Towards a 3-D Plasma Fluid Modelling for the W7-X Divertor

M. Borchardt[1], A. Mutzke[1], J. Nührenberg[1], J. Riemann[1],
R. Schneider[1], S. Weber[2]

[1] *IPP-Teilinstitut Greifswald, EURATOM Association, D–17489 Greifswald, Germany*

[2] *LULI, Ecole Polytechnique, F–91128 Palaiseau Cedex, France*

Introduction

The divertor concept is considered a well developed scheme for power and particle exhaust from the outer regions of a fusion plasma referred to as scrape-off layer (SOL). Divertor configurations have been successfully tested and modelled with tokamaks (JET, ASDEX-Upgrade) and are designed for experimental use with stellarators (W7-AS, W7-X, LHD) as well.
For W7-X a divertor is planned using the existing island topology of the magnetic field, where configurations with and without strong stochastic effects are possible. Comparing stellarator results to tokamaks the latter case is of particular interest.
In contrast to the highly sophisticated tools applied to the studies of tokamaks there is an urgent need for an adequate numerical description of stellarators with their inherent three-dimensionality.
Based on the same plasma fluid approach as used for codes like B2-Eirene a transport code is under development taking account of the full three-dimensional geometry by solving the strongly anisotropic SOL transport equations in magnetic coordinates. This ansatz allows to use standard discretization methods with higher order schemes still enabling full geometric flexibility. Experience and techniques of 2-D SOL modelling can be directly adapted including the extension to multi-fluid formulations.
The general design of the code enables benchmarking with characteristic 2-D situations covered by other codes (B2, UEDGE, EDGE2D), studies of 3-D effects in tokamaks such as localized gas puffs or recycling and the description of SOL physics of stellarators without an initial restriction to W7-X applications.

General concept and model equations

Due to the choice of magnetic coordinates the entire plasma can be described by a conjunction of independent coordinate systems each of them representing a different region as long as stochastic regions are negligible [1, 2]. In a first step this scheme is used to describe the magnetic surfaces within an isolated island flux tube winding around the

core plasma with and without intersection by target plates [3]. Subsequently the coupling between neighbouring islands and the core will be introduced via appropriate boundary conditions for each of the separate magnetic coordinate systems thus leading to a description of the entire SOL. The general design of the code contains no assumption about the mesh with respect to local grid refinement and allows arbitrarily shaped computational cells due to indirect addressing.
In addition to the actual SOL modelling adequate tools for grid generation and post-processing are under development.
The general equations describing the SOL plasma fluid follow from the full set of multi-fluid Braginskii equations [4].
Due to the extreme anisotropy between parallel and perpendicular heat flux the anisotropic Laplace equation for the electron temperature T_e is considered an appropriate test case and is solved for several situations of increasing complexity.
The equation to be solved reads

$$\nabla \cdot \vec{q}_e = -\nabla \cdot \left(\kappa_\parallel \nabla_\parallel T_e + \kappa_\perp \nabla_\perp T_e \right) = 0 \tag{1}$$

with the heat flux $\vec{q}_e$ being given in magnetic coordinates s, θ, ϕ [5]

$$\begin{aligned} \vec{q}_e &= \vec{q}_{e,\parallel} + \vec{q}_{e,\perp_1} + \vec{q}_{e,\perp_2} \\ &= -\kappa_\parallel \frac{F_T'}{F_T' I + F_P' J} \vec{B} \left(\iota \partial_\vartheta + \partial_\phi \right) T_e \\ &\quad -\kappa_{\perp_1} \nabla s \left(\partial_s + \frac{g^{\vartheta s}}{g^{ss}} \partial_\vartheta + \frac{g^{\phi s}}{g^{ss}} \partial_\phi \right) T_e \\ &\quad +\kappa_{\perp_2} \frac{\nabla s \times \vec{B}}{(F_T' I + F_P' J) g^{ss}} \left(I \partial_\vartheta - J \partial_\phi \right) T_e \end{aligned} \tag{2}$$

with the left-handed system $\vec{B}, \nabla s, \nabla s \times \vec{B}$ $(\parallel, \perp_1, \perp_2)$, the poloidal and toroidal currents I and J, the radial derivatives of the flux functions F_P' and F_T' and the metric coefficients g^{ij}. The latter ones are numerically available with VMEC/JMC [5, 6]. The anisotropy $\kappa_\parallel / \kappa_\perp \gg 1$ can be seen according to the conductivities

$$\kappa_\parallel = \chi_\parallel^{class.} T_e^{5/2} \,, \quad \kappa_\perp = \left(\chi_\perp^{anom.} + \chi_\perp^{class.} \right) n \,, \qquad \frac{\kappa_\parallel}{\kappa_\perp} \approx 10^6 \quad . \tag{3}$$

An extension of the code successively implementing the equations for continuity, momentum transport and complex heat flow is underway and the corresponding work will be presented in future contributions.

Numerical aspects

Taking advantage of Gauss' theorem the above equation (1) is integrated and can then be discretized applying the Finite Volume method

$$\int_V \nabla \cdot \vec{q}_e \, \mathrm{dV} = \int_{\partial \mathrm{V}} \vec{\mathrm{q}}_\mathrm{e} \cdot \mathrm{d}\vec{\sigma} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \int_{\partial \mathrm{V_i}} \vec{\mathrm{q}}_\mathrm{e} \cdot \mathrm{d}\vec{\sigma}_\mathrm{i} = 0 \quad . \tag{4}$$

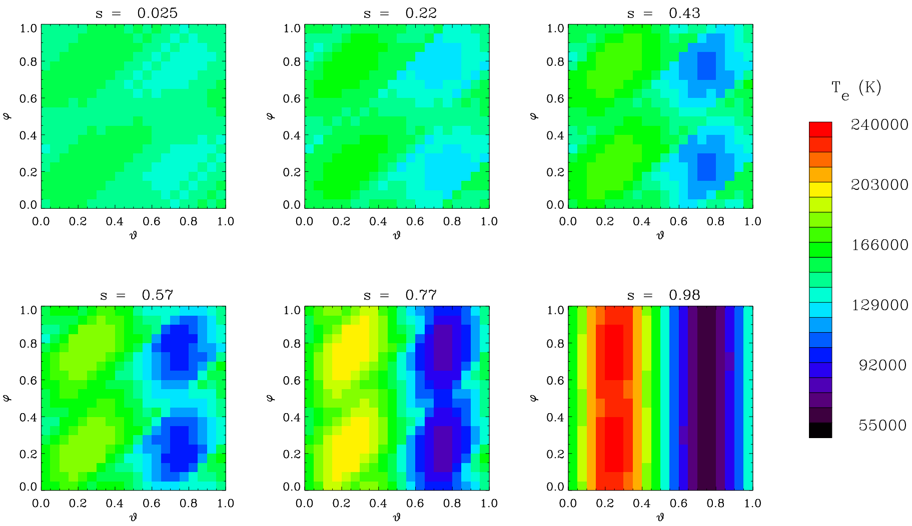

Figure 1: Temperature profile for $s = const.$

The resulting system of N non-linear equations is iteratively solved via a Newton method, i.e. by approximating the total heat flux balance for each computational cell V_i successively, using a direct sparse matrix solver [7]. Properties which need to be derived from grid points are available by a numerical interpolation scheme which is capable of accounting for different computational molecules depending on a maximum order of neighbourhoods to be considered.

Results

First calculations performed with the new 3-D SOL code were to determine the temperature profile for a single island flux tube winding around the core of a W7-X plasma. For the boundary conditions we used poloidal (θ) and toroidal (ϕ) periodicity, a no-power flow condition at $s = \varepsilon > 0$ and a prescribed temperature profile ($T_{s=1} = [12.5 + 7.5\sin(2\pi\theta)]\,\mathrm{eV}$) at the outermost flux surface $s = 1$. The latter condition was strongly influenced by the results already presented in [3]. As in [3] an analytical metric model was used for simplicity. The following figures are results for a regular grid with 20 grid points in each direction.

Figure 1 scans through different radii and shows how the initial pattern prescribed at $s = 1$ becomes more and more toroidally structured due to the metric and the dominant parallel transport along the magnetic field lines of rotational transform $\iota = F_P'/F_T' = \Delta\theta/\Delta\phi \approx 1$. In order to study the influence of the 3-D magnetic field the contravariant metric coefficient $g_{\theta s}$ within the radial heat flow across a surface was artificially set equal to zero. Due to this the spatial pattern of radial heat flow is actually governed by the remaining coefficient g^{ss} alone. The evolution of g^{ss} with the radial coordinate s is shown in Figure 2 and clearly reflected by the temperature profiles given in Figure 3. This result appears as a convolution of the initial temperature profile prescribed at $s = 1$ with the function g^{ss}.

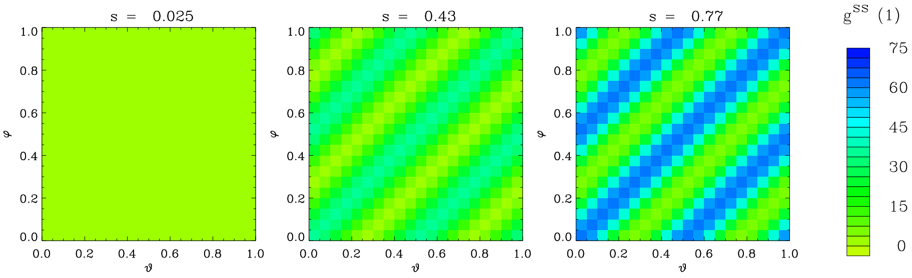

Figure 2: Metric coefficient g^{ss} for $s = const$.

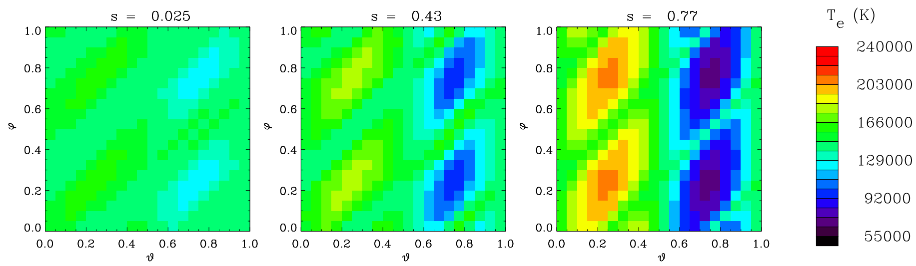

Figure 3: Temperature profile for $s = const$.

Acknowledgement

Our thanks are due to C. Nührenberg for providing us with numerical data concerning the formation of equilibrium flux surfaces and the derivation of a metric model for a W7-X vacuum configuration.

References

[1] Boozer A H 1982 *Phys. Fluids* **25** 520–521.

[2] D'haeseleer W D et al. 1991 *Flux Coordinates and Magnetic Field Structure*, (Springer)

[3] Weber S 1998 Czechoslovak Journal of Physics **48** 345–50.

[4] Braginskii S I 1965 Rev. Plas. Phys. **1** 205–311.

[5] Nührenberg J and Zille R 1987 in: *Theory of Fusion Plasmas (EUR 12149 EN)* (Editrice Compositori, Bologna) 3–23.

[6] Hirshman S P et al. 1986 Comput. Phys. Commun. **39** 161
Hirshman S P et al. 1986 Comput. Phys. Commun. **43** 143

[7] Harwell Subroutine Library (HSL), sparse matrix solver *MA28*, www.netlib.org/harwell/