An Extension of Relaxed State Principle to Tokamak Plasmas with ITBs

A.B. Kukushkin, V.A. Rantsev-Kartinov

INF RRC "Kurchatov Institute", 123182 Moscow, Russia

1. Introduction

The signs of the electric current filamentation and networking found [1,2] in the visible light and soft X-ray images, and magnetic probing data, in extension of the identification of the plasma percolating networks in Z-pinches [3] (and other inertially confined plasmas), suggest the necessity to append conventional picture of plasmas which are nearly a fluid described by the conventional MHD, with a «network» component which is formed by the strongest long-living filaments (LLFs) of electric current and penetrate the «fluid» component (for the probable microscopic picture of the LLFs and its basic element, a microsolid tubular skeleton, see [4] and this conference proceedings).

The networking-based approach gives a novel qualitative view [2,5] on the MHD equilibria and nonlocal component of heat transport in magnetically confined plasmas. In particular, the internal transport barriers (ITBs) may be associated with the radial sectioning of the network component [1,2]. The approach [2,5] suggests the lines of extending the Relaxed States principle [6] to the case when besides a finite particle pressure and stable plasma flows, an ITBs are present. This includes an alternative view on (or substantiation of) the success of the principle [6] in the case of low-β plasmas with a weak/moderate external toroidal magnetic field (like, e.g., in reverse field pinches), which may be given in terms of the long-livingness of the network component. Indeed, the networking-based approach, in fact, replaces the identity of reconnectable magnetic flux tubes, suggested in [6], with the identity of LLFs with non-reconnectable microsolid skeletons. In the case of magnetically confined toroidal (MCT) plasmas this implies the integrity of the network component of the plasma in each radial section (see [1,2]). The identity of LLFs is provided by the intrinsic mechanisms of networking, namely by the symmetrization of sub-filaments within observed individual filaments and by the similar mechanisms of symmetrization of networking processes in a broad range of length scales (see [1,3] for details). The introduction of a skeleton changes nothing in the final result when a single skeleton covers the entire plasma volume and the internal symmetrization of the networking of LLFs does occur: the system, if plasma pressure gradient is small, happily reaches the force-free magnetic configuration. However, this doesn't hold true when the skeleton is sectioned.

A number of solutions to the problem of the MHD equilibrium for a finite pressure gradient have been given in literature for the case of a smooth profile of plasma pressure and electric current (i.e. plasmas without ITBs). In tokamak case, the substantiation of the «profile consistency» phenomenon [7] has been independently given in [8-10]. All these schemes minimize magnetic energy W_m (of poloidal B_p [8,9] or total magnetic field [10]) and thermal energy W_{th} [8,10] under condition of the conserved total (toroidal) electric current [8-10] and one more constraint (in [9,10], the Grad-Shafranov equation; in [8], a particular choice of dependencies $j(q)$ and $p(q)$). Interestingly, in [8] an extended scheme, with the addition of helicity conservation, has also been considered in the variational procedure.

The fruitfulness of the principle of magnetic helicity conservation in describing a non-force-free equilibrium has been demonstrated in [11]. Here, the variational principle minimizes W_m and W_{th}, with conserved magnetic helicity (rather than total electric current), and introduces the Grad-Shafranov equation directly to a variational procedure. The results for cylindrical geometry, with the first-order toroidal corrections, managed to give a peaked profile of the electric current.

The present paper describes an extension of the approaches [6,11] to the case of MCT plasmas with ITBs, via introducing the effect of radial sectioning of plasmas directly in the helicity conservation constraint(s) and testing the problem in cylindrical approximation.

2. Reformulation of Relaxed State principle in a sectioned plasma with LLFs

The analysis [2] suggests that the helicity conservation constraints may draw the most natural bridge between force-free and non-force-free cases. An extension of the approach [6,11] to the case of radial sectioning of the network component of MCT plasmas gives minimization of the following functional:

$$\delta(W_m + W_{th} + \sum_i \lambda_i K_i + W_{eq}) = 0\,, \tag{1}$$

where magnetic, W_m, and thermal, W_{th}, energies are integrated over entire volume V of plasma; K_i stands for the partial magnetic helicity where integration goes over the *i*-th radial section of plasma volume between *(i-1)*-th and *i*-th ITBs. For a thin layer of toroidal plasma,

$$dK = (\Phi_p)_{out}\, d\Phi_t + (\Phi_t)_{in}\, d\Phi_p\ , \tag{2}$$

where Φ_p and Φ_t are, respectively, poloidal and toroidal magnetic fluxes, and $(\Phi_p)_{out}$ and $(\Phi_t)_{in}$ are, respectively, the fluxes in the outer and inner volume with respect to the layer. Also, W_{eq} in Eq. (1) stands for the contribution of the force equilibrium equation, with a variable, vectorial Lagrangian multiplier (see [11]).

In what follows, we treat the problem in the cylindrical approximation for the case of a strong external magnetic field. We simplify the problem by integrating W_{th} in Eq.(1) by parts and using the force equilibrium equation explicitly. This reduces Eq. (1) to variations of the poloidal field only.

We assume ITBs to be located at low-number magnetic flux surfaces (for the reliable identification of the presence of ITBs at these surfaces, regardless of any external heating, see [12]). The minimization gives a freedom for the plasma macroscopic parameters to loose their smoothness at the ITBs, because the ITBs are considered as a thin layers at certain magnetic surfaces [1,2]. Such interpretation doesn't contradict to the widely used interpretation of an ITB as a thick layer between two magnetic surfaces at which the radial derivative of the temperature(s) changes dramatically.

We are to note that, strictly speaking, plasma flow effects should also be introduced everywhere: in total energy, force equilibrium equation, probably even in the helicity (cf. e.g. the generalized helicity approach [13]). However, our preliminary analysis suggests that the role of plasma flows is maximal in sustaining a strong shear (or even the jumps) of the rotation velocity at ITBs and, therefore, at the first step one can take plasma flow effects into account only implicitly, via assuming the radial sectioning as itself.

The above-mentioned integration by parts in Eq. (1) gives a relationship between the jumps of the pressure at ITBs and the pressure value at the internal surface of the outer barrier:

$$p^{(N)}(x_N - 0) = \sum_{i=1}^{N-1} x_i^2 \Delta p_i \qquad \Delta p_i \equiv p^{(i+1)}(x_i + 0) - p^{(i)}(x_i - 0);\ \ x = r/a\,,\ \ x_N \equiv 1\,, \tag{3}$$

where N is the number of sections, and x_i is the normalized radius of the i-th ITB. This relation works as a boundary condition for the problem.

The positions of ITBs, $\{x_i\}$, are considered here to be an input data together with the values $\{q_i\}$ of safety factor q at ITBs. Strictly speaking, $\{x_i\}$ are to be varied together with the polodial magnetic field B_p. However, we consider them to be determined by the mechanisms which are not explicitly dependent on the tendency of the plasma to minimize its magnetic and thermal energy. This assumes the buildup of the network component by its separate, rather independent mechanisms so that the q profile follows, to a large extent, the structuring of the network component rather than predetermines it. Probably, it would be possible to derive $\{x_i\}$ from a self-consistent minimization procedure once one includes certain parameters of the network of LLFs as an independent variable. Now we restrict ourselves to reconstruction of q profile from a set of points $\{q_i\}$ at positions $\{x_i\}$ and, further, we determine all other profiles, B_p and j, and p, including their jumps at ITBs.

Within above frames, Eq. (1) gives a system of linear equations on $\{\lambda_i\}$. For the first section, one has obvious universal solution:

$$\tilde{B}_p^{(1)}(x) = \lambda_1 \cdot x, \quad \lambda_1 = 1/q_0, \qquad \tilde{B}_p \equiv B_p R / aB_T \ . \tag{4}$$

The set $\{q_i\}$ should be related to the values of B_p somewhere in the ITBs. If one takes it at inner surface of the ITB, this gives a senseless result, namely uniform q and j profiles. Therefore, we establish this relation at the outer surface of ITBs:

$$\tilde{B}_p^{(i+1)}(x_i + 0) = x_i / q_i \ . \tag{5}$$

Note that Eq. (5) implies that the family of sawtooth-like oscillations observed in [12] at the ITBs, are located probably at the inner surfaces of ITBs, similar to the «father» of this family, the inner crashes at $q=1$ surface.

The typical results for the two cases of interest, $q_0 < q_1$ and $q_0 > q_1$, are shown in Fig. 1 and Fig. 2. Here, B_{pol} is the normalized $\tilde{B}_p$ from Eq. (4), $q_{rel} \equiv q(x)/q(a)$, and $p(x)$ is taken in units $p(0)$, and $\{q_i\}$ are taken from [12], Fig. 8, discharges A and D, respectively. The jumps of q at ITBs give, as a rule, the counter directed surface electric currents (respective values I_i are given in Tables 1 and 2, in units of total electric current). The pressure profile is derived from the force equilibrium equation while its absolute value, $p(0)$, is found from Eq. (3).

If one applies the above procedure to the case when the magnetic configuration approaches the force free equilibrium (this requires variation of the toroidal field though), the values of the jumps, for the vanishing pressure gradient, obviously tend to zero.

3. Conclusions

The extension of the Relaxed State principle [6] to the case of magnetically confined toroidal plasmas with ITBs requires, to our mind, a reformulation of the problem. The introduction of the effect of radial sectioning of the network component to the variation procedure gives new constraints (the conservation of partial magnetic helicities). The results of testing the minimization procedure in cylindrical approximation give the jumps of safety factor and pressure at ITBs, and respectively, surface electric currents of which most essential appear to be counter directed to total current. The appearance of new degrees of freedom (either positions of ITBs or surface electric currents at ITBs) might be resolved self-consistently within a formalism which allows for the fine structure of ITBs. We believe [2] the latter problem to be determined by the formation of the networks of long-living filaments. In any case, the proposed minimization procedure is able to give one-to-one dependence between the fine structure of the profile at ITBs (namely, the respective jumps) with the entire smoothened profile.

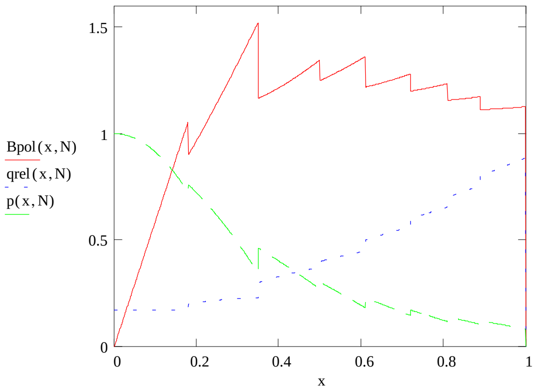

Figure 1

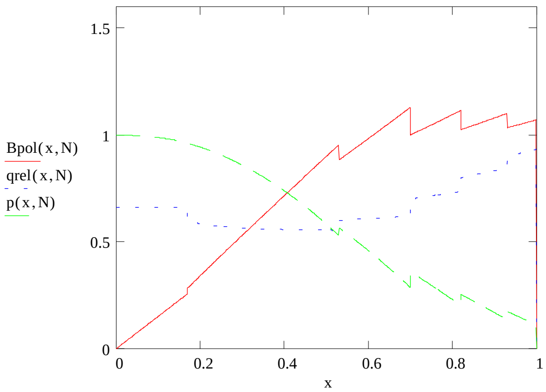

Figure 2

Table 1

x_i	0	0.18	0.35	0.50	0.61	0.72	0.81	0.89	1.0
q_i	0.85	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5
I_i	0	0.029	0.125	0.049	0.087	0.058	0.064	0.055	0.127
$\sum I_i$	0.595								

Table 2

x_i	0	0.17	0.53	0.70	0.82	0.93	1.0
q_i	3.3	3.0	3.0	3.5	4.0	4.5	5.0
I_i	0	-0.0044	0.039	0.091	0.074	0.063	0.072
$\sum I_i$	0.333						

REFERENCES

[1] Kukushkin A.B., Rantsev-Kartinov V.A., Rev. Sci. Instrum., **70,** 1392 (1999); Proc. 1998 Int. Congress on Plasma Phys. & 25th EPS Conf., Prague, ECA 22C, p. 2216.

[2] Kukushkin A. B. Rantsev-Kartinov V. A. Preprint of Kurchatov Institute, IAE 6095/6, Moscow, May 1998, 48 pp.

[3] Kukushkin A.B., Rantsev-Kartinov V.A., Laser and Part. Beams, **16,** 445 (1998).

[4] Kukushkin A.B., Rantsev-Kartinov V.A., (a) Proc. 17th IAEA Fusion Energy Conf., Yokohama, Japan, October 1998, IAEA-F1-CN-69/IFP/17; (b) Proc. 3rd Symposium «Current Trends in Int. Fusion Research: Review and Assessment», Washington D.C., March 1999, Ed. E. Pararella, NRC Research Press, Ottawa, Canada (to appear).

[5] Kukushkin A.B., Rantsev-Kartinov V.A., Proc. Int. Toki Conference (ITC-9), Toki, Japan, December 1998, P-III-44.

[6] Taylor J.B., Phys.Rev.Lett., **33,** 1139 (1974); Rev. Mod. Phys., **58**, 741 (1986).

[7] Coppi B., Comments Plasma Phys. Contr. Fusion, **5**, 261 (1980).

[8] Kadomtsev B.B. Sov. J. Plasma Phys. **13**, 443 (1987).

[9] Biskamp D., Comments Plasma Phys. Contr. Fusion, **10**, 165 (1986).

[10] Hsu J.Y., Chu M.S., Phys. Fluids, **30**, 1221 (1987).

[11] Edenstrasser J.W., Schuurman W., Phys. Fluids, **26**, 500 (1983).

[12] Lopez Cardoso N. J., et. al., Plasma Phys. and Contr. Fusion, **39 (**Suppl. 12B), 303 (1997).

[13] Sudan R.N., Phys. Rev. Lett., **42**, 1277 (1979).