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Nonlinear neoclassical transport in toroidal edge plasmas

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Introduction

In this paper, the theory of neoclassical transport in an impure, collisional plasma with arbitrary cross section and aspect ratio is extended to allow for steeper pressure and temperature gradients than are usually considered in the conventional theory. This is an extension of recent work [1,2] to the Pfirsch-Schlüter (high-collisionality) regime, which frequently prevails in the tokamak edge. Contrary to conventional neoclassical theory, the gradients are allowed to be so large that the friction force between the bulk ions and heavy impurities is comparable to the parallel impurity pressure gradient. In this case the impurity ions are found to undergo a spontaneous rearrangement on each flux surface. This reduces their parallel friction with the bulk ions and causes the neoclassical ion flux to become a non-monotonic function of the gradients for plasma parameters typical of the tokamak edge. Thus, the neoclassical confinement is improved in regions where the gradients are large, such as in the edge pedestal. Most tokamak H-mode edge plasmas are in this nonlinear non-classical regime.

Conventional theory of neoclassical transport in tokamaks [3-5] is not applicable to the pedestal region at the plasma edge, where the pressure and temperature profiles are very steep. Neoclassical theory requires the expansion parameter $\delta \equiv \rho_\theta / L_\perp \ll 1$ where ρ_θ is the poloidal ion gyroradius and $L_\perp$ the radial scale length associated with the density and temperature profiles. When δ is infinitesimally small, all plasma parameters are approximately constant on flux surfaces. When δ is larger, poloidal asymmetries become possible [6-8]. Typically the first plasma parameter to develop a poloidal variation is the density, n_z, of highly charged impurity ions [9], whose poloidal modulation is of the order $\tilde{n}_z/n_z \sim \delta_z \equiv \delta \hat{\nu}_{ii} z^2$, where $\hat{\nu}_{ii} \equiv L_\| / \lambda_{ii}$ is the collisionality, with λ_{ii} the mean-free path for the bulk ions and $L_\|$ the connection length. In conventional neoclassical theory, δ_z is assumed to be small. Here, we adopt the ordering $\delta \ll 1, \delta_z = O(1)$, which is more realistic for the tokamak edge.

For simplicity, we restrict our attention to the case of a hydrogen plasma (i), with a single species of highly charged ($z \gg 1$) impurity ions. Both these species are taken to be collisional, as is typical of a tokamak edge plasma somewhat inside the last closed flux surface. We evaluate the particle and heat fluxes in two opposite limits: that of trace impurities, $Z_{\rm eff} - 1 = n_z z^2/n_i \ll 1$, and the Lorentz limit $n_z z^2/n_i \gg 1$. In the first case the impurities do not affect the kinetics of the bulk plasma, while in the second case the frequency of ion-impurity collisions exceeds that of ion-ion collisions.

Poloidal impurity distribution

The poloidal impurity distribution is governed by the parallel momentum equation

$$z n_z e \nabla_\| \tilde{\Phi} + T_i \nabla_\| n_z = R_{zi\|}, \tag{1}$$

where we have neglected inertia and parallel viscosity, being smaller than the parallel pressure gradient by the factor of $\delta/(z\hat{\nu}_{ii}) \ll 1$, [1]. From the impurity continuity equation,

$\nabla \cdot (n_z \boldsymbol{V}_z) = 0$, it follows that there is a parallel impurity return flow

$$V_{z\|} = -\frac{I}{B}\frac{d\Phi}{d\psi} + \frac{K_z B}{n_z}, \tag{2}$$

where K_z is proportional to the poloidal flow velocity and $\boldsymbol{B} = I\nabla\varphi + \nabla\varphi \times \nabla\psi$ is the magnetic field, so that ψ is the poloidal flux.

The ion-impurity parallel friction force $R_{zi\|}$ can be calculated from

$$R_{zi\|} = \int m_i v_\| C_{iz}(f_{i1})\, d^3v = \int m_i v_\| \nu_{iz}\left(f_{i1} - \frac{m_i v_\|}{T_i} V_{z\|} f_{i0}\right) d^3v, \tag{3}$$

where we assumed that the mass ratio is large, $m_z/m_i \gg 1$, and f_{i0} is Maxwellian.

Trace impurities in a collisional plasma

The bulk-ion distribution function is obtained following Hazeltine [10], by solving the drift-kinetic equation

$$v_\| \nabla_\| f_i + \boldsymbol{v}_D \cdot \nabla f_i = C_{ii}(f_i) \tag{4}$$

by a double expansion in two small parameters, δ and $\Delta \equiv \hat{\nu}_{ii}^{-1}$. To order $(\delta^1 \Delta^0)$ we have

$$f_{i1}^0 = -\frac{v_\| I}{15\Omega_i}\frac{d\ln T_i}{d\psi}(1-b^2)\left(x^2 + 2x^4 - 20\right) f_{i0} + \left(\frac{p_{i1}^0}{p_i} + \frac{m_i v_\| V_{i\|}}{T_i} + \frac{T_{i1}^0}{T_i}\left(x^2 - \frac{5}{2}\right)\right) f_{i0}, \tag{5}$$

where the subscript and superscript on f_i refer to the orderings in δ and Δ, respectively, $x = v/v_{Ti}$ with $v_{Ti} = (2T_i/m_i)^{1/2}$ and the magnetic field strength has been normalised so that $b \equiv B/\left\langle B^2\right\rangle^{1/2}$, where $\langle \ldots \rangle$ denotes flux-surface averaging.

Inserting this result in (3) gives

$$R_{zi\|} = \frac{m_i n_i}{\tau_{iz}}\left(V_{i\|} - V_{z\|} + \frac{25}{16}\frac{\tau_i \nabla_\| T_i}{m_i}\right), \tag{6}$$

where

$$V_{i\|} - V_{z\|} = -\frac{IT_i}{m_i \Omega_i}\left(\frac{d\ln p_i}{d\psi} + \frac{d\ln T_i}{d\psi}\left(1.8b^2 + 0.05\frac{b^2\left\langle\left(\nabla_\| \ln b\right)^2\right\rangle}{\left\langle\left(\nabla_\| b\right)^2\right\rangle}\right)\right) - \frac{K_z B}{n_z}. \tag{7}$$

It follows from the orderings that the ion temperature varies less over the flux surface than the magnetic field strength,

$$\nabla_\| T_i = 1.2\frac{IT_i}{\Omega_i \tau_i}\frac{d\ln T_i}{d\psi}(1 - b^2). \tag{8}$$

Using this result in the impurity parallel momentum equation (1) then gives an equation for the normalised impurity density $n = n_z/\langle n_z \rangle$

$$\frac{\partial n}{\partial \vartheta} = g\left(n - b^2 + \gamma\left(n - \left\langle nb^2\right\rangle\right) b^2\right), \tag{9}$$

where $\gamma = 2.8(\ln T_i)'/(\ln n_i)'$ is a constant, the poloidal angle has been normalised by $d\vartheta/d\theta \equiv \langle \mathbf{B}\cdot\nabla\theta\rangle/\mathbf{B}\cdot\nabla\theta$ and

$$g = -\frac{m_i n_i I}{e\tau_{iz} n_z \langle \mathbf{B}\cdot\nabla\theta\rangle}\left(\frac{d\ln p_i}{d\psi} - \frac{d\ln T_i}{d\psi}\right) = O(\delta_z)$$

measures the steepness of the bulk ion density and temperature profiles.

Equation (9) has the same form as was found in the mixed-collisionality regime, [1], only the constants g and γ are slightly different. As in the banana regime, the impurities accumulate on the inboard side of the tokamak when the gradients are steep, $g \gtrsim 1$, thereby reducing the impurity-ion parallel friction. Conventional neoclassical theory assumes $g \ll 1$ and therefore neglects these effects.

Lorentz limit: $Z_{\text{eff}} \gg 1$

In the Lorentz limit, impurity-ion collisions are more frequent than ion-ion collisions; $C_{ii} \ll C_{iz}$. The solution of the drift-kinetic equation

$$v_{\|}\nabla_{\|} f_i + \boldsymbol{v}_D\cdot\nabla f_i = C_{ii}(f_i) + C_{iz}(f_i) \simeq C_{iz}(f_i) \tag{10}$$

to order $(\delta^1\Delta^0)$ is then

$$f_{i1}^0 \simeq \frac{m_i v_{\|}}{T_i} V_{z\|} f_{i0} - \frac{4\tau_{iz}}{3\sqrt{\pi}} v_{\|} x^3 \left[\frac{\nabla_{\|}\tilde{p}_i}{p_0} + \frac{e\nabla_{\|}\tilde{\Phi}}{T_0} + \left(x^2 - \frac{5}{2}\right)\frac{\nabla_{\|}\tilde{T}_i}{T_0}\right] f_{i0}, \tag{11}$$

and the ion-impurity collision time is $\tau_{iz} = 3(2\pi)^{3/2}\epsilon_0^2 m_i^{1/2} T_i^{3/2}/n_z z^2 e^4 \ln\Lambda = (n_i/n_z z^2)\tau_{ii}$. This result can be used to calculate the ion-impurity friction force

$$R_{zi\|} = -\frac{75\pi}{512}\frac{m_i}{\tau_{iz}}\left[\frac{I p_i}{eB}\left(1 - \frac{b^2}{\langle nb^2\rangle}\right)\left(\frac{d\ln p_i}{d\psi} - \frac{3}{5}\frac{d\ln T_i}{d\psi}\right) + \frac{K_z n_i}{\langle n_z\rangle}\left(\frac{1}{\langle nb^2\rangle} - \frac{1}{n}\right) B\right]. \tag{12}$$

The remaining unknown quantity K_z governs the poloidal impurity rotation and is determined from the parallel viscosity, which is derived from f_{i1}^1 in the next order kinetic equation. The parallel viscosity constraint

$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\boldsymbol{\pi}}\right\rangle \simeq \left\langle (\nabla_{\|} B)\int m v^2 P_2(\xi) f_{i1}^1 d^3v\right\rangle = 0 \tag{13}$$

where $P_2(\xi)$ denotes the Legendre polynomial $P_2(\xi) = (3\xi^2 - 1)/2$, with $\xi = v_{\|}/v$, gives

$$K_z = -\frac{I\langle n_z\rangle T_0}{\langle B^2\rangle e}\left(\frac{d\ln p_i}{d\psi} + \frac{d\ln T_i}{d\psi}\right). \tag{14}$$

Inserting these results in (1) and using quasineutrality, $n_e = n_i + z n_z$, gives the equation for the poloidal impurity distribution

$$(1 + \alpha n)\frac{\partial n}{\partial\vartheta} = \hat{g}\left[n\left(1 - \frac{b^2}{\langle nb^2\rangle}\right) + \hat{\gamma}\left(1 - \frac{n}{\langle nb^2\rangle}\right) b^2\right]. \tag{15}$$

Here $\alpha \equiv \langle Z_{\text{eff}} - 1\rangle T_e/(T_e + T_i)$, $\hat{\gamma} = ((\ln p_i)' + (\ln T_i)')\,/\,((\ln p_i)' - (3/5)(\ln T_i)')$ and

$$\hat{g} = -\frac{75\pi}{512}\frac{m_i n_i I}{e\tau_{iz} n_z \langle \mathbf{B}\cdot\nabla\theta\rangle}\left(\frac{d\ln p_i}{d\psi} - \frac{3}{5}\frac{d\ln T_i}{d\psi}\right) = O(\delta_z).$$

Neoclassical transport

We now proceed to solve (9) and (15) and evaluate the classical and neoclassical particle fluxes

$$\left\langle \left(\mathbf{\Gamma}_i^{\rm cl} + \mathbf{\Gamma}_i^{\rm neo} \right) \cdot \nabla\psi \right\rangle = \left\langle \frac{R^2 \nabla\varphi \cdot \left(\mathbf{R}_{iz_\perp} + \mathbf{R}_{iz\|} \right)}{eB} \right\rangle .$$

The numerical solution of (15) for different magnetic equilibria (START, Alcator C-Mod and large aspect ratio tokamaks) indicates that for large gradients ($g \gtrsim 1$) the impurities undergo a poloidal redistribution. In the case of trace impurities, $\alpha \ll 1$, the impurities are pushed to the inboard side of the flux surface, while in the opposite limit, $\alpha \gg 1$, they accumulate on the outboard side of the flux surface. This result is similar to the one obtained in the mixed-collisionality regime [1,2], except for the sign of the in-out asymmetry in the Lorentz limit.

Using the friction force $R_{zi\|}$ from Eq (3) gives the average neoclassical particle flux across a flux surface

$$\langle \mathbf{\Gamma}_i^{\rm neo} \cdot \nabla\psi \rangle = \frac{I \langle p_z \rangle \langle \boldsymbol{B} \cdot \nabla\theta \rangle}{e \langle B^2 \rangle} \left\langle \frac{(1+\alpha n)}{b^2} \frac{\partial n}{\partial \vartheta} \right\rangle . \tag{16}$$

In a plasma with small inverse aspect ratio, $\epsilon \ll 1$, and circular cross section, (15) can be solved by making the expansions $b^2 = 1 - 2\epsilon\cos\theta + O(\epsilon^2)$, $n = 1 + n_c \cos\theta + n_s \sin\theta + O(\epsilon^2)$, The particle flux then becomes

$$\left\langle \left(\mathbf{\Gamma}_i^{\rm cl} + \mathbf{\Gamma}_i^{\rm neo} \right) \cdot \nabla\psi \right\rangle = \frac{\epsilon^2 p_z}{q^3 e} \left[1 + \frac{2q^2}{1 + g^2\gamma^2/(1+\alpha)^2} \right] g, \tag{17}$$

where the first term is the classical flux (which is not much affected by the impurity redistribution) and the second term represents the neoclassical flux. The latter exceeds the former by the Pfirsch-Schlüter factor $2q^2$ when the gradients are weak, $g \ll 1$. If the pressure gradient becomes sufficiently steep ($g \gg 1$) the neoclassical flux is suppressed since the denominator in the second term of Eq (17) depends quadratically on g. Classical transport then dominates, and the total flux is a non-monotonic function of the gradients, which gives rise to the possibility of a transport bifurcation. That this is the case in the mixed-collisionality regime was found in Refs. [1,2] and here we conclude that it is not affected by the collisionality.

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