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Influence of Helical Magnetic Field Inhomogeneity on the Properties of Alfven and Fast Magnetosonic Waves

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<u>Description of the confining magnetic field.</u> The stellarator magnetic field $\vec{B}_0(r,\vartheta,z)=\vec{e}_r B_{0r}+\vec{e}_\vartheta B_{0\vartheta}+\vec{e}_z B_{0z}$ can be represented (in cylindrical coordinates) in the paraxial approximation, for $l\geq 2$, through the single harmonic (see [1]):

$$B_{0r}=\bar{\Delta}_l B_0 \alpha r\, sin(l\theta),\quad B_{0\vartheta}=\Delta_l B_0\, cos(l\theta)\quad,\quad B_{0z}=B_0-\Delta_l B_0 \alpha r\, cos(l\theta), \tag{1}$$

$$\bar{\Delta}_n=\frac{nb_n I_n'(k_s r)}{B_0 \alpha r},\quad \Delta_n=\frac{nb_n I_n(k_s r)}{B_0\alpha r},\quad b_n=8Jak_s^2 K_n'(k_s a)(nc)^{-1},\quad \theta=\vartheta-\alpha z. \tag{2}$$

In (2), a is the radius of the cylindrical surface carrying a thin helical winding with the current J, $K_l(\xi)$ is McDonald function, $I_l(\xi)$ is modified Bessel function, the prime devotes the derivative with respect to the argument, $\alpha=2\pi/L$, L is the pitch length of the helical winding, l is the poloidal number of the helical-field periods, $k_s= l\,\alpha$.

<u>Influence on the dispersion properties in the general case</u>. Inhomogeneity of the magnetic field $\vec{B}_0(r,\vartheta,z)$ (1) often may be omitted in fusion applications due to its smallness, $\bar{\Delta}_l<<1, \Delta_l<<1$. The effect of this nonuniformity on dispersion properties of Alfven waves (AW) and fast magnetosonic waves (FMSW) was studied in [2] in the nonresonant case. The inhomogeneity of $\vec{B}_0$ was shown to result in the weak shift $\delta\omega$ of the waves eigen frequency ω, $\omega=\omega_0+\delta\omega$, where ω_0 is the eigen frequency known from the zero-th approximation, and the shift is the second-order quantity in the ripple amplitude, $\delta\omega \propto \Delta_l^2$.

<u>Influence on the dispersion properties in the resonant case</u>. In the present report, the influence of the $\vec{B}_0$ nonuniformity on the dispersion properties of AW and FMSW is investigated in the resonant case such that the period $2\pi/k_z$ of the fundamental mode of electromagnetic oscillations in the axial direction is twice as large as the pitch length L and poloidal number m of the fundamental mode is twice as small as l,

$$k_s=2k_z,\qquad l=2m. \tag{3}$$

Perturbation theory developed for the case with the degenerate spectra is used to solve the problem.

In the resonant case, the $\vec{B}_0$ inhomogeneity causes the splitting of the eigen frequencies of AW and FMSW, $\omega=\omega_0\pm\delta\omega$. The corrections $\delta\omega$ to the eigen frequencies are the first-order quantities, $\delta\omega\propto\Delta_l$. Splitting of the spectra is shown to take place for oscillations with weak gyrotropy (for which difference in eigen frequencies of oscillations with opposite signs of poloidal numbers in straight magnetic field, $\Delta_l=0$, is small, $\omega_0(|m|)-\omega_0(-|m|)<<\omega_0$). In particular, in a stellarator with $l=2$, splitting of the spectra is expected to be pronounced for the small-scale ($k_A a_p\approx\pi n_r>>1$, here k_A is Alfven wave number, a_p is average radius of plasma column, n_r is radial wave number) FMSW propagating nearly perpendicular to steady magnetic field, $k_A>>k_z$, with the poloidal number $m=\pm 1$ and the axial wave number $k_z=\mp\alpha$. If the chamber is fully filled with plasma with uniform radial density profile, then eigen frequency of FMSW is equal,

$$\omega_0 = j_{1,s} c\omega_{ci} \Big/ \left(a\omega_{pi}\right), \quad \delta\omega = 0{,}25\,\bar{\Delta}_l(a)\omega_0, \tag{4}$$

here $j_{1,s}$ is the value of the *s*-th root of the first-order Bessel function, $J_1(j_{1,s})=0$, ω_{ci} and ω_{pi} are the cyclotron and plasma frequencies of ions. The gyrotropy can be weak in this case,

$$\left[\omega_0(+|m|) - \omega_0(-|m|)\right]/\,\omega \approx 2|m|\left(1+\omega/\omega_{ci}\right)\left(\pi n_r\right)^{-2} << \Delta_l. \tag{5}$$

The splitting of the eigen frequency by the helical magnetic field can be observed experimentally. The similar phenomenon was discovered in experiment [3] with tokamaks: the longitudinal electric current eliminated the degeneration of the spectra of FMSW with respect to the sign of their axial wavenumber.

Distribution of electromagnetic field of eigen oscillations of a plasma column with a nonuniform radial density profile is determined in the following form

$$B_z = \Big[\left(C_0^{(+)}\psi_1^{(0)}(r) + C_1^{(+)}\psi_1^{(+)}(r)\right)e^{i\theta} + \left(C_0^{(-)}\psi_1^{(0)}(r) + C_1^{(-)}\psi_1^{(-)}(r)\right)e^{-i\theta} + C_3^{(+)}\psi_3^{(+)}(r)e^{3i\theta} + C_3^{(-)}\psi_3^{(-)}(r)e^{-3i\theta}\Big]exp(-i\omega t), \tag{6}$$

In the resonant case, the natural modes are standing waves, $C_0^{(+)} = \pm C_0^{(-)}$. The positions of the nodes (antinodes) of the standing wave with the lower frequency $\omega = \omega_0 - \delta\omega$ (with the higher frequency $\omega = \omega_0 + \delta\omega$) coincide with the surfaces at which the confining magnetic field is minimum (maximum). The reason is that, in a uniform magnetic field, the eigen frequencies of AW and FMSW increases with the magnetic field $|\vec{B}_0|$.

The superposition of two natural standing waves whose frequencies are close to each other leads to the arise of a beat wave. The beat frequency $\delta\omega$ depends on the external parameters ($\Delta_l, \omega_{ci}, L, a$) and the plasma density <*n*>. Consequently, the measurements of the $\delta\omega$ can serve to diagnose the <*n*>.

Satellite Alfven resonances in the plasma core. Possibility of additional plasma heating within satellite Alfven resonances (SAR) $r = r_A^{(\pm)}$, for which the condition

$$\varepsilon_1 \equiv 1 - \sum_i \omega_{pi}^2(r)/\left(\omega^2 - \omega_{ci}^2\right) = \left(N_z \mp N_s\right)^2, \tag{7}$$

holds, was shown in [4] for the case, when SAR is located in the plasma core. In (7), $N_z = ck_z/\omega$ is the axial refractive index of the fundamental mode, $N_s = ck_s/\omega$. The arising of SARs is explained as follows. Due to the helical inhomogeneity of $\vec{B}_0$ (1), electromagnetic waves propagate in stellarator in the form of envelope. Side by side with the fundamental mode $\propto exp\,i\left(k_z z + m\vartheta - \omega t\right)$, this envelope contains the satellite harmonics $\propto exp\,i\left[\left(k_z \mp k_s\right)z + (m \pm l)\vartheta - \omega t\right]$. In the vicinity of SARs (7), just the amplitudes of the satellite harmonics rapidly increase, and these waves convert into small-scale “kinetic” AW.

Even weak phenomena are known to affect significantly on the conversion of electromagnetic waves within AR regions and to provide for their efficient absorption there.

SARs at the plasma periphery. In the present report, the distribution of *rf* fields and the *rf* power absorbing in the vicinity of SARs are found with taking into account the following weak phenomena: thermal motion of particles, finite inertia of electrons, helical curvature of magnetic surfaces, striction nonlinearity, reversed effect of kinetic parametric ion cyclotron plasma instabilities on pumping wave and dissipation.

Conditions are determined under which effect of the $\vec{B}_0$ inhomogeneity on the structure of SAR is more essential than of the other weak phenomena mentioned above. In particular, the influence of the $\vec{B}_0$ nonuniformity is more essential than that of finite ion Larmor radius $\rho_{Li} = v_{Ti} / \omega_{ci}$ (where $v_{Ti} = \sqrt{T_i} / m_i$ is the thermal velocity, and T_i is the ion temperature) if the following inequality holds,

$$\bar{\Delta}_l^2 >> \left(\rho_{Li} / r_A^{(+)}\right)^2 \left(N_z / N_s - 1\right) l^2 . \quad (8)$$

These conditions may hold at the plasma periphery, where the noncircular shape of the magnetic surfaces is the most pronounced and the plasma is the most cold.

Under these conditions, satellite harmonics affect weakly on the distribution of the fundamental harmonic. Within SAR, satellite harmonics incorporate small terms $\propto \Delta_l^{2/3}$ into the equation for the amplitude $E_r^{(0)}$ of the fundamental harmonic of the radial electric field, this contribution being less then outside of SAR, where it is $\propto \Delta^2$.

Behavior of the amplitude $E_r^{(+1)}$ of the nearest satellite harmonic within SAR is governed by the nonuniform Eiry equation. The width of SAR δr is determined by the ripple parameter $\bar{\Delta}_l$ in this case,

$$\delta r \sim a^* \bar{\Delta}_l^{2/3} , \quad (9)$$

here $a^* = \left| d \ln \varepsilon_1^{(0)} / dr \right|^{-1}$ is characteristic length at which plasma density varies.

Outside of SAR, amplitude $E_r^{(+2)}$ of the second satellite harmonic $\propto exp\, i\left[\left(k_z - 2k_s\right)z + (m + 2l)\vartheta - \omega t\right]$ is known [2] to be the small value of the second order, $E_r^{(+2)} \sim \Delta_l E_r^{(+1)} \sim \Delta_l^2 E_r^{(0)}$. When coming to the SAR region $r = r_A^{(+)}$, the amplitude $E_r^{(+2)}$ grows more rapidly, than $E_r^{(+1)}$, namely,

$$E_r^{(+2)} = -\frac{r\bar{\Delta}_l}{2l}\frac{dE_r^{(+1)}}{dr} \propto \left[\varepsilon_1^{(0)} - \left(N_z - N_s\right)^2\right]^{-2} . \quad (10).$$

This growth results in restriction of the $E_r^{(+1)}$ growth. Characteristic values of satellite harmonics amplitudes are as follows by the order of magnitude within SAR,

$$E_r^{(+2)} \sim \Delta_l^{1/3} E_r^{(+1)} \sim \Delta_l^{2/3} E_r^{(0)} . \quad (11)$$

Poloidal component of the electric field varies slowly within SAR,

$$\frac{dE_\vartheta^{(+1)}}{dr} \approx \frac{m+l}{r} E_r^{(+1)} . \quad (12)$$

The order of magnitude of the satellite harmonic amplitude $E_\vartheta^{(+1)}$ retains within SAR the same as outside of this region, $E_\vartheta^{(+1)} \sim \Delta_l E_\vartheta^{(0)}$.

Amplitude $E_z^{(+1)}$ of the nearest satellite harmonic of axial electric field grows within SAR even rapidly than $E_r^{(+1)}$,

$$E_z^{(+1)} \approx \frac{ic^2\left(k_z - k_s\right)}{\omega^2 \varepsilon_3}\frac{dE_r^{(+1)}}{dr} . \quad (13)$$

Nevertheless this amplitude retains smaller than $E_r^{(+1)}$ that is confirmed by the following relation being valid by the order of magnitude,

$$E_z^{(+1)} \sim \left(\frac{\varepsilon_1}{\varepsilon_3}\right)^{2/3} \left(\frac{\rho_{Li} l}{r_A^{(+)} \overline{\Delta_l}}\right)^2 \left(\frac{N_z}{N_s} - 1\right) E_r^{(+1)} << E_r^{(+1)} \ . \tag{14}$$

The damping rate γ_s caused by the ions scattering on turbulent pulses increases sufficiently within SAR as compared with its value γ outside of this region,

$$\gamma_s = \gamma(1 + \delta\gamma) \, , \quad \delta\gamma \sim \overline{\Delta_l}^{2/3} \, . \tag{15}$$

Weak striction nonlinearity is shown to result in moving of the SAR region away from the plasma axis for the small distance.

Account for the $\vec{B}_0$ nonuniformity does not change the value of *rf* power absorbed in the vicinity of the SAR.

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