

Kinetic Modeling of ECRH in Stellarators Using the Stochastic Mapping Technique*

Sergei V. Kasilov¹, **W. Kernbichler**², Viktor V. Nemov¹, Martin F. Heyn²

¹*Institute of Plasma Physics, National Science Center “Kharkov Institute of Physics and Technology”, Ul. Akademicheskaya 1, 310108 Kharkov, Ukraine*

²*Institut für Theoretische Physik, Technische Universität Graz, Petersgasse 16, A-8010 Graz, Austria*

Introduction

The Stochastic Mapping Technique (SMT) developed recently [1], is used for Monte Carlo (MC) modeling the electron distribution function of a stellarator plasma with 2nd harmonic X-mode ECRH. This technique allows one to resolve the suprathermal electron distribution function in velocity and coordinate space with sufficient statistics for realistic stellarator magnetic fields. The MC code has been tested earlier for the case of a point source in phase space (Green’s function approach [2]) where it is sufficient to model electron drift motions and Coulomb collisions. In the present report this method is extended by incorporating a non-local quasi-linear (QL) diffusion MC operator into SMT in order to enable the study of the QL interaction of electrons with an ECRH beam.

Nonlinear δf method

In order to separate the problem of finding the distribution function of suprathermal electrons generated by localized ECRH from the problem of computing the particle and energy balance, the electron distribution function f is formally split into two parts, $f = f_0 + \delta f$, where f_0 is the unperturbed distribution function of bulk electrons and δf is the non-Maxwellian part of special interest. The kinetic equation for the electron distribution f is

$$\frac{\partial f}{\partial t} + V^\alpha \frac{\partial f}{\partial z^\alpha} = \hat{L}_C f + \hat{L}_{QL} f, \quad (1)$$

where z^i and V^i are guiding center variables and phase space velocity respectively, \hat{L}_C is the Coulomb collision operator describing collisions with background ions and with electrons and \hat{L}_{QL} is the quasilinear operator. Using the separation of f into f_0 and δf in (1), the equation for δf is given as

$$\frac{\partial \delta f}{\partial t} + V^\alpha \frac{\partial \delta f}{\partial z^\alpha} - \hat{L}_C \delta f - \hat{L}_{QL} \delta f + \nu_{\text{eff}} \delta f = \hat{L}_{QL} f_0, \quad (2)$$

where ν_{eff} is some effective exchange frequency which is taken to be small in the suprathermal range of energies. If the total amount of suprathermal electrons is small, the kinetic equation for f_0 together with the assumption of a local Maxwellian distribution function leads to a set of one-dimensional neoclassical transport equations, which contain additional magnetic surface averaged, $\langle \dots \rangle_\Psi$, source terms of particles $S_{n,QL}$ and energy $S_{w,QL}$,

$$\begin{aligned} S_{n,QL} &= \left\langle \int d^3p (\hat{L}_C \delta f + \nu_{\text{eff}} \delta f) \right\rangle_\Psi, \\ S_{w,QL} &= \left\langle \int d^3p (\hat{L}_C \delta f + \nu_{\text{eff}} \delta f) \times (\mathcal{E} - m_0 c^2 - e\phi_0) \right\rangle_\Psi. \end{aligned} \quad (3)$$

*This work was partly supported by the Association EURATOM-OEAW under contract number ERB 5004 CT 96 0020.

Without the quasilinear operator on the left hand side of (2), this procedure reduces to the linear Green's function approach given in Ref. [2]. This approach is valid if $\delta f \ll f_0$ in the whole phase space and leads to a linear scaling of δf with the ECRH input power.

Modeling of the perturbed distribution function

Equation (2) for the perturbation of the distribution function is solved using the Monte-Carlo (MC) method based on SMT [1,3]. Instead of directly modeling the drift orbits of the test particles, $\mathbf{z} = \mathbf{z}(\mathbf{z}_0, t)$, using the equations of motion and local MC operators for the description of collisions, in SMT the test particle orbits are followed only on Poincarè cuts, i.e., surfaces where the magnetic field has a minimum along the magnetic field lines. The Lagrangian coordinates of the test particles are denoted with \mathbf{u}_m , where m labels the cut. Each new position \mathbf{u}'_m is obtained from the stochastic map, $\mathbf{u}'_m = \mathbf{U}(\mathbf{u}_m) + \delta\mathbf{u}_m$, where $\mathbf{U}(\mathbf{u}_m)$ is the regular map reconstructed from the precomputed drift orbits and $\delta\mathbf{u}_m$ is a random change. These random changes accumulate the effects of Coulomb collisions and rf diffusion during one bounce period, the time between subsequent crossings of the cuts. The covariances and deviations of the displacements $\delta\mathbf{u}_m$ must satisfy

$$\bar{D}_m^{ij} = \langle \delta u_m^i \delta u_m^j \rangle / 2 = \bar{D}_C^{ij} + \bar{D}_{QL}^{ij}, \quad \mathcal{F}_m^i = \langle \delta u_m^i \rangle. \quad (4)$$

The SMT procedure is valid in the weak collisional regime and with sufficiently small displacements $\delta\mathbf{u}$ induced by rf wave-particle interaction.

We consider 2nd harmonic X-mode ECRH assuming that the rf field amplitude is highly localized in space and has a Gaussian shape along the magnetic field lines, $\mathbf{E} \propto \exp(-\frac{1}{2}\alpha_1 s^2)$, where s is the distance along the field line and α_1 is the inverse area of the beam. In this case, the variance and deviation of the particle energy after one pass through the beam is given as

$$\bar{D}_{QL}^{\mathcal{E}\mathcal{E}} = \frac{\pi e^2 |E^-|^2 k_\perp^2 B_{\text{res}}^2 p_\perp^4}{2 m_0^2 \omega^2 \alpha_1 B^2 p_\parallel^2} \exp(-\eta), \quad \eta = \frac{m_0^2 \omega^2}{\alpha_1 p_\parallel^2} \left(\gamma - \frac{B}{B_{\text{res}}} - \frac{k_\parallel p_\parallel}{m_0 \omega} \right)^2, \quad (5)$$

$$\mathcal{F}_{QL}^{\mathcal{E}} = \left(\frac{\partial}{\partial \mathcal{E}} + \frac{1}{B_{\text{res}}} \frac{\partial}{\partial \mu} \right) \bar{D}_{QL}^{\mathcal{E}\mathcal{E}} \quad (6)$$

with \mathcal{E} , μ the energy and magnetic moment, p_\perp , p_\parallel the perpendicular and parallel momentum in the beam center, E^- the right-polarized component of the wave electric field, B the magnetic field in the beam center, $B_{\text{res}} = m_0 c / 2e\omega$, m_0 the electron mass, ω the wave frequency, k_\perp , k_\parallel the perpendicular and parallel wave number, and γ the relativistic factor. With (5) and (6) known, the components of \bar{D}_m^{ij} and \mathcal{F}_m^i can be obtained from

$$\bar{D}_m^{ij} = \beta^i \beta^j \bar{D}_{QL}^{\mathcal{E}\mathcal{E}}, \quad \mathcal{F}_m^i = \beta^i \mathcal{F}_{QL}^{\mathcal{E}}, \quad \beta^\mathcal{E} = 1, \quad \beta^\mu = \frac{1}{B_{\text{res}}}. \quad (7)$$

For slow particles with $\lambda = p_\parallel / p \ll 1$ the quasilinear theory is violated. The nonlinear effects during the wave-particle interaction will reduce the energy absorption by those particles. In order to describe this reduction qualitatively, $\bar{D}_{QL}^{\mathcal{E}\mathcal{E}}$, is replaced by the model coefficient

$$[\bar{D}_{QL}^{\mathcal{E}\mathcal{E}}]^* = \frac{\bar{D}_{QL}^{\mathcal{E}\mathcal{E}}}{\sqrt{1 + (\alpha \bar{D}_{QL}^{\mathcal{E}\mathcal{E}})^2}}, \quad \alpha = \frac{\partial^2 \eta}{\partial \mathcal{E}^2}. \quad (8)$$

Such a model assumes 'heating out of resonance' [4] when the energy gain by the particle passing through the beam is assumed to stop outside the resonance zone ($\eta \gg 1$).

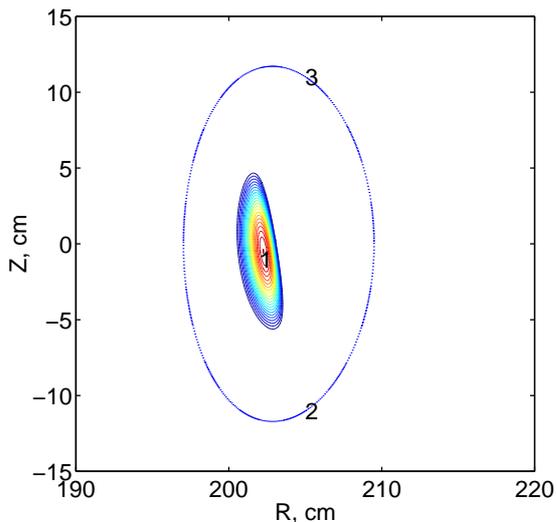


Figure 1. Absorbed rf power distribution.

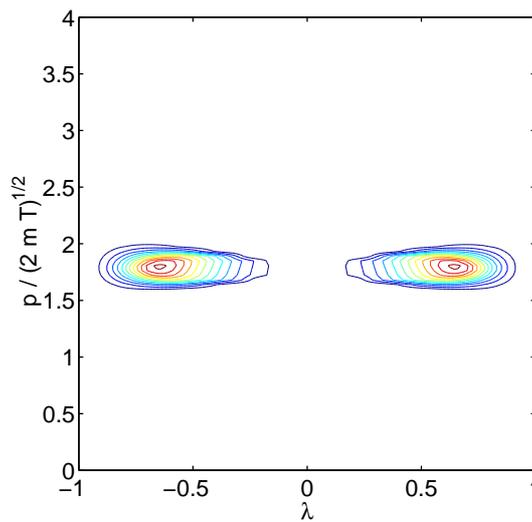


Figure 2. Contours of rf diffusion coefficient.

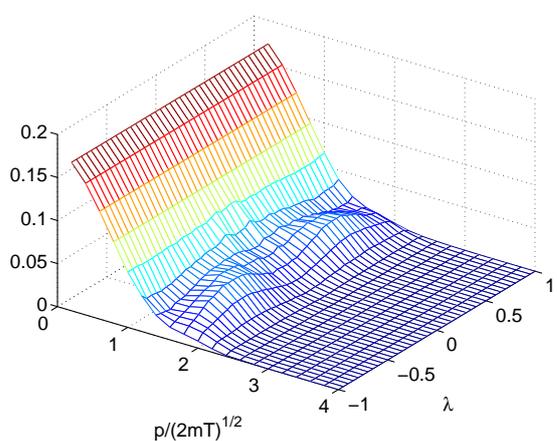


Figure 3. Total distribution function in the beam area, point 1.

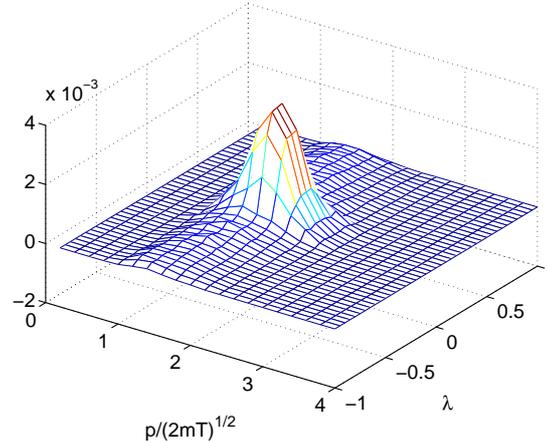


Figure 4. Perturbed distribution function below the beam, point 2.

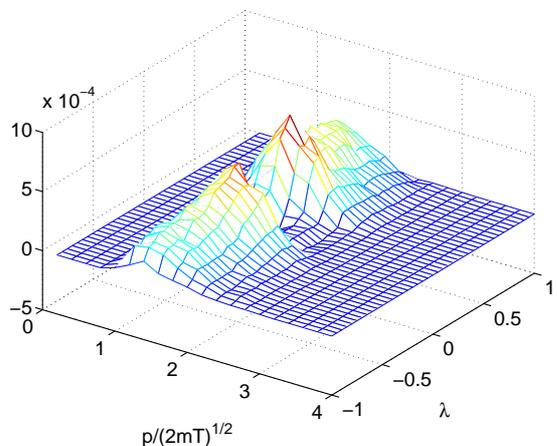


Figure 5. Perturbed distribution function above the beam, point 3.

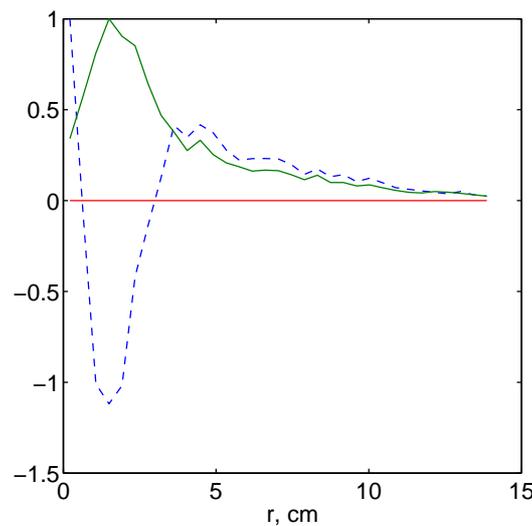


Figure 6. Profiles of particle and energy sources.

Results

The computation is performed for a medium size stellarator with $l=2$, five magnetic field periods, $R = 2$ m, $B = 2.5$ T, and a constant plasma density $n_e = 10^{19} \text{ m}^{-3}$ and temperature $T_e = 3$ keV. A localized ECRH beam with $P = 100$ kW and $\alpha_1 = 2.5 \cdot 10^3 \text{ m}^{-2}$ is assumed to be perpendicularly injected into the magnetic field minimum. The variation of the beam amplitude due to absorption is not taken into account. The distribution of the absorbed power over the poloidal cross section is shown in Fig. 1. The cold resonance position is exactly on the magnetic axis. One magnetic surface is shown and three points (1, 2, 3) are indicated, where the distribution function is reconstructed. The direction of the electron drift is downwards. In Fig. 2 the effective energy diffusion coefficient (8) is shown. Compared to the conventional QL diffusion operator, the absorption region in phase space is shifted towards the region of passing particles because this coefficient is reduced for slow trapped particles due to nonlinear effects. In Fig. 3 the total distribution function is shown for point 1 (inside the absorption region). The formation of a quasilinear plateau in can be observed mainly in the passing particle region. The perturbed distribution function in Fig. 4 corresponds to the point below the beam (in the direction of the drift). It is formed mainly by ripple trapped particles, which contribute to convective particle and energy losses. In turn, the perturbed distribution function above the beam (Fig. 5) is formed mainly by passing particles detrapped in the lower region of the configuration. Note that despite the peaked perturbed distribution function δf , the total distribution function for this parameter set has negative derivatives over perpendicular energy everywhere.

In Fig. 6 the radial profiles of particle (dashed) and energy (solid) source densities (3) are shown. The rf beam region corresponds to $r < 5$ cm. About 30 % of the energy of suprathermal particles is deposited outside the heating region. Also, the convection of electrons away from the heating region is clearly observed. In the present computation there is no radial electric field included. In the presence of such a field, the losses of energy outside the heating region will be reduced [2].

Summary

The stochastic mapping technique together with the nonlinear δf method has been applied to model the electron distribution function during ECRH in a stellarator. With this method, the distribution function can be resolved very well both in coordinate and velocity space. The ECRH modeling shows significant convective energy losses from the heating region despite the fact that most of the energy first goes to the passing particles. The latter follows from the nonlinear reduction of the energy absorption by slow trapped particles which is taken into account only qualitatively. A proper account of nonlinear wave-particle interaction [5] is required.

References

- [1] S.V. Kasilov, W. Kernbichler, V.V. Nemov, and M.F. Heyn, ECA, **22C**, 1726–1729, 1998.
- [2] M. Romé et al., Plasma Phys. Contr. Fusion, **39**, 117–158, 1997.
- [3] S.V. Kasilov, V.E. Moiseenko, and M.F. Heyn, Phys. Plasmas, **4**(7), 2422–2435, 1997.
- [4] M.R. O'Brien, M. Cox, and D. F. H. Start, Nucl. Fusion, **26**(12), 1625–1640, 1986.
- [5] M.F. Heyn, et al., same conference, paper P4.067, 1999.