

Thermal Equilibrium of Annular Nonneutral Plasmas

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A peculiar characteristic of one component plasmas is that they can approach thermodynamic equilibrium while remaining confined by static electric and magnetic fields in Penning-Malmberg traps [1]. For the confinement geometry of these experiments, the class of thermal equilibrium density distributions which are consistent with Poisson's equation has been determined in Ref. [2], and has been extensively discussed in Ref. [3]. Here, we consider a different thermal equilibrium: a hollow plasma column in a Penning-Malmberg trap where a conducting axial rod, raised at a given potential, is present. In this system new plasma configurations occur with cross section shapes ranging from almost circular to very elongated. For definiteness, we consider a simple configuration where a pure electron plasma is confined in a Penning-Malmberg trap with one central grounded conducting cylinder (of radius R) and two end plug cylinders at negative potential, as shown in Fig. 1. A uniform magnetic field B is directed along the axis of the cylinder where a central rod (of radius r_d) is present. Taking into account the cylindrical symmetry of the system, the thermal equilibrium electron density, n_e , obeys the Boltzmann's law

$$n_e = n_{e0} \exp \left[-\frac{1}{T_e} \left(-e\varphi + \frac{m_e}{2} \omega_E (\Omega - \omega_E) r^2 \right) \right]. \quad (1)$$

Here φ represents the electrostatic potential, $\Omega = eB/m_e c$ is the electron cyclotron frequency, ω_E the azimuthal plasma rotation frequency, and the other notations are standard. The parameters n_{e0} , T_e , and ω_E are constant. They are determined by the total number of particles, the total energy, and the total canonical angular momentum of the system. The electrostatic potential must be determined self-consistently from the Poisson's equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2} = 4\pi e n_e. \quad (2)$$

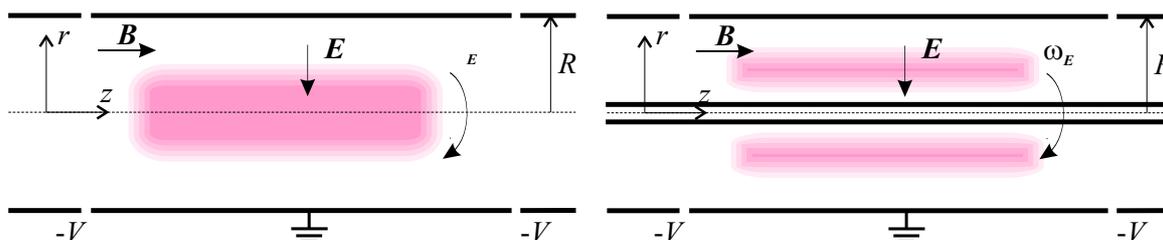


Figure 1: Schematics of a Penning-Malmberg trap: **Left**) without central rod, **Right**) with central rod.

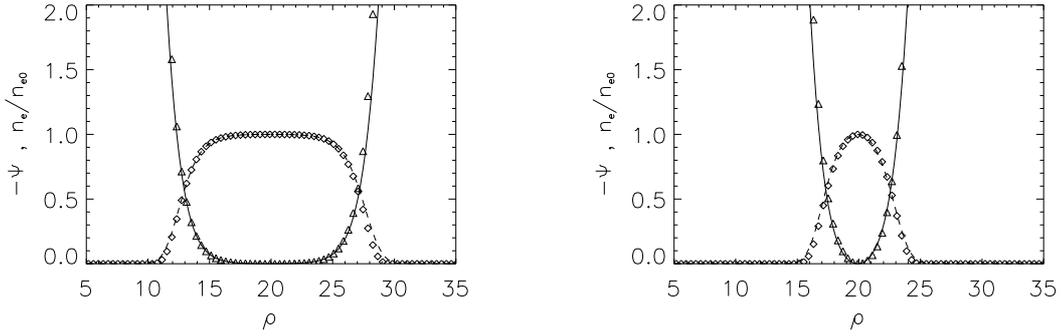


Figure 2: Plot of the normalized effective potential $-\psi$ (solid line) and the normalized electron density $n_e/n_{e0} = \exp \psi$ (dashed line) vs normalized radius: **Left**) $\bar{\rho} = 20$, $\gamma = 10^{-3}$; **Right**) $\bar{\rho} = 20$, $\gamma = 10^{-1}$. The approximate analytical solution (see Eq. (4)) is shown by triangles and squares, respectively.

This equation must be solved subject to the boundary conditions on the walls of the cylinder and on the central rod. In terms of the dimensionless (effective) potential $\psi = e\varphi/T_e - m_e\omega_E(\Omega - \omega_E)r^2/2T_e$, and dimensionless coordinates $\rho = r/\lambda_D$, $\zeta = z/\lambda_D$, Eq. (2) takes the form

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \psi}{\partial \zeta^2} = e^\psi - 1 - \gamma, \quad (3)$$

where $\lambda_D = \sqrt{T_e/4\pi e^2 n_{e0}}$, $\gamma = 2\omega_E(\Omega - \omega_E)/\omega_p^2 - 1$, and $\omega_p = \sqrt{4\pi e^2 n_{e0}/m_e}$. Since the electron density is given by $n_e = n_{e0} \exp(\psi)$, the plasma is located near the maximum of the effective potential ψ . Assuming the maximum value equal to 0, the parameter n_{e0} represents the maximum value of the plasma density.

We first consider the one dimensional equilibrium neglecting the ψ dependence on ζ . The maximum electron density will be located at some radius $\bar{\rho} > 0$, where $\psi' = 0$ (the prime denotes the derivative with respect to ρ). A solution of Poisson's equation can be found using $\psi(\bar{\rho}) = 0$, $\psi'(\bar{\rho}) = 0$ as "boundary" conditions. A localized solution exists for $\gamma > 0$. Its behavior depends on the two dimensionless parameters γ and $\bar{\rho}$. Numerical solutions for given values of γ and $\bar{\rho}$ are presented in Fig. 2. In general, it turns out that the density profile becomes step-like as $\gamma \rightarrow 0$, while it becomes smoother as γ is growing. Density profiles with an annulus width of the order of many Debye lengths λ_D occur if $\gamma \rightarrow 0$. In this case, it results

$$\psi \approx \gamma \left[1 - \frac{K_1(\bar{\rho})I_0(\rho) + I_1(\bar{\rho})K_0(\rho)}{K_1(\bar{\rho})I_0(\bar{\rho}) + I_1(\bar{\rho})K_0(\bar{\rho})} \right]. \quad (4)$$

If $\bar{\rho} \rightarrow 0$, this solution recovers that found in Ref. [2]: $\psi \approx \gamma[1 - I_0(\rho)]$.

In the case of finite length plasma, to find a thermodynamic equilibrium solution the self-consistent two-dimensional nonlinear Poisson's equation (2) has to be solved numerically. An axisymmetric half-equilibrium with reflection symmetry about the midplane $z = 0$ is considered here, with the following boundary conditions for the electrostatic potential: $\varphi(r_d, z) = V_{rod}$ on the central rod, $\partial\varphi/\partial z = 0$ at the midplane and at the end plate. On the conducting external wall, $\varphi(R, z) = V_{wall}$ except at the location of the plug section ($z_{plug1} \leq z \leq z_{plug2}$) where $\varphi(R, z) = V_{plug}$. Eq. (3), written in terms of the

dimensionless effective potential ψ is solved numerically using finite differences with an overrelaxation method. The boundary conditions for the effective potential ψ are easily computed from those relevant to φ by means of the relation $\psi = e\varphi/T_e - (\gamma + 1)\rho^2/4$.

In the situation treated here, the two-dimensional solution depends on γ (which represents a combination of the characteristic frequencies of the system: cyclotron frequency, plasma frequency, azimuthal rotation frequency), R/λ_D and r_d/λ_D , i.e., the normalized internal radii of the two cylindrical conductors, $z_{\text{plug1}}/\lambda_D$, i.e., the axial position of the plug cylinders, and $e(V_{\text{plug}} - V_{\text{wall}})/T_e$ and $e(V_{\text{rod}} - V_{\text{wall}})/T_e$, i.e., the (normalized) bias voltage on the internal conductor and the confining potential on the plug cylinders (assuming as reference the potential on the external conductor). In the numerical simulations, the (normalized) position $\bar{\rho}$ of the radial maximum of the density in the midplane has been fixed, instead of assigning the potential on the central rod. The values assigned to V_{rod} and V_{wall} are obtained from the one-dimensional solution for ψ . A few results of the computations are shown in Figs. 3–4. In general the one-dimensional solution provides a really good approximation to the density profile $n_e(r, 0)$ in the midplane of the trap, computed with the two-dimensional relaxation procedure, as shown in Fig. 3 on the right. Deviations of the central density profile from that predicted by the one-dimensional theory can be observed only for very short plasma clouds, as shown in Fig. 4. In general, while the value of $e(V_{\text{plug}} - V_{\text{wall}})/T_e$ is related with the radial position of the plasma, the value of $e(V_{\text{rod}} - V_{\text{wall}})/T_e$ turns out to influence the end profile shape of the plasma. As in the one-dimensional case, the radial profile of the density is strongly influenced by the value of the parameter γ . In particular, it is still true that the density profile becomes step-like as $\gamma \rightarrow 0$, while it becomes smoother as γ is increasing. A nonneutral plasma in the state of global thermodynamic equilibrium is to be stable against any type of *small* perturbations. This is valid in particular for the diocotron instability. Earlier experiments found an exponentially unstable $m = 1$ diocotron mode for an hollow electron column [5]. In fact, a study devoted to the analysis of the diocotron instability in nonneutral annular plasmas [6] contained in a trap with a biased central conductor already identified a parameter region where all azimuthal modes are stable. In fact, the state of global thermodynamic equilibrium of an annular nonneutral plasma studied here turns out to be located well inside this region (to the left of the unstable region shown in the Fig. 7 of Ref. [6]).

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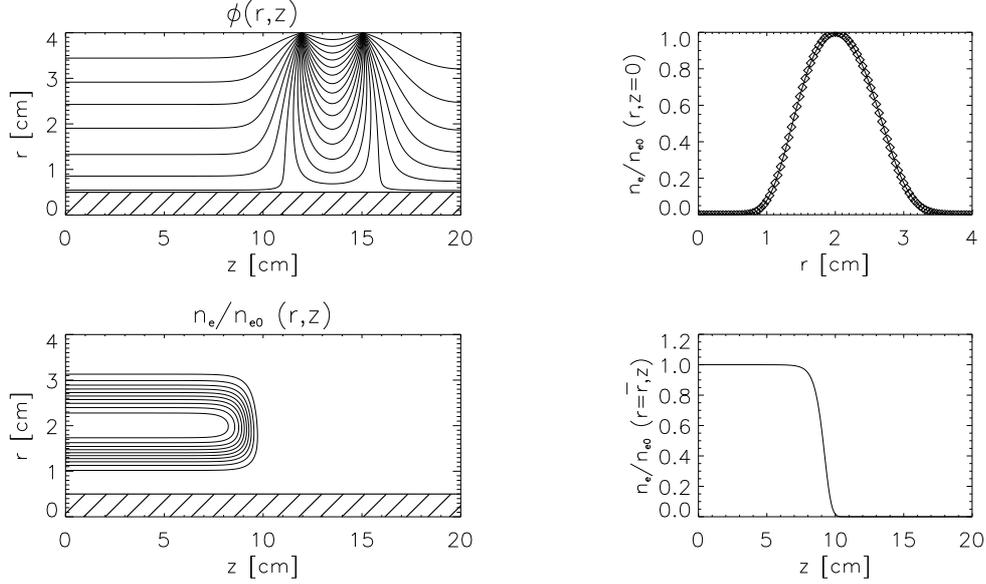


Figure 3: **Left)** Contour plot of φ (top) and normalized plasma density (bottom). The area filled with a pattern of lines at 45 degrees denotes the region occupied by the central rod. The parameters are: $\lambda_D = 0.35$ cm, $T_e = 1.0$ eV, $\gamma = 0.3$, $r_d = 0.5$ cm, $R = 4.0$ cm, $z_{\text{plug1}} = 12$ cm, $z_{\text{plug2}} = 15$ cm, $V_{\text{wall}} = 32.16$ V, $V_{\text{rod}} = -8.31$ V ($\bar{r} = 2.0$ cm), $V_{\text{plug}} = -100$ V. **Right)** Radial density profile (top), and axial density profile (bottom) for the radius of maximum central density, $r = \bar{r}$. The one-dimensional approximation to the central density profile is denoted by small squares.

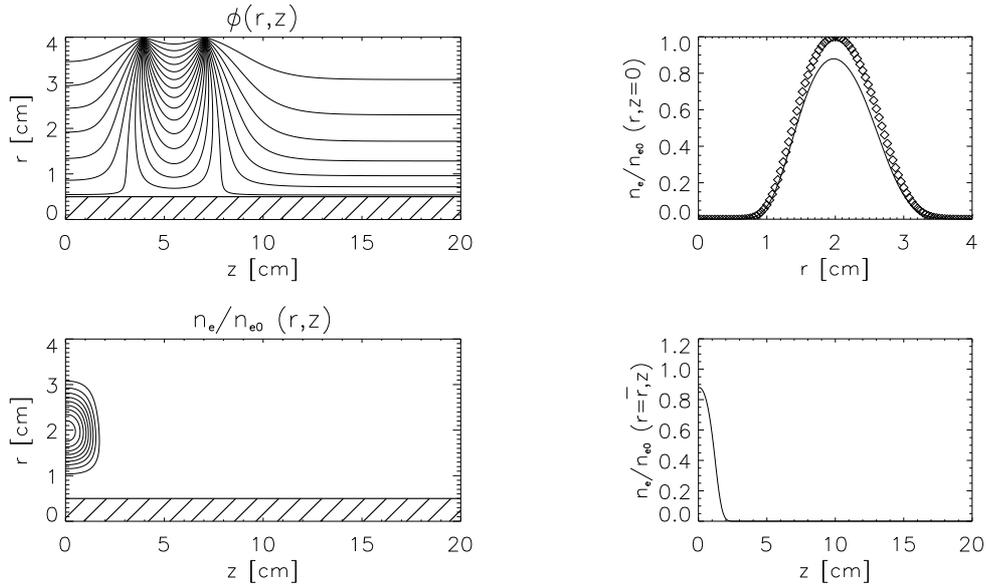


Figure 4: The same as in Fig. 3. The parameters are the same as in Fig. 3 except for the position of the plug cylinder: $z_{\text{plug1}} = 4$ cm, $z_{\text{plug2}} = 7$ cm.