26^{th} EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 1681 - 1684

The Mercier Criterion in Reversed Shear Tokamak Plasmas

M. S. Chance, C. Kessel and S. C. Jardin

Princeton University Plasma Physics Laboratory
P.O. Box 451, Princeton, NJ 08543, USA

1 Introduction

A recent study[1] has found that the Mercier criterion[2, 3] was violated in the inner region of a reversed shear plasma when the safety factor on axis, $q_{\rm axis}$, exceeded a threshold. This has contradicted the notion that negative shear is robustly Mercier and balloon stable, both from semi-analytical results[4] and favorable numerical[5] and experimental[6, 7] evidence from several tokamak devices.

For profiles similar to those in Ref. [1], in a fixed boundary circular cross-section plasma, we have found no Mercier instability for a representative range of $q(\psi)$ with negative shear. We recast the Mercier criterion in terms of V^{tt}, i.e., the second derivative of the plasma volume within a flux surface with respect to the *toroidal*, rather than with respect to the *poloidal* flux, V'', showing that the dominant shear term is absorbed in V^{tt}. We also show that the significantly different local magnetic shear properties is more beneficial in reversed configurations.

2 Circular configurations

We investigated a variety of reversed shear configurations similar to the circular case of Ref. [1], using representative values of $q_{\rm axis}$ of 4.5 and 8.0, holding $q_{\rm min}$, $r_{\rm min}$ and $q_{\rm edge}$ approximately fixed. With the magnetic axis at $R = 9$, and a minor radius, a, of 3 the aspect ratio is fixed at 3.0. For completeness we also studied a non-reversed shear case with $q_{\rm axis} = 1.1$. The safety factor profiles for a case with $\beta_N = 2.0$, and $q_{\rm axis}$ equal to 1.1, 4.5 and 8.0 are shown as a function of the poloidal flux surface number in Fig. 1 together with the pressure profile which is used for all the cases studied.

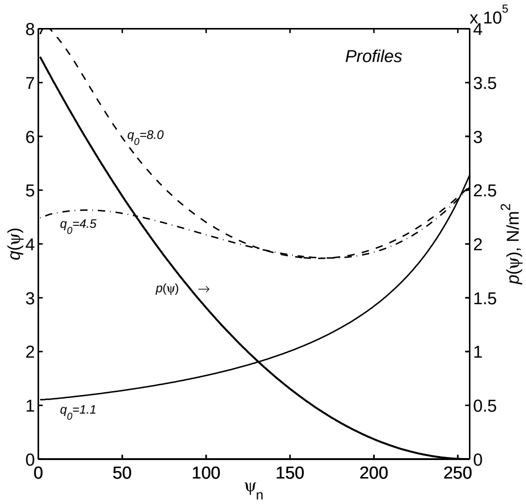

Figure 1: *Safety factor profiles (lighter lines) for the reversed monotonic shear cases.*

3 The Mercier criterion

Mercier criterion with the terms grouped similarly to those given in Ref. [1], but using our notation and normalization can be written as,

$$
\begin{aligned}
q'^2 D_I \quad = \quad & -\frac{q'^2}{4} + g^2 p'^2 \left[\left\langle \frac{\mathcal{J} B^2}{|\nabla\psi|^2} \right\rangle \left\langle \frac{\mathcal{J}}{B^2 |\nabla\psi|^2} \right\rangle - \left\langle \frac{\mathcal{J}}{|\nabla\psi|^2} \right\rangle^2 \right] \\
& + g q' p' \left\langle \frac{\mathcal{J}}{|\nabla\psi|^2} \right\rangle - \left\langle \frac{\mathcal{J} B^2}{|\nabla\psi|^2} \right\rangle \left[p' \frac{V''}{4\pi^2} - p'^2 \left\langle \frac{\mathcal{J}}{B^2} \right\rangle \right], \qquad (1)
\end{aligned}
$$

where $q'^2 D_I \le 0$ for stability. Here, $q'(\psi)$ denotes the global magnetic shear, and we have substituted the parallel current, $\sigma \equiv \mathbf{J}\cdot\mathbf{B}/B^2 = -gp'/B^2 - g'$, into the original form of [1]. $\mathbf{B} = \nabla\phi\times\nabla\psi + g(\psi)\nabla\phi$, so that the poloidal flux, Ψ, is $2\pi\psi$ within the volume, V, and $g(\psi) = 2\pi R B_\phi$. The coordinate system used is such that $\mathbf{r} = (R, \phi, Z) = (\psi, \theta, \phi)$, with the Jacobian, $\mathcal{J} = (\nabla\psi\times\nabla\theta\cdot\nabla\phi)^{-1}$. The averages, $\langle\cdots\rangle \equiv 1/2\pi \oint \cdots d\theta$, are are evaluated numerically using the trapezoidal rule because of the extremely high accuracy offered by the Euler-Maclaurin effect for periodic functions.

For the case where $q_{\rm axis} = 8.0$, the groups of terms comprising Eq. (1) are plotted in Fig. 2 using a *modified* logarithmic scale useful for plotting data which smoothly span large and small values of either sign. It is defined by $y = \log_{10}\left(\gamma f/2 + \sqrt{(\gamma f/2)^2 + 1}\right)$. For $|y| \gtrsim 1$, $y \sim {\rm sign}(\gamma f)\log_{10}|\gamma f|$, for both positive and negative values. For $\gamma f \ll 1$, $f \sim (2y/\gamma)\log_e 10$. In this study the stretching factor γ is fixed at 10.

Considering the right hand side of Eq. (1), the first term, labeled by 's^2' in the figure, is stabilizing, and, because of the *Schwartz inequality*, the second group of terms, labeled 'scz' is destabilizing. The third term is the so called "shear" term and as pointed out in [1] it is destabilizing for q' negative. It was considered by [1] to be the dominant cause for the instability of reversed shear configurations. In the fourth group of terms, labeled 'V''', the less dominant term with p'^2 is destabilizing. The expression involving $V''(= \partial^2 V/\partial\psi^2)$, the "well" term, is by far the more dominant one and is found to be *stabilizing* in regions where the global shear is negative, and vice-versa. This counterintuitive change in the sign of V'' within the cross-section of the plasma suggests that it is not a good indication of the stabilizing tokamak magnetic well. The curves in the figure representing V'' and the "shear" terms are emphasized with the darker lines. Note that D_I, labeled 'D_I' is negative, (i.e. stable) over the whole plasma.

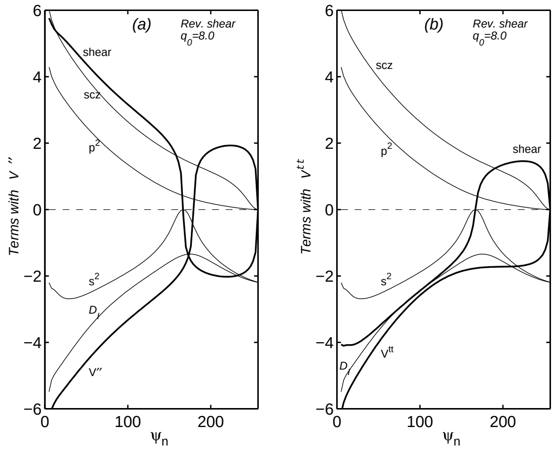

Figure 2: *The terms of D_I using V'' (a), and V^{tt} (b), for a reversed shear profile. $q_{\rm axis} = 8.0$. The heavier curves of V'' and the shear term labeled "shear" in (a) combine to give the curves of V^{tt} and the shear term labeled "shear" in (b).*

4 The Mercier criterion with V^{tt}

An examination of Fig. 2a shows that the value of the shear term is approximately equal and opposite to that of the "well" term. This suggests that a more appropriate use of V (albeit roughly) for describing the tokamak magnetic well is $V^{tt}(\equiv \partial^2 V/\partial\Phi^2)$, where Φ is the *toroidal* flux [on average, magnetic field lines circulate q times around the long way of the torus before making it once around the short way]. We can thus write,

$$V'' \quad = \quad 4\pi^2 q^2 V^{tt} + \frac{q'}{q}V'; \tag{2}$$

the term involving q', a "shear" term, is largely responsible for the change of sign in V''. It will oppose the dominant (toroidal) contribution of the shear term of Eq. (1). Substituting Eq. (2) and combining the shear terms, the Mercier criterion then takes the form:

$$\begin{aligned} q'^2 D_I \quad = \quad & -\frac{q'^2}{4} + g^2 p'^2 \left[\left\langle \frac{\mathcal{J}B^2}{|\nabla\psi|^2} \right\rangle \left\langle \frac{\mathcal{J}}{B^2|\nabla\psi|^2} \right\rangle - \left\langle \frac{\mathcal{J}}{|\nabla\psi|^2} \right\rangle^2 \right] \\ & - \frac{q'p'}{q} g^2 \left[\left\langle \mathcal{J} \right\rangle \left\langle \frac{\mathcal{J}}{R^2|\nabla\psi|^2} \right\rangle - \left\langle \frac{\mathcal{J}}{R^2} \right\rangle \left\langle \frac{\mathcal{J}}{|\nabla\psi|^2} \right\rangle \right] \\ & - \frac{q'p'}{q} \left\langle \mathcal{J} \right\rangle \left\langle \frac{\mathcal{J}}{R^2} \right\rangle - \left\langle \frac{\mathcal{J}B^2}{|\nabla\psi|^2} \right\rangle \left[p' q^2 V^{tt} - p'^2 \left\langle \frac{\mathcal{J}}{B^2} \right\rangle \right], \end{aligned} \tag{3}$$

where $q'^2 D_I \le 0$ for stability.

The groups of these terms are plotted in Fig 2b for the same configuration as was used for Fig 2a. The residuals from the shear terms, i.e., the third and fourth groups on the right, plotted together and labeled 'shear' in the figure, are now reduced to the poloidal field scale; in the cylindrical limit the former now also vanish. In the reversed shear cases studied, these residuals are both stabilizing in the negative shear region. This is obviously true of the fourth group. Heuristically assuming that $R^{-2} \sim 1 - 2\varepsilon \cos\theta$, $|\nabla\psi|^{-2} \sim 1 - \Delta\cos\theta$, and $\mathcal{J} \sim 1 + \tau\cos\theta$, with ε, Δ and $\tau \ll 1$, then the third group of terms becomes $\sim -(p'q'/q)\varepsilon\Delta(1 - \tau^2/2)$. If there is outward shifting of the magnetic surfaces ($\Delta > 0$), this group is stabilizing for negative shear (with p' negative), and is independent of $\mathcal{J}$ to leading order.

The "well" term containing V^{tt}, labeled 'V^{tt}', is now negative throughout the plasma cross-section for all the positive and reversed shear cases studied. This is more appropriate to describe the intrinsic property of tokamak wells and tends to $\sim (1 - q_0^2)$ at the magnetic axis.[2]

5 Monotonic $q(\psi)$ profile

The terms of D_I given by Eq. (1) for the circular configuration with a monotonic $q(\psi)$ profile in is shown in Fig. 3a. Although D_I is negative throughout the cross section we see again that the destabilizing "well" term is now opposed by the shear term which is now stabilizing. When the terms are grouped with V^{tt} according to Eq. (3) we see from Fig. 3b that the shear term is destabilizing but V^{tt} is negative throughout the cross section.

Note that there is approximately an order of magnitude cancellation between the large "well" and shear terms in the two forms of D_I, in configurations with either reversed or monotonic shear profiles.

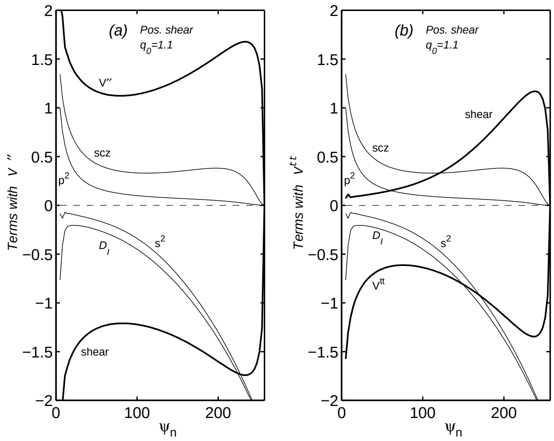

Figure 3: *The terms of D_I using V'' (a), and V^{tt} (b), for a monotonic shear profile.* $q_{\rm axis} = 1.1$.

6 Local magnetic shear

The local magnetic shear, [3, 4] $S(\psi,\theta)$, defined in normalized form, as

$$\hat{S}(\psi,\theta) = \frac{V}{2\pi^2 q\mathcal{J}}\left[\left(\frac{\mathcal{J}g}{R^2}\right)' + \frac{\partial}{\partial\theta}\left(\frac{\nabla\psi\cdot\nabla\theta}{|\nabla\psi|^2}\frac{\mathcal{J}g}{R^2}\right)\right], \tag{4}$$

is comprised under averaging, of the averaged global shear, $2Vq'/V'q$, and a residual oscillating part. The intrinsic outward shift of the magnetic axis in tokamak configurations redistributes the poloidal flux so as to reduce the local magnetic shear at the outer major radius side. This opposition to imposed positive global shear in conventional non-reversed configurations results in a low shear neighborhood about a null in the local shear near the magnetic axis. On the left of Fig. 4 for the non-reversed shear case, Seven equally distributed contour levels each for positive and negative $\hat{S}$ are shown. Examination of the density of the lines shows that the local shear is negative (solid lines) out to about $R = 11$ and is very small and positive in the region of unfavorable curvature. This lack of shear tends to breed interchanges. In reversed shear plasmas, on the other hand, the imposed negative global shear enhances the negative local shear tendency and render the plasma more stable in the reversed region by strengthening the (negative) shear and expanding the radial extent of the negative shear region. The null moves out towards the plasma edge where the pressure gradient is small. See Fig. 4(right). Ballooning modes are found to be least stable near the edge of reversed shear plasmas where the local shear is small. A fully reversed shear profile would cause the null to migrate beyond the plasma edge. In both cases it is an unfortunate trade off that most of the shear resides in the region of favorable curvature. Although these arguments involving the poloidal dependencies are more relevant to ballooning modes, they are pertinent to interchanges as well since the physical mechanisms of the instability are similar and ballooning stability is a sufficient

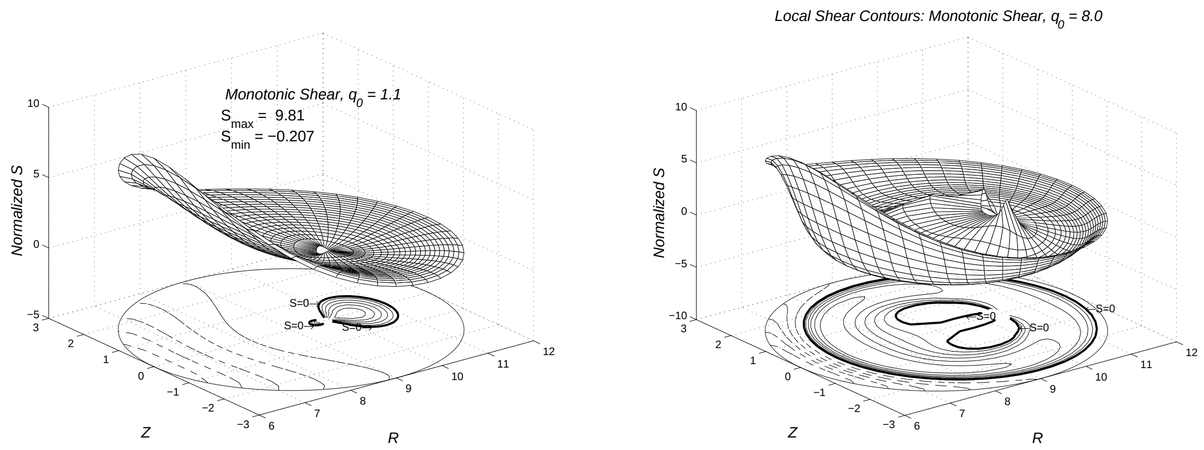

Figure 4: *Contours of the normalized local shear on the R-Z plane together with a 3D display for a monotonic q profile for which $q_{\rm axis} = 1.1$ (left) and $q_{\rm axis} = 8.0$ (right). Seven contour levels each for positive and negative $\hat{S}$ are shown. Dotted lines signify $\hat{S} \geq 0$, solid lines $\hat{S} \leq 0$, and the heavy solid lines, $\hat{S} = 0$, which are labeled. Most of the shear resides in the region of favorable curvature. The outermost circle is the plasma boundary.*

condition for achieving interchange stability.

7 Conclusions

In this work we have attempted to clarify the role of magnetic shear in the pressure driven stability properties of reversed shear configurations. Our results which shows that these configurations are robustly stable provided that the safety factor remains above unity and the triangularity is positive, contradict the recent results of Ozeki, et al. [1]. A regrouping of the terms in the Mercier criterion demonstrates that the shear terms are actually stabilizing and supports conclusions of both the numerical analyses and the semi-analytic model of Ref. [4]. The local magnetic shear analyses suggest that reversed shear configurations have more favorable stability properties. We have also examined plasmas with the ITER reversed shear plasma shape (with elongation $\kappa = 2.0$, and triangularity $\delta = 0.5$) and found no Mercier instability for $q_{\rm axis}$ ranging from 4.5 to 10.0, and with β_N ranging from 3.5 to 5.5. Ballooning instability occurred for $\beta_N > 4.5$ where the magnetic shear begins to rise outside the $q_{\rm min}$ location, consistent with the local shear observations.

Possible reasons for the contradiction could be distortion of the equilibrium surfaces, errors in the analysis and inaccuracies in the numerical algorithms.

8 Acknowledgments

Assistance from Dr. Stephan Brunner in the use of the *MATLAB* graphics package for the figures is gratefully acknowledged. This work was supported by U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

References

[1] T. Ozeki, M. Azumi, S. Ishida, and T. Fujita. *Plasma Physics and Controlled Fusion*, 40(6):871–877, June 1998.
[2] C. Mercier. Commission of the European Communities, Luxembourg, 1974.
[3] J. M. Greene and J. L. Johnson. *Plasma Physics*, 10(8):729–745, August 1968.
[4] J. M. Greene and M. S. Chance. *Nuclear Fusion*, 21(4):453–464, April 1981.
[5] Y. Nakamura, M. Wakatani, M. Furukawa, and K. Ichiguchi. In IAEA, editor, *Seventeenth International Conference on Fusion Energy*, volume (to be published), Yokohama, Japan, October 1998. International Atomic Energy Agency, Vienna, International Atomic Energy Agency.
[6] J. Manickam, E. D. Fredrickson, Z. Chang, M. Okabayashi, et al. In IAEA, editor, *Sixteenth International Conference on Fusion Energy*, volume 1, pages 453–462, Montreal, October 1996. International Atomic Energy Agency, International Atomic Energy Agency.
[7] E. J. Strait, T. A. Casper, M. S. Chu, J. R. Ferron, et al. DIII-D tokamak. *Physics of Plasmas*, 4(5):1783–1791, May 1997.