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Cross-Field Plasma Acceleration and Potential Formation in a Relativistic Magnetized Plasma

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Abstract

It has been proved theoretically that particle acceleration along and across a magnetic field and electric field across a magnetic field can be induced by almost perpendicularly propagating electrostatic waves in a relativistic magnetized plasma.

1. Introduction

Particle acceleration along and across a magnetic field and electric field across a magnetic field which are induced by almost perpendicularly propagating electrostatic waves in a relativistic magnetized plasma have been investigated theoretically based on relativistic quasilinear transport equations derived from relativistic Vlasov-Maxwell equations[1]. The electrostatic waves accelerate plasma particles and the ratio of parallel and perpendicular drift velocities $v_{s\parallel}/v_d$ can be verified to be proportional to $k_\parallel/k_\perp$. As a result, the strong plasma acceleration or transport across a magnetic field appears. Simultaneously the intense cross-field electric field $\boldsymbol{E}_0=\boldsymbol{B}_0\times\boldsymbol{v}_d/c$ is generated via the dynamo effect of perpendicular particle drift to satisfy the generalized Ohm's law, that is , the electrostatic waves can produce the cross-field particle drift that is identical to $\boldsymbol{E}\times\boldsymbol{B}$ drift[2-4]. Moreover, the relativistic quasilinear transport equations for the relativistic cross-field particle acceleration were derived by means of Lorentz transformation of the relativistic quasilinear momentum-space diffusion equation in the frame moving with the cross-field drift velocity. They can be applied usefully to the theoretical investigation of the relativistic cross-field particle acceleration or transport which may occur possibly in space and fusion plasmas.

2. Nonrelativistic Cross-Field Particle Acceleration

2.1. Relativistic quasilinear momentum-space diffusion equation

We consider the nonrelativistic cross-field particle acceleration which arises from quasilinear momentum-space diffusion due to electrostatic waves propagating in a uniform relativistic plasma immersed in the uniform magnetic and electric fields $\boldsymbol{B}_0=(0,0,B_0)$ and $\boldsymbol{E}_0=(0,E_0,0)$. The dielectric constant $\varepsilon_{\boldsymbol{k}}=1+\sum_s\varepsilon_{\boldsymbol{k}}^{(s)}=\varepsilon_{\boldsymbol{k}}'+i\varepsilon_{\boldsymbol{k}}''(\varepsilon_{\boldsymbol{k}}^{(s)}=\varepsilon_{\boldsymbol{k}}'^{(s)}+i\varepsilon_{\boldsymbol{k}}''^{(s)})$ is obtained as

$$\varepsilon_{\boldsymbol{k}}^{(s)}=-\frac{\omega_{ps}^2 m_s}{k^2}\sum_{r=-\infty}^{\infty}\int d\boldsymbol{p}\,\frac{J_r^2(\mu_{\boldsymbol{k}})U_r(\boldsymbol{k})g_{s0}}{k_\parallel v_\parallel+k_\perp v_d-\omega_{\boldsymbol{k}}+r\omega_{cs}}\quad, \tag{1}$$

$$U_r(\boldsymbol{k})=k_\parallel\frac{\partial}{\partial p_\parallel}+\frac{r\omega_{cs}}{v_\perp}\frac{\partial}{\partial p_\perp}\quad, \tag{2}$$

$$g_s = a_s \sum_{m=0}^{\infty} \frac{1}{m!} p_x^m \left(-\frac{p_{ds}}{p_\perp}\frac{\partial}{\partial p_\perp}\right)^m g_{s0}(p_\perp, p_\parallel, t)$$
$$= a_s g_{s0}((p_\perp^{\,2} - 2p_x p_{ds})^{1/2}, p_\parallel, t) \quad , \tag{3}$$

where $\boldsymbol{p}=\boldsymbol{p}'+\boldsymbol{p}_{ds}$, $\boldsymbol{p}'=\gamma_s m_s \boldsymbol{v}'$, $\boldsymbol{p}_{ds}=\gamma_s m_s \boldsymbol{v}_d$, $\gamma_s=(1-v'^2/c^2)^{-1/2}=(1+p'^2/m_s^{\,2}c^2)^{1/2}$, J_r is the Bessel function of the rth order, $\mu_{\boldsymbol{k}}=k_\perp v_\perp/\omega_{cs}$, $\omega_{ps}=(4\pi n_s e_s^{\,2}/m_s)^{1/2}$, $\omega_{cs}=|e_s|B_0/\gamma_s m_s c$, $p_x=p_\perp\cos\theta+p_{ds}$, $p_y=p_\perp\sin\theta$, $p_z=p_\parallel$, $p_\perp=\gamma_s m_s v_\perp$, $p_\parallel=\gamma_s m_s v_\parallel$, The momentum-space integration in Eq. (1) is performed in the displaced cylindrical coordinate ($\mathrm{d}\boldsymbol{p}=p_\perp \mathrm{d}p_\perp \mathrm{d}p_\parallel \mathrm{d}\theta$), $\boldsymbol{v}_d=(v_d , 0, 0)=c\boldsymbol{E}_0\times\boldsymbol{B}_0/B_0^{\,2}$ equals $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity, $v_d/c<<1$, $\boldsymbol{k}=(k_\perp , 0, k_\parallel)$, g_s is the background momentum distribution function containing the fluctuation-induced cross-field drift velocity $v_d=\int \mathrm{d}\boldsymbol{p} v_x g_s$ and a_s is the normalization constant which is determined such that $\int \mathrm{d}\boldsymbol{p} g_s=\int \mathrm{d}\boldsymbol{p} g_{s0}=1$. Equation (3) is the solution of the unperturbed relativistic Vlasov equation

$$(\boldsymbol{E}_0 + \boldsymbol{v}\times\boldsymbol{B}_0/c)\cdot(\partial g_s/\partial \boldsymbol{p})=0 \quad , \tag{4}$$

which leads to the generalized Ohm's law for a collisionless plasma

$$\boldsymbol{E}_0 + \boldsymbol{v}_d\times\boldsymbol{B}_0/c = 0 \quad , \tag{5}$$

by means of the momentum-space integration of Eq. (4) multiplied by $\boldsymbol{p}$. This means that $\boldsymbol{E}_0=\boldsymbol{B}_0\times\boldsymbol{v}_d/c$ is produced via the dynamo effect of the cross-field particle drift arising from the acceleration due to electrostatic waves. Namely, $\boldsymbol{v}_d=c\boldsymbol{E}_0\times\boldsymbol{B}_0/B_0^{\,2}$ is identical to $\boldsymbol{E}\times\boldsymbol{B}$ drift. When $\boldsymbol{v}_d=0$, g_s is reduced to to g_{s0} being symmetric with respect to the magnetic field.

We can derive the relativistic quasilinear momentum-space diffusion equation by the same manner as the previous works[1-3] as follows:

$$\frac{\partial g_s}{\partial t} = \sum_{\boldsymbol{k}\neq 0} |E_{\boldsymbol{k}}|^2 Q_{\boldsymbol{k}} g_{s0} \quad , \tag{6}$$

with the θ-dependent momentum-space diffusion coefficient

$$Q_{\boldsymbol{k}} = \mathrm{Im}\left[\frac{e_s^{\,2}}{k^2}\sum_{n,r=-\infty}^{\infty} Y_{n,r}(\boldsymbol{k})\frac{e^{-i(r-n)\theta}J_r(\mu_{\boldsymbol{k}})U_r(\boldsymbol{k})}{k_\parallel v_\parallel + k_\perp v_d - \omega_{\boldsymbol{k}} + r\omega_{cs}}\right] \quad , \tag{7}$$

$$Y_{n,r}(\boldsymbol{k}) = U_n(\boldsymbol{k})J_n(\mu_{\boldsymbol{k}}) + \frac{k_\perp}{v_\perp}(r-n)J_n'(\mu_{\boldsymbol{k}}) \quad , \tag{8}$$

where $Q_{\boldsymbol{k}}$ is expressed in the displaced cylindrical coordinate in the momentum-space, and hence g_{s0} appears in the right-hand of Eq. (6) and J_n' is the first derivative of J_n with respect to the argument. The azimuthal dependence of $Q_{\boldsymbol{k}}$ represents the anisotropy of the momentum-space diffusion around the magnetic field and the resulting cross-field particle drift in the $k_\perp$-direction (x-direction).

2.2 Relativistic quasilinear transport equations

The momentum-space integration of the momentum-space diffusion equation multiplied by $w_s= n_s\gamma_s(1+ v_d v_\perp\cos\theta/c^2)m_s c^2$ or $\boldsymbol{p}_s=n_s\boldsymbol{p}$ leads to the relativistic quasilinear transport equations as described in Refs.1-3. Thus the transport equations indicating the temporal development of the energy and momentum densities of magnetized particles of s-species are given by

$$\frac{\partial U_s}{\partial t} = -\sum_{k\neq 0} 2\gamma_{\boldsymbol{k}}^{(s)} U_{\boldsymbol{k}} \quad , \tag{9}$$

$$\frac{\partial \boldsymbol{P}_s}{\partial t} = -\sum_{k\neq 0} \frac{2\gamma_{\boldsymbol{k}}^{(s)}\boldsymbol{k}}{\omega_{\boldsymbol{k}}} U_{\boldsymbol{k}} \quad , \tag{10}$$

where $U_{\boldsymbol{k}}=(1/8\pi)((\partial\varepsilon_{\boldsymbol{k}}'\omega_{\boldsymbol{k}})/\partial\omega_{\boldsymbol{k}})|E_{\boldsymbol{k}}|^2$ is the wave energy density, $\boldsymbol{k}U_{\boldsymbol{k}}/\omega_{\boldsymbol{k}}$ is the wave momentum density, $U_s=\int d\boldsymbol{p} w_s g_s$ and $\boldsymbol{P}_s=\int d\boldsymbol{p}\boldsymbol{p}_s g_s$ are the energy and momentum densities of particles of species s , $\gamma_{\boldsymbol{k}}^{(s)}=-\varepsilon_{\boldsymbol{k}}''^{(s)}/(\partial\varepsilon_{\boldsymbol{k}}'/\partial\omega_{\boldsymbol{k}})$ ($\gamma_{\boldsymbol{k}}=\sum_s\gamma_{\boldsymbol{k}}^{(s)}$) is the linear damping rate ascribed to particles of species s, and $\boldsymbol{P}_s=(P_{s\perp}, 0, P_{s\parallel})$, $P_{s\perp}=n_s p_{ds}$, $P_{s\parallel}=n_s\gamma_s m_s v_{s\parallel}$. The transport equations (9) and (10) predict clearly that the electrostatic waves can generate strong particle acceleration along and across the magnetic field via their Landau or cyclotron damping. The relation of $P_{s\parallel}/P_{s\perp}=k_\parallel/k_\perp$ ($\boldsymbol{P}_s \parallel \boldsymbol{k}$) is seen from (10) with $\boldsymbol{P}_s(0)=0$ and implies that the small parallel and large perpendicular particle acceleration appears simultaneously. Namely the same result as the nonrelativistic case was obtained[2-4].

In the absence of nonlinear wave-wave and wave-particle interaction the kinetic wave equation for the electrostatic waves becomes

$$\frac{\partial U_{\boldsymbol{k}}}{\partial t} = 2\gamma_{\boldsymbol{k}} U_{\boldsymbol{k}} \quad . \tag{11}$$

From Eqs. (9)-(11), as has been obtained similarly in the nonrelativistic case[2-4], the conservation laws for the total energy and momentum densities of waves and particles are provided as

$$\frac{\partial}{\partial t}\left(\sum_{k\neq 0} U_{\boldsymbol{k}} + \sum_s U_s\right) = 0 \quad , \tag{12}$$

$$\frac{\partial}{\partial t}\left(\sum_{k\neq 0} \frac{\boldsymbol{k}}{\omega_{\boldsymbol{k}}} U_{\boldsymbol{k}} + \sum_s \boldsymbol{P}_s\right) = 0 \quad . \tag{13}$$

It can be also verified by adding the appropriate nonlinear terms to Eqs. (9)-(11) that these conservation laws hold in the presence of nonlinear wave-wave and wave-particle interaction.

3. Relativistic Cross-Field Particle Acceleration

Next we investigate the relativistic cross-field particle acceleration in a relativistic magnetized plasma. The relativistic quasilinear transport equations in the laboratory (stationary) frame of reference C with the electric field and the $\boldsymbol{E}\times\boldsymbol{B}$ drift can be obtained by means of Lorentz transformation for the momentum-space integration of the relativistic quasilinear momentum-space diffusion equation in the frame of reference C' moving with the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity $\boldsymbol{v}_d$. The stationary electric and magnetic fields $\boldsymbol{E}_0$' and $\boldsymbol{B}_0$' in the moving frame of reference C' are deduced from Lorentz transformation and are given by

$$\boldsymbol{E}_0' = 0 \quad , \quad \boldsymbol{B}_0' = \boldsymbol{B}_0/\gamma_d \quad , \tag{14}$$

where $\gamma_d=(1-\beta^2)^{-1/2}$ and $\beta=v_d/c$. In the moving frame C' the stationary electric field vanishes and hence no $\boldsymbol{E}\times\boldsymbol{B}$ drift exists. Further the relation of the frequencies and wave numbers in C and C' is represented by $\tilde{k}_\perp = \gamma_d(k_\perp - \beta\omega_{\boldsymbol{k}}/c)$, $\tilde{k}_\parallel = k_\parallel$, $\tilde{\omega}_{\tilde{\boldsymbol{k}}} = \gamma_d(\omega_{\boldsymbol{k}} - k_d v_d)$ (or $k_\perp = \gamma_d(\tilde{k}_\perp + \beta\tilde{\omega}_{\tilde{\boldsymbol{k}}}/c)$, $k_\parallel = \tilde{k}_\parallel$, $\omega_{\boldsymbol{k}} = \gamma_d(\tilde{\omega}_{\tilde{\boldsymbol{k}}} + \tilde{k}_\perp v_d)$. The electrostatic waves propagating in C' become electromagnetic in C, that is, $\boldsymbol{E}_{\boldsymbol{k}}=(\tilde{E}^{x'}_{\tilde{\boldsymbol{k}}}, 0, \gamma_d\tilde{E}^{z'}_{\tilde{\boldsymbol{k}}})$ and $\boldsymbol{B}_{\boldsymbol{k}}=(0, -\beta\gamma_d\tilde{E}^{z'}_{\tilde{\boldsymbol{k}}}, 0)$. Thus the similar relativistic quasilinear transport equations in C can be derived as follows:

$$\frac{\partial U_{\mathrm{Rs}}}{\partial t} = -\sum_{\boldsymbol{k}\neq 0} 2\gamma^{(s)}_{\mathrm{Rs}}\, U_{\mathrm{R}\boldsymbol{k}} \quad , \tag{15}$$

$$\frac{\partial \boldsymbol{P}_{\mathrm{Rs}}}{\partial t} = -\sum_{\boldsymbol{k}\neq 0} \frac{2\gamma^{(s)}_{\mathrm{R}\boldsymbol{k}}\boldsymbol{k}}{\omega_{\boldsymbol{k}}} U_{\mathrm{R}\boldsymbol{k}} \quad , \tag{16}$$

where $U_{\mathrm{R}\boldsymbol{k}}=(1/8\pi)(\partial(\varepsilon'_{\mathrm{R}\boldsymbol{k}}\omega_{\boldsymbol{k}})/\partial\omega_{\boldsymbol{k}})|\tilde{E}_{\tilde{\boldsymbol{k}}}|^2$ and $\boldsymbol{k}U_{\mathrm{R}\boldsymbol{k}}/\omega_{\boldsymbol{k}}$ are the effective wave energy and momentum densities in C, respectively, U_{Rs} and $\boldsymbol{P}_{\mathrm{Rs}}$ are the energy and momentum densities of particles of species s in C, respectively, $\tilde{\boldsymbol{E}}_{\tilde{\boldsymbol{k}}}$ is the wave electric field in C', and $\varepsilon_{\mathrm{R}\boldsymbol{k}} = 1 + \sum_s \varepsilon^{(s)}_{\mathrm{R}\boldsymbol{k}} = \varepsilon'_{\mathrm{R}\boldsymbol{k}} + i\varepsilon''_{\mathrm{R}\boldsymbol{k}}$ is the dielectric constant in C' which is expressed in terms of $\boldsymbol{k}$ and $\omega_{\boldsymbol{k}}$. That is, $\varepsilon_{\mathrm{R}\boldsymbol{k}}$ is obtained by means of replacement of $k_\perp \to \gamma_d(k_\perp - \beta\omega_{\boldsymbol{k}}/c)$, $\omega_{\boldsymbol{k}} - k_\perp v_d \to \gamma_d(\omega_{\boldsymbol{k}} - k_\perp v_d)$ and $\omega_{cs} \to \omega'_{cs} = \omega_{cs}/\gamma_d$ in $\varepsilon_{\boldsymbol{k}}$ of Eq. (1). The linear damping rate in C is given by $\gamma^{(s)}_{\mathrm{R}\boldsymbol{k}} = -\varepsilon''^{(s)}_{\mathrm{R}\boldsymbol{k}}/(\partial\varepsilon'_{\mathrm{R}\boldsymbol{k}}/\partial\omega_{\boldsymbol{k}})$ ($\gamma_{\mathrm{R}\boldsymbol{k}} = \sum_s \gamma^{(s)}_{\mathrm{R}\boldsymbol{k}}$, $\varepsilon^{(s)}_{\mathrm{R}\boldsymbol{k}} = \varepsilon'^{(s)}_{\mathrm{R}\boldsymbol{k}} + i\varepsilon''^{(s)}_{\mathrm{R}\boldsymbol{k}}$). When the linear kinetic wave equation holds, we also find the same conservation laws as Eqs. (12) and (13).

4. Conclusion

It was verified theoretically that particle acceleration along and across the magnetic field and electric field across the magnetic field can be induced by almost perpendicularly propagating electrostatic waves in a relativistic magnetized plasma. The relativistic quasilinear transport equations show that the electrostatic waves produce strong particle acceleration in the $\boldsymbol{k}$-direction. Simultaneously the intense cross-field electric field $\boldsymbol{E}_0 = \boldsymbol{B}_0\times\boldsymbol{v}_d/c$ is created via the dynamo effect of the cross-field particle drift to satisfy the generalized Ohm's law. The obtained transport equations can be available for the theoretical investigation of the relativistic and nonrelativistic cross-field particle acceleration and transport which may occur possibly in space and fusion plasmas.

References

[1] R. Sugaya, J. Plasma Phys. **56**, 193 (1966); Phys. Plasmas **1**, 2768 (1996); Phys. Plasmas **3**, 3485 (1996).

[2] R. Sugaya, in *Proceedings of the Seventh European Fusion Theory Conference, Jülich,* Vol. 1, p. 207 (1997).

[3] R. Sugaya, in *Proceedings of International Congress on Plasma Physics combined with the 25th EPS Conference on Controlled Fusion and Plasma Physics, Praha,* Vol. 22C, p. 209 (1998).

[4] R. Sugaya, in *Proceedings of the 26th EPS Conference on Controlled Fusion and Plasma Physics, Maastricht,* Vol. 23 J, p. 497 (1999).