis by using CAS3D code.[4]

Categorization of MHD equilibria $[1-3]$

From the viewpoint of the ideal MHD stability, a vacuum configuration of a planar axis $L = 2/M = 10$ heliotron/torsatron system is helicity-dominant, which comes from the helical coils. Both the local magnetic shear and normal magnetic curvature are mainly determined by helicity. The characteristics of the finite-β equilibria are determined by an essentially axisymmetric large Shafranov shift. The change in the local structures of the local magnetic shear (integrated along the magnetic field line) and the normal magnetic curvature by the Shafranov shift is related to toroidicity. The Shafranov shift reduces the (integrated) local magnetic shear on the outside of the torus, leading to the reduction of the field line bending stabilizing effect on ballooning modes. On the other hand, the Shafranov shift enhances (reduces) the local unfavorable normal magnetic curvature on the inner (outer) side of the torus.

According to the degree of the reduction of the local magnetic shear by the Shafranov shift, the Mercier-unstable equilibria can be categorized into two types, namely, toroidicity-dominant Mercier-unstable equilibria and helicity-dominant Mercier-unstable equilibria. The toroidicity-dominant Mercier-unstable equilibria are characterized by properties that the local magnetic shear is strongly reduced by a relatively large Shafranov shift, so that ballooning modes are easily destabilized. These equilibria are created with a peaked pressure profile either with zero net toroidal current or with net toroidal current such that the rotational transform increases slightly. The helicity-dominant Mercier-unstable equilibria are characterized by properties that the local magnetic shear is not reduced so much by a relatively small Shafranov shift, so that ballooning modes are not easily destabilized. These equilibria are created with a broad pressure profile with zero net toroidal current. The toroidicity-dominant Mercier-unstable equilibria tend to be more Mercier stable than the helicity-dominant Mercier-unstable equilibria, for the same β value at the magnetic axis, because the average magnetic curvature due to the Shafranov shift is favorable (unfavorable) in the region where the pressure gradient is large, for the former (latter) equilibria. From the same reason, the

Mercier-stable equilibria belong to toroidicity-dominant ones.

Global mode analysis in the Mercier-unstable equilibria [3]

Since the local magnetic curvature due to helicity has the same period M in the toroidal direction as the toroidal field period of the equilibria, the characteristics of the pressure-driven modes in such Mercier-unstable equilibria dramatically change according to how much the local magnetic shear is reduced (whether the equilibrium is toroidicity-dominant or helicity-dominant) and also according to the relative magnitude of the typical toroidal mode numbers n of the perturbations compared with the toroidal field period M of the equilibria.

For the toroidicity-dominant Mercier-unstable equilibria, the pressure-driven modes change from interchange modes with negligible toroidal mode coupling for low toroidal mode numbers $n < M$, to tokamak-like poloidally localized ballooning modes with weak toroidal mode coupling for moderate toroidal mode numbers $n \sim M$, and finally to both poloidally and toroidally localized ballooning modes purely inherent to three-dimensional systems with strong poloidal and toroidal mode couplings for fairly high toroidal mode numbers $n \gg M$. Strong toroidal mode coupling, in cooperation with the poloidal mode coupling, makes the perturbation localize to flux tubes.

For the helicity-dominant Mercier-unstable equilibria, the pressure-driven modes change from interchange modes, with negligible toroidal mode coupling for $n < M$ or with weak toroidal mode coupling for $n \sim M$, directly to poloidally and toroidally localized ballooning modes purely inherent to three-dimensional systems with strong poloidal and toroidal mode couplings for $n \gg M$.

Since the equilibria are Mercier unstable, interchange modes with low toroidal mode numbers $n < M$, experiencing the unfavorable magnetic curvature with its local structure averaged out, occur for both toroidicity-dominant and helicity-dominant equilibria. For fairly high toroidal mode numbers $n \gg M$, the perturbations can feel the fine local structure of the magnetic curvature due to helicity and also the local magnetic shear is reduced more or less in both types of equilibria, and consequently poloidally and toroidally localized ballooning modes inherent to three-dimensional systems are destabilized for both toroidicity-dominant and helicity-dominant Mercier-unstable equilibria. The situation for moderate toroidal mode numbers $n \sim M$ is different between toroidicity-dominant and helicity-dominant equilibria. The local magnetic shear is more reduced in toroidicity-dominant Mercier-unstable equilibria than in helicity-dominant Mercier-unstable equilibria, and also the modes with moderate toroidal mode numbers $n \sim M$ can not feel the local structure of the normal magnetic curvature due to helicity effectively. Thus, tokamak-like poloidally localized ballooning modes with a weak toroidal mode coupling can be easily destabilized for toroidicity-dominant Mercier-unstable equilibria, and interchange modes, driven by the average unfavorable magnetic curvature and not experiencing the affect of toroidal mode coupling, can be destabilized for helicity-dominant Mercier-unstable equilibria. Since the normal magnetic curvature becomes more unfavorable on the inner side of the torus than on the outer side of the torus by the Shafranov shift, the interchange modes are localized on the inner side of the torus for both types of equilibria. This type of interchange mode is anti-ballooning with respect to the poloidal mode coupling.

In both types of Mercier-unstable equilibria, the pressure-driven modes are destabilized around Mercier-unstable flux surfaces, where the average normal magnetic curvature is unfavorable (magnetic hill) and the global magnetic shear is stellarator-like. Modes become more unstable and more localized both on flux tubes and in the radial direction, and have stronger toroidal mode coupling, as the typical toroidal mode numbers increase.

Global mode analysis in the Mercier-stable equilibria

In Mercier-stable MHD equilibria, which are toroidicity-dominant equilibria, only ballooning modes inherent to the three-dimensional MHD equilibria occur for $n \sim M$ or $n \gg M$ as the most unstable modes. The properties of ballooning modes with $n \gg M$ are quite similar to ones with $n \gg M$ in Mercier-unstable equilibria. The ballooning modes with $n \sim M$ consist of two diminant group of Fourier modes with different toroidal mode numbers, which leads to the weak toroidal localization. Those modes are localized in the region with the average unfavorable magnetic curvature and the stellarator-like global magnetic shear.

Moreover, new type of modes with smaller growth rates are destabilized in the region with the average favorable magnetic curvature (magnetic well) and the tokamak-like global magnetic shear. These modes are driven by both the kink term and a part of the pressure-driven term, namely, the geodesic curvature term. The pressure-driven term consists of both the normal curvature and geodesic curvature terms:

$$\frac{1}{2}\int d\tau \left\{-2(\vec{\xi}_\perp \cdot \nabla P)(\vec{\xi}_\perp \cdot \vec{\kappa})\right\} = \frac{1}{2}\int d\tau \left\{-2\frac{dP}{d\psi}\kappa_n (\xi^\psi)^2 + J_\parallel \hat{b}\cdot\nabla(\xi^\psi \eta)\right\}$$

where $\kappa_n = \vec{\kappa}\cdot\partial_\psi \vec{r}$, $\xi^\psi = \vec{\xi}\cdot\nabla\psi$, and $\eta = (\vec{\xi}\times\vec{B})\cdot\partial_\psi\vec{r}$. In the region where new modes are destabilized, the normal magnetic curvature acts as a stabilizing term, and both the kink and geodesic curvature terms:

$$\frac{1}{2}\int d\tau \left\{-J_\parallel(\vec{\xi}_\perp \times \hat{b})\cdot\vec{Q}_\perp + J_\parallel \hat{b}\cdot\nabla(\xi^\psi\eta)\right\}$$

drive these new mode, so that the Mercier criterion is not applicable to these new modes. **Discussions**

The characteristics of the ideal pressure-driven modes in currentless $L = 2/M = 10$ heliotron/torsatron MHD equilibria are clarified, through the global mode analysis. The properties of the pressure-driven modes are consistent with the conjecture from local mode analysis given in Ref. 2. New modes driven by both the kink and geodesic curvature terms are found, to which the Mercier criterion is not applicable. The stabilizing effects due to a diamagnetic rotation and a finite Larmor radius will be reported elsewhere.

References

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