

The visualization of turbulent fluctuations in tokamaks

E. Mazzucato,¹ C. W. Domier,² N. C. Luhmann Jr.² and H. K. Park¹

¹Princeton Plasma Physics Laboratory, P. O. Box 451, Princeton, New Jersey 08543

²Department of Applied Science, University of California at Davis, Davis, California 95616

Introduction

Recent experiments in tokamaks with reversed magnetic shear¹⁻³ have revealed a synergetic dependence of plasma confinement on plasma equilibrium. This phenomenon, that could have important implications for fusion reactors, was explained with the suppression of turbulence by the ExB velocity shear. However, even though this explanation is in agreement with the fluctuation measurements of Ref. 4, it is not completely satisfactory since it is based neither on a self-consistent theory of plasma turbulence, nor on a comprehensive set of measurements. In fact, the relationship between transport and turbulence in tokamaks is still shrouded by serious uncertainties.

A major obstacle to the development of a theory of turbulence in tokamaks is the scarcity of experimental observations. The common remedy is therefore to rely on numerical simulations. Unfortunately, this is also not satisfactory because, given the enormous complexity of the problem, any numerical simulation of plasma turbulence must be driven by direct experimental observations. A case in point is that of hydrodynamics,⁵ where many advances in the theory of turbulence were stimulated by the visualization of the turbulent flow with optical techniques. In this paper, we describe the conceptual design of a similar technique for the visualization of turbulent fluctuations in tokamaks.

Microwave Imaging Reflectometry

The method described in this paper is based on microwave reflectometry⁶ – a radar technique used extensively in tokamaks for inferring the plasma density from the reflection of waves by a plasma cutoff. In these experiments, a probing wave is launched into the plasma from the equatorial plane using a small antenna. The measurement is essentially a point measurement and does not provide any direct information on the spatial structure of plasma fluctuations.

The method of correlation reflectometry has been so far the most sophisticated application of reflectometry to the study of turbulence in tokamaks. In these measurements, the radial structure of fluctuations is inferred from correlation measurements using several probing waves with closely spaced cutoff layers. Unfortunately, even this limited information is very difficult to extract from the experimental data when plasma fluctuations have a 2D spatial structure. This is because the reflected wave is usually allowed to propagate freely to the detector plane, which very often is located at a large distance from the reflecting layer. As a result, what is sampled is a complicated interference pattern with characteristics that may differ from those of the plasma fluctuations under investigation. This phenomenon is illustrated in Fig. 1, that shows some results from a series of numerical

simulations⁷ where a plane stratified plasma density ($n_e(r)$) is perturbed by a field of 2D fluctuations ($\delta n_e(r, x)$) with a Gaussian power spectrum $\propto \exp[-(k_x / \Delta k_x)^2 - (k_r / \Delta k_r)^2]$. These simulations indicate that if the amplitude of fluctuations is not too large, the field of reflected waves arises from the phase modulation (ϕ) of the probing wave with a magnitude given by 1D geometric optics (i.e., neglecting the bending of rays caused by fluctuations). To an outside observer, the reflected wave appears to arrive from a virtual location behind the cutoff, at a distance that corresponds to the average round-trip group delay. After reflection, the electromagnetic field separates into a wave propagating along the direction of specular reflection, and into a group of scattered waves propagating in different directions. The amplitude of the former decreases quickly to an insignificant level ($\approx \exp(-\sigma_\phi^2 / 2)$) as the variance (σ_ϕ^2) of ϕ becomes larger than one. The spectral width of scattered waves increases with the amplitude of fluctuations and becomes a factor of σ_ϕ larger than the spectral width of ϕ . Thus, if we assume that $\sigma_\phi \Delta k_x$ is much smaller than the vacuum wave vector (k_0) of the probing wave, the scattered waves will spread over a range of radial wave numbers $\delta k_r \approx (1 + \sigma_\phi^2) \Delta k_x^2 / 2k_0$. As a result, beyond a distance $D \approx 1 / \delta k_r$ from the virtual reflecting location, the backward field has a complicated interference pattern with large amplitude variations and random phases. This is indeed what Fig. 1 shows for the case of density fluctuations with an average relative amplitude of 0.5%, $\Delta k_r = 1 \text{ cm}^{-1}$ and $\Delta k_x = 0.5 \text{ cm}^{-1}$. On the contrary, near the virtual reflecting point, where the reflected wave can be calculated by propagating backwards in vacuum the spectrum of reflected waves, the amplitude is almost constant and the phase agrees with that of 1D geometric optics. In other words, reflectometry can be modeled by a thin random phase screen or by a conducting surface with a shallow random corrugation. 8,9

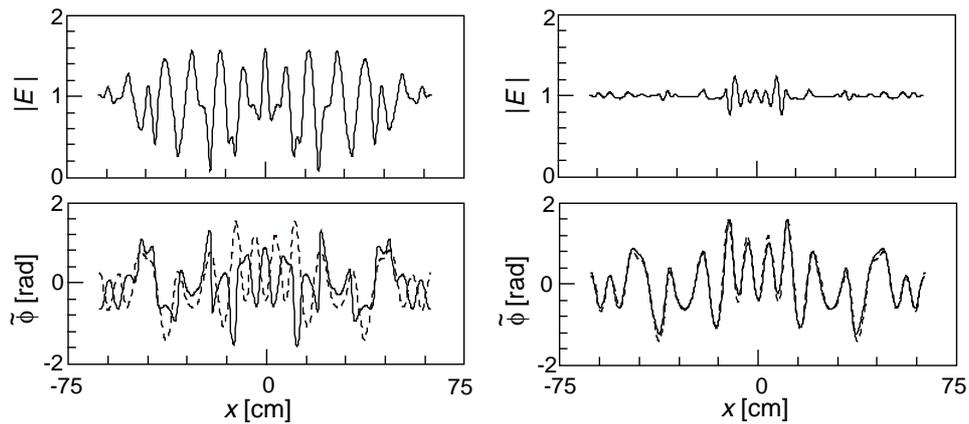


Fig. 1. Amplitude (top) and phase (bottom) of reflected wave at plasma boundary (left) and virtual reflecting point (right) for 2D fluctuations with $\sigma_n = 5 \times 10^{-3}$, $\Delta k_r = 1 \text{ cm}^{-1}$ and $\Delta k_x = 0.5 \text{ cm}^{-1}$. Probing wave with O-mode and $k_0 = 15.7 \text{ cm}^{-1}$; dashed line is the phase of 1D geometric optics.

Obviously, this model of reflectometry must fail for large amplitude fluctuations. In fact, since each spectral component of the backward field originates near the corresponding reflecting point, the model fails when these points are distributed over a distance Δr_c that is

comparable to the radial scale length of fluctuations $1/\Delta k_r$, i.e., when $\Delta k_r \Delta r_c > 1$. Since $\Delta r_c / L_\epsilon \approx \sigma_\phi^2 \Delta k_x^2 / k_0^2$, where L_ϵ is the radial scale length of the average plasma permittivity at the cutoff location, we may conclude that a condition for the validity of the above description of reflectometry is $\sigma_\phi^2 < k_0^2 / L_\epsilon \Delta k_r \Delta k_x^2$, or by expressing σ_ϕ^2 in terms of the density variance σ_n^2 , it is also given by $\sigma_n < 1/\pi^{3/4} L_n \Delta k_x$, where L_n is the plasma density scale length.⁷ Since both theory and experiments indicate that the amplitude of density fluctuations in tokamaks obeys the *mixing length criterion* $\sigma_n < 1/L_n \Delta k_r$, we conclude that a sufficient condition for the validity of the model is $\Delta k_r > \pi^{3/4} \Delta k_x$ (besides the condition $\Delta k_r < k_0 / (k_0 L_\epsilon)^{1/3}$ that is needed in the case of 1D fluctuations as well).

These arguments emphasize the importance of performing reflectometry measurements as close as possible to the virtual cutoff, since it is only by sampling the backward field at this location that it is possible to interpret the data. Experimentally, this can be done by collecting the reflected waves with a wide aperture antenna, and by imaging the virtual cutoff onto the detector plane. This is indeed the first novelty of the scheme proposed in this paper. A second

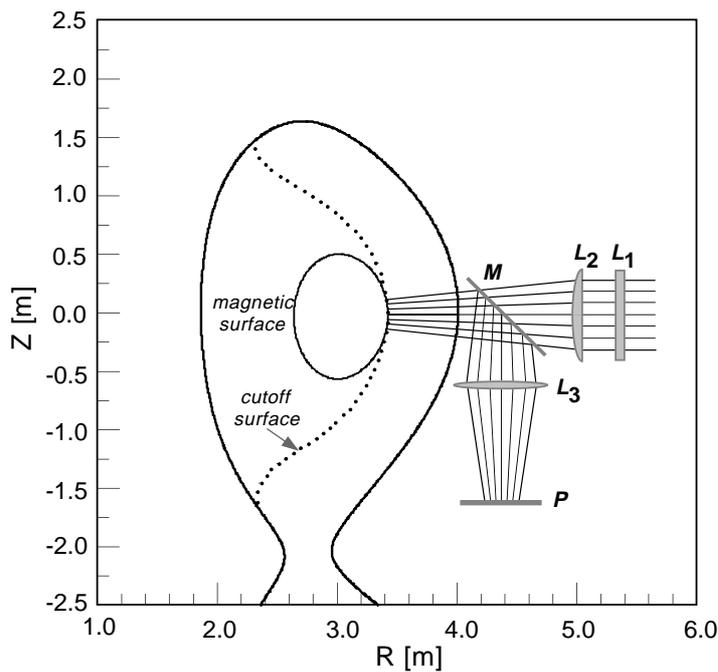


Fig. 2. Microwave imaging reflectometer.

novelty is the simultaneous sampling of an extended portion of magnetic surface, which requires the use of large microwave beams and 2D arrays of detectors. Such arrays are technically feasible, as demonstrated by Ref. 10 that describes a microwave camera employing a focal plane array (4 rows of 64 elements) for the detection of the human body thermal emission at 94 GHz. Its resolution is approximately 1 K with a video frame rate of 30 Hz. Another example can be found in Ref. 11, that describes the measurement of the electron cyclotron emission in the TEXT tokamak using a wide band 20 channel array in the range 90-110 GHz. More recently, similar measurements have been repeated in the RTP tokamak using a 16 channel array in the range 100-140 GHz.¹²

Figure 2 illustrates the conceptual design of a microwave imaging reflectometer for the visualization of turbulence in tokamaks. In this scheme, the probing wave is launched using two cylindrical lenses (L_1 and L_2) whose function is to tailor the wave front to the shape of the cutoff surface. This is done by focusing the probing wave onto the two principal centers of curvature of the cutoff surface. For the case considered in Fig. 2, which is that of a JET-

like plasma, the poloidal center of curvature is at $R=2.38$ m, while obviously the toroidal center of curvature is at $R=0$. Since the former depends on the wave frequency, the position of L_2 must be adjustable. Outside of the plasma, the backward wave is reflected by the semitransparent reflector M and an image of the cutoff is formed by the spherical lens L_3 onto the plane P , where the electromagnetic field is measured with a 2D array of microwave receivers.

The proposed method can use either the ordinary or the extraordinary mode of propagation, but the latter must be preferred because of a better spatial resolution. The following table gives the frequency range of operation in some of the existing tokamaks

	JET	DIII-D	TEXTOR
O-mode [GHz]	30-70	30-70	30-70
X-mode [GHz]	90-140	50-90	70-120

In conclusion, the microwave imaging reflectometer described in this paper is a first attempt at developing techniques for the global visualization of turbulent and coherent structures in tokamaks. Undoubtedly, the practical implementation of the proposed method presents serious difficulties, such as the need for a large port and for 2D arrays of microwave detectors. Nevertheless, the proposed technique has the potential for providing new information on the spatial structure of turbulent fluctuations, that could be used for advancing the theory of plasma turbulence in tokamaks or for checking the results of numerical simulations.

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