Access to Optimised Shear Equilibria in Spherical Tokamaks

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1. Introduction. High β regimes in which the magnetic equilibrium has flat or non-monotonic safety factor profile near the magnetic axis ("Optimised Shear") offer the prospect of steady-state operation in both conventional and spherical tokamaks (ST). This is because the plasma current could consist mainly of pressure-driven current with only a modest amount of non-inductive current drive. Experimental and theoretical demonstration of these regimes is therefore very important - this is, for example, an objective of the new spherical tokamaks MAST and NSTX. We present the results of a numerical investigation into how these regimes can be accessed in spherical tokamaks, concentrating on the use of neutral beam heating and current drive in the START and MAST devices.

Previous work showed that high pressure regimes with optimised shear can be accessed in qualitative accord with experimental indications from START plasmas with neutral beam heating [1]. In this paper these simulations have been extended with improved representations of the experimental conditions, especially the plasma pressure profile which now allows for the contribution of the centrally peaked fast ions from neutral beam heating. Flat $q(r)$ can be accessed, even in regimes with small pressure-driven current, in accord with experimental reconstructions using EFIT (for somewhat different plasma conditions). When the fast ion pressure is broadened the effect becomes more pronounced.

If these regimes are to be maintained in true steady-state then the addition of non-inductive current drive is required. In the MAST simulations the effects of neutral beam current drive (which is insignificant in START) have been included and the paper discusses how NBCD could be used in MAST to investigate whether regimes with steady-state potential can be accessed and sustained.

2. Formulation of the problem. The code SCoPE has been developed by the authors for the computational simulation of equilibrium evolution. The coupled equilibrium (Grad-Shafranov) and evolution (parallel Ohm's law) equations in free boundary geometry underly the mathematical model [2,3]:

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2} = -\mu_0 R j_\varphi, \tag{1}$$

$$j_\varphi = \begin{cases} R\dfrac{\partial p(t,\psi)}{\partial \psi} + \dfrac{1}{2\mu_0 R}\dfrac{\partial F^2(t,\psi)}{\partial \psi} & \text{in } \Gamma_p \\ \displaystyle\sum_{i=1}^{L} J_i(t)\delta(R - R_i)\delta(Z - Z_i) & \text{outside } \Gamma_p \end{cases},$$

$$\frac{\partial}{\partial t}\left(\int_0^Z \tilde{F}\,dZ'\right)\frac{\partial\psi}{\partial Z} - \frac{\partial\psi}{\partial t}\tilde{F} = \frac{1}{\mu_0\sigma_\parallel}\left(\frac{\partial\tilde{F}}{\partial R}\frac{\partial\psi}{\partial R} + \frac{\partial\tilde{F}}{\partial Z}\frac{\partial\psi}{\partial Z}\right) -$$

$$-\frac{1}{\mu_0\sigma_\parallel}\tilde{F}\left(R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) + \frac{\partial^2\psi}{\partial Z^2}\right) - \frac{R^2}{\sigma_\parallel}\vec{j}_{\text{add}}\vec{B}, \quad \text{in } \Gamma_p \qquad (2)$$

$$F(t,\psi) = \int_{\psi=\text{const}} \tilde{F}(t,R,Z)B_{\text{pol}}^{-1}\,dl\bigg/\int_{\psi=\text{const}} B_{\text{pol}}^{-1}\,dl. \qquad (3)$$

Both of the equations (1) and (2) are written in cylindrical (R,Z) co-ordinates where R and Z are the major radius and the vertical co-ordinate. Here B_{pol} is the poloidal magnetic field, $p(t,\psi)$ is the plasma pressure determined from transport equations that are included in the model, $J_i(t)$ are currents in the vessel wall, solenoid and poloidal field coils, $\vec{j}_{\text{add}}$ are non-Ohmic plasma currents, and Γ_p is the free plasma boundary (defined as the closed flux surface of maximum width) or a prescribed boundary. Expressions for bootstrap, Pfirsch-Schluter and diamagnetic currents and neo-classical conductivity $\sigma_\parallel$ are taken from Refs. [4,5]. The unknown functions are two scalar functions - the poloidal magnetic flux ψ and the current F (which gives the toroidal field $B_{\text{tor}} = F/R$).

The system (1) - (3) is completed by the following initial and boundary conditions

$$\psi(0,R,Z) = \psi_0(R,Z), \quad F(0,\psi) = F_0(\psi),$$

$$\lim_{R\to 0}\psi = \lim_{\substack{R\to\infty \\ Z\to\infty}}\psi = 0, \quad \tilde{F}(t,R,Z)\big|_{\Gamma_p} = \frac{\mu_0}{2\pi}I_{\text{rod}}(t) \qquad (4)$$

Here $I_{\text{rod}}(t)$ is the current down the central rod, which produces the toroidal field.

Usage of this equilibria evolution model instead of fixed time equilibria (based on the equilibrium equation only) is necessary for the study of how regimes such as the optimised shear regime can be accessed, and how long they can be sustained. The model includes intrinsic pressure gradient driven currents (bootstrap, Pfirsch-Schluter and diamagnetic), ad hoc models for currents driven by lower hybrid waves and neutral beam injection, neo-classical resistivity, effects of the vessel wall, and programmed currents in the solenoid and poloidal field coils. For these studies, transport equations were replaced with prescribed time-dependent temperature and density profiles. Lower hybrid current was switched off. Details about numerical methods used for eqs. (1)-(4) are in Ref. [6].

3. Equilibrium evolution modelling in START. The main aim of the calculations was to study the access to optimised shear equilibria in START taking account of the contribution from fast ions (from NBI) to the plasma pressure. The initial and time dependent plasma parameters were as in Ref. [1]: major and minor radii $R_{mag\ axis} = 0.34$ m and $a = 0.23$ m, elongation $\kappa = 1.6$, $I_{\text{rod}} = 500$ kA ($B(R_{mag\ axis}) = 0.3$T), density $n_{e,i} = 0.4(0.5(1-(r/a)^3)+0.5)10^{20}$ m^{-3}, and temperature $T_{e,i} = 0.3(0.7(1-(r/a)^3)+0.3)$ keV, current density profile $j_0(1-(r/a)^2)$ with j_0 adjusted to give 200 kA total current (r is the flux surface half-width in the poloidal plane). The temperature and density were each increased by a factor of $\sqrt{6}$ over 10 ms. The leftmost and rightmost edges of the plasma were controlled with 5% accuracy by adjusting currents in poloidal field coils. NBI contribution to pressure was based on results of Ref. [7], but different shapes near the magnetic axis have been considered. It was found that depending on the shape of the NBI pressure the negative shear regime can become more pronounced (than without

account of NBI) or disappear. With flatter pressure near the axis, the q rise at the magnetic axis is more noticeable than in [1].

4. Equilibria evolution modelling in MAST. The purpose of calculations for MAST-like plasma was to investigate whether regimes with steady-state potential can be accessed and sustained in the presence of pressure-driven and NBI currents. The following initial parameters were used: major and minor radii $R_{mag\ axis} = 0.73$ m and $a = 0.53$ m, elongation $\kappa = 1.7$, $I_{\rm rod} = 1.5$ MA ($B(R_{mag\ axis}) = 0.48$T), central density and temperature $n^o_{e,i} = 0.4 \cdot 10^{20}$ m^{-3}, $T^o_{e,i} = 0.3$ keV, total current 650 kA. The initial plasma shape is shown in Fig. 1 with dashed lines. Density, temperature and model current density profiles, normalised to values at the axis, are given in Fig. 2. A fixed plasma boundary problem was considered for simplicity.

A simple energy balance model was used $dW/dt = Q - W/\tau_E$ with $\tau_E = 0.03$ s and $Q(t)$ linearly rising over 0.1 s from 1 to 5 MW. In the case of time independent central density and normalised radial profiles this model gives central plasma temperature variation over time $T^o_{e,i}(t) = T^o(0)W(t)/W(0)$. This dependence was used in the simulations. The NBI current increased over time according to $\sim Q(t)T^o_{e,i}(t)$. The NBI contribution to the plasma pressure was not considered in these first calculations. The loop voltage at the plasma boundary was kept zero.

The final flux surfaces are specified in Fig. 1 by solid lines. Fig. 3 presents initial and final toroidal current density and safety factor q over major radius. Fig. 4 gives total toroidal currents. It was found that over ~0.5s the plasma reached a regime close to a steady-state. Plasma parameters at $t = 0.5$ s were: total current 757 kA, central electron and ion temperature 2.3 keV, $\beta_{\rm tor} = 2\mu_0 \int pdV/(B^2_{\rm tor,vac}(R_{\rm geom}) \int dV) \approx 20\%$, $B(R_{mag\ axis}) = 0.45$T, 85% fraction of additional non-inductive currents: 25% of bootstrap and Pfirsch-Schluter, 60% of NBI (by adjusting p profile bootstrap fraction can be made $\sim 50\%$). In this regime, the final q profile is flatter than the initial one (see Fig. 3). The rise of j_φ near the edge is mainly due to $dp/d\psi$ (see eq. (1)).

Calculations have also been done for exponential over time dependence of plasma temperature and NBI current. This is appropriate to the case when $Q(t) \approx$const. These runs confirmed that the plasma can be brought to a close to a steady-state regime with a large fraction of additional currents $\sim85\%$ and a very flat or even reversed shear q profile.

Finally, we have performed semi-self-consistent calculations using a combination of codes SCoPE and LOCUST. The main aim of these calculations was to check how fast ion pressure can influence access to steady-state optimised shear regimes in MAST. The problem is that $q(0)$ may drop below 1, if the fast ion pressure gradient contribution to the toroidal current near the axis is too high and too peaked. SCoPE's time evolving equilibria was used in LOCUST to calculate the contribution of fast ions to the plasma pressure and NBI current density which were then used again in SCoPE. (In true self-consistent calculations SCoPE and LOCUST should be solved every time step. This will be done in future after overcoming some numerical problems.) Optimised shear regimes with $q(0) > 1$ were again found.

5. Conclusion. Calculations show that at high pressures in spherical tokamak START-like plasmas, a reversed or very low shear regimes could be achieved and sustained. This is in qualitative agreement with equilibrium reconstruction of high β plasmas on START, though for somewhat different conditions. Simulations for MAST allow one to conclude that it should be possible to access and sustain near-steady-state "optimised

shear" regimes in MAST. Although regimes with a high fraction of non-inductive current drive have been identified, and therefore the prospects of achieving "steady-state" regimes in MAST are good, further work needs to be done on minimising the fraction driven by NBI so as to minimise the additional heating required in a steady-state ST fusion device.

References.
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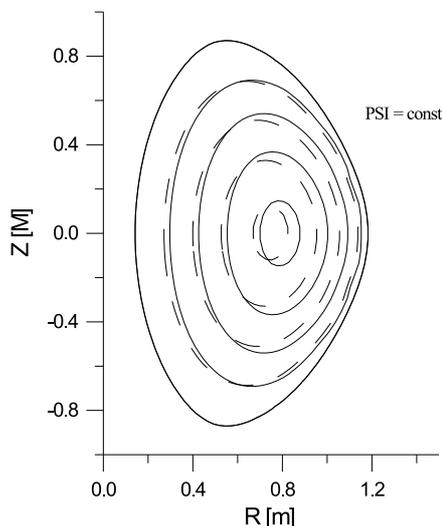7. *R.J. Akers et al, 25th EPS Conference, Prague 1998, p. 2016.*

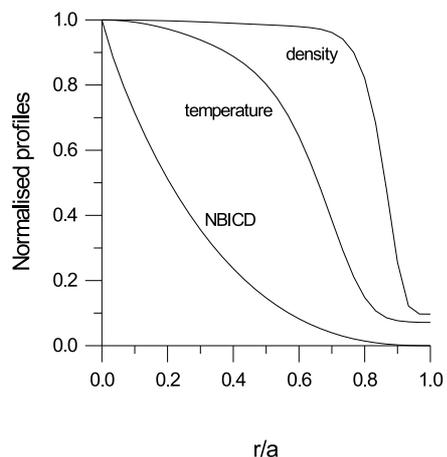Figure 1: Surfaces ψ=const at t=0 - dashed lines and t=0.5 s - solid lines.

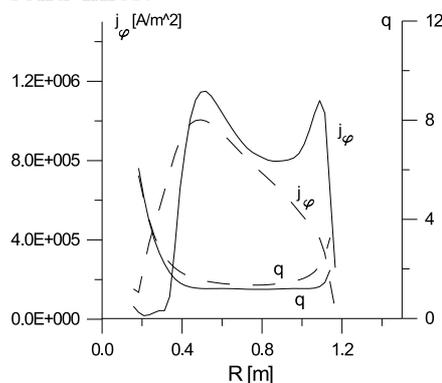Figure 2: Temperature, density and NBICD, normalised to central values.

Figure 3: Total current density and q profile at t=0 - dashed lines and t=0.5 s - solid lines.
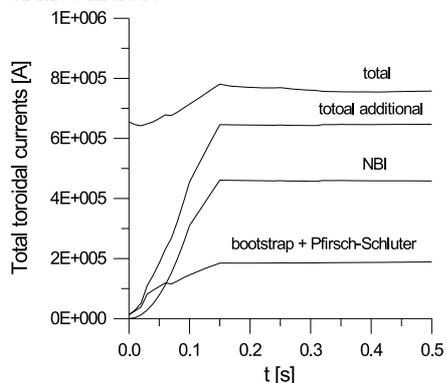

Figure 4: Time dependence of different components of toroidal current.