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Collisionless reconnection in the whistler frequency regime

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In the last years processes occurring at spatial scales below the ion skin depth and on time scales faster than the ion cyclotron period have been studied extensively. Ions are known not to contribute significantly to the plasma dynamics, which is mainly determined by the electron motion. With the additional assumption that the charge separation effects and the displacement current can be neglected, this regime is generally known as electron-magnetohydrodynamics [1]. In this high frequency regime, plasmas are normally collisionless or weakly collisional. The EMHD regime appears to be very interesting for magnetic field line reconnection [2]-[3] because, in the presence of very sharp current gradients below the ion skin depth, the reconnection instability develops characteristically on fast electron dynamical time scales. Moreover, EMHD reconnection is attractive because of its single species nature. This makes it possible to extend into the kinetic regime the investigation of magnetic reconnection in this high frequency range using a single species fully kinetic Vlasov code, which avoids the problem of having to resolve both the electron and the ion time and length scales. In spite of its theoretical interest, the EMHD reconnection instability has been mostly investigated within the restricted framework of the standard incompressible fluid "tearing-like" regime.

The simplest configuration in which reconnection can be studied is the slab model, where curvature effects are neglected and a Cartesian, one dimensional geometry is assumed. The equilibrium magnetic field has a Harris-like form $\mathbf{B}_0(x) = B_{0\mathrm{y}}(x)\mathbf{e}_\mathrm{y} + B_{0\mathrm{z}}\mathbf{e}_\mathrm{z}$, where the shear field $B_{0\mathrm{y}}(x) = B_\infty F(x/L_\mathrm{B})$ is described by the odd profile function F that has a null line at $x = 0$ and saturates $[F(\pm\infty) = \pm 1]$ when its argument goes to infinity. Here L_B is the characteristic inhomogeneity scale length around the null line and B_∞ is the asymptotic value of the shear magnetic field. Since the ions are immobile, the whole plasma current is borne by the electrons which causes an equilibrium electron flow in the z direction, related to the shear of the magnetic field in the y direction. The equilibrium poloidal field is an important parameter in determining the reconnection rate, because it couples with the density perturbations.

In MHD reconnection, plasma quasi-neutrality can always be preserved, even if compressibility effects are taken into account. On the contrary, since the ions are static, in EMHD electron compressibility corresponds to a violation of quasi-neutrality and *vice versa*. Thus neglecting electron compressibility effects completely is consistent only if the Langmuir frequency is much larger than the electron cyclotron frequency, i.e., if the plasma is not strongly magnetized. If a strong component of the magnetic field in the z direction is present, small density fluctuations can reduce the growth rate of EMHD reconnection instability. The differential system for the high frequency reconnection involves the $\tilde{B}_\mathrm{x}$ and $\tilde{B}_\mathrm{z}$ components of the magnetic field

$$\gamma[1 + \lambda_\mathrm{e}^2(k^2 - \frac{d^2}{dx^2})]\tilde{B}_\mathrm{z} = -F(x)(k^2 - \frac{d^2}{dx^2})\tilde{B}_\mathrm{x} - F''(x)\tilde{B}_\mathrm{x}\ , \qquad (1)$$

$$\gamma[1 + d_{\rm e}^2(k^2 - \frac{d^2}{dx^2})]\tilde{B}_{\rm x} = k^2[F(x) - d_{\rm e}^2 F''(x)]\tilde{B}_{\rm z} \ , \tag{2}$$

where lengths are adimentionalized to $L_{\rm B}$ and times to the whistler time scale $\tau_{\rm W} = L_{\rm B}^2/\alpha B_\infty$ ($\alpha = c/4\pi n_{0{\rm e}} e$ is the Hall constant). Two length scales appear in the EMHD eigenmode equations taking into account charge separation: the skin depth $d_{\rm e} = c/\omega_{pe}$ and the "renormalized skin depth" $\lambda_{\rm e}^2 \equiv d_{\rm e}^2(1 + \omega_{\rm ge}^2/\omega_{\rm pe}^2) \geq d_{\rm e}^2$, introduced by Kuvshinov *et al.* [4]. Obviously, if $\omega_{\rm ge} \lesssim \omega_{\rm pe}$, we have $\lambda_{\rm e} \gtrsim d_{\rm e}$ and the two scales are of the same order of magnitude (weakly magnetized case). On the contrary, if the plasma is strongly magnetized $\omega_{\rm ge} \gg \omega_{\rm pe}$, the two scales are quite different ($\lambda_{\rm e} \gg d_{\rm e}$), leading to a different ordering of the terms in Eq. (1) and (2).

We use a boundary layer approach to solve the system of Eqs. (1) and (2) in the incompressible case ($\lambda_{\rm e}/d_{\rm e} = 1$) in the boundary layer limit on the full range of k values, even outside the region of validity of the constant-ψ solution (see Ref. [5]). For the sake of simplicity, we use a simple linear form for the equilibrium magnetic field profile [$F(x) = x$ for $|x| < 1$ and $F(x) = \text{sgn}\ x$ for $|x| > 1$]. Following the standard singular perturbation technique with respect to the small parameter $d_{\rm e}$, we separate the problem in the external problem, where $\partial_{\rm x} \sim 1$ and $d_{\rm e}^2\,(k^2 - \partial_{\rm x}^2) \ll 1$ and the internal problem, where $d_{\rm e}$ is retained but the profile of the equilibrium magnetic field is linearized.

In the external region the differential equation to be solved is

$$\tilde{B}_{\rm x}'' - \left[k^2 + \frac{F''(x)}{F(x)} + \frac{(\gamma/k)^2}{F^2(x)}\right]\tilde{B}_{\rm x} = 0 \ . \tag{3}$$

The solution of the internal problem must be matched with the small x behavior of Eq. (3), which is of the form $B_{\rm x}(x) \sim |x|^{\alpha_+} + (1/2)\Lambda\,(k, \gamma/k)\,|x|^{\alpha_-}$, where the relation $\alpha_+ + \alpha_- = 1$ holds. Λ is a parameter reminiscent of the Δ' parameter of the constant-ψ solution and leading back to it in the case that the ratio γ/k can be neglected in the external region.

In the internal region we linearize the equilibrium magnetic field $F(x) \approx x$ and neglect y derivatives with respect to x derivatives. It is well recognized that these differential systems can be more easily solved in Fourier space. The differential system reduces to

$$\frac{d}{dq}\left(\frac{q^2}{1+q^2}\frac{d\tilde{B}_{\rm z}}{dq}\right) = \left(\frac{\gamma}{k}\right)^2 \left(1 + q^2\right)\tilde{B}_{\rm z} \ . \tag{4}$$

This differential equation is quite complex, and can be solved only with an approximation of the coefficients in the q-space. The matching of this asymptotic behavior gives the dispersion relation for EMHD reconnecting modes.

In Fig. 1a various plots of this analytical dispersion relation for various values of the electron skin depth $d_{\rm e}$ are shown. In the boundary layer approach the growth rate is an increasing function of $d_{\rm e}$ and, when $d_{\rm e}$ is hold fixed, the dispersion relation $\gamma = \gamma\,(k)$ is a one humped curve with a maximum for a certain k_0 depending on $d_{\rm e}$. When $k > k_0$ there is the constant-ψ regime, while for $k < k_0$ there is the low-k regime for EMHD tearing mode. The growth rate for the constant-ψ regime is exactly recovered in the framework of our approximation of the internal equations [$\gamma\tau_{\rm W} \sim (d_{\rm e}\Delta')^2$]. In the opposite limit, taking γ/k fixed and small k we obtain $\gamma\tau_{\rm W} \sim (d_{\rm e}/L_{\rm B})^{2/3}\,kL_{\rm B}$. With respect to MHD magnetic reconnection due to electron inertia, in EMHD the growth rates are more fast and show a more rapid scaling with respect to the inertial skin depth $d_{\rm e}$ both in the constant-ψ and

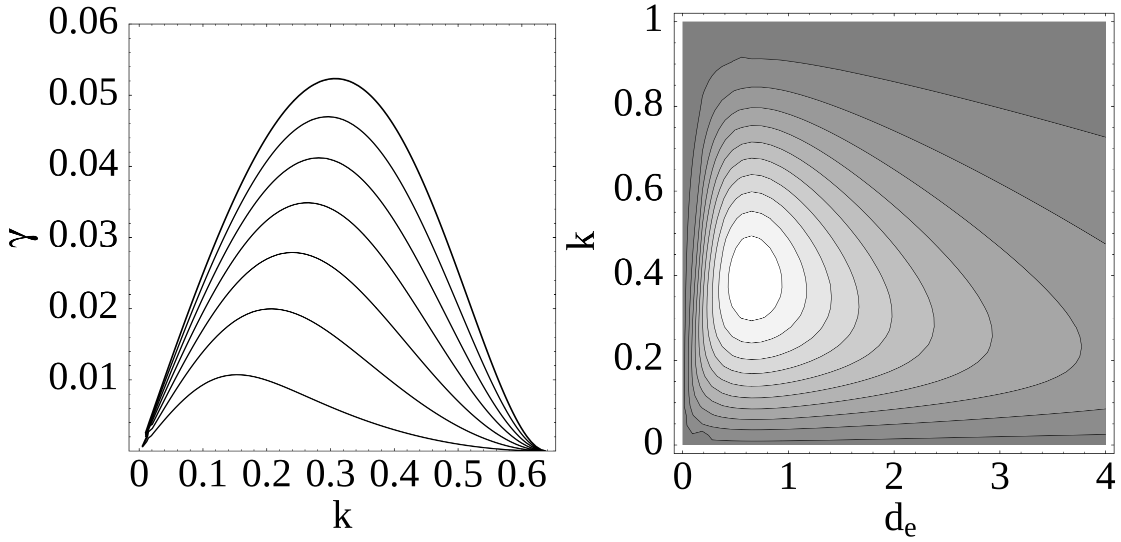

Figure 1: *(a) $\gamma = \gamma(k)$ from the boundary layer theory (d_e from 0.1 (lower curve) to 0.7 (upper curve) with step 0.1). (b) Contour-plot of $\gamma(d_e, k)$ from the numerical solution.*

in the low-k regime. MHD predicts $\gamma \sim d_e^3$ in the constant-ψ, and $\gamma \sim d_e$ in the low-k regime, while EMHD gives $\gamma \sim d_e^2$ in constant-ψ and $\gamma \sim d_e^{2/3}$ in low-k.

We solve numerically the differential system obtained from Eq. (1) and (2) with the same relaxation code used in Ref. [6]. This code uses a refinement of the integration step in the regions where strong spatial gradients develop. In order to verify the analytical scaling laws and for the sake of (numerical) simplicity we take the equilibrium magnetic field profile $F(x) = \tanh(x)$. The dependence of the eigenvalue $\gamma(d_e, k)$ on the parameters d_e and k is determined within the rectangular region with d_e ranging from 0.01 to 4.0 and k from 0.01 to 0.99. The contourplot of $\gamma(d_e, k)$ obtained from the numerical calculations is shown in Fig. 1b. We see that γ is a single-maximum function with the maximum in $k^* \sim 0.40$ and $d_e^* \sim 0.65$, where $\gamma^* \sim 0.068$. Scaling laws in the boundary layer regime are also verified.

The introduction of a strong perpendicular component of the magnetic field ($\omega_{ge} \gg \omega_{pe}$) force us to consider the effect of charge-separation. These effects leads to a more difficult analytical problem in comparison with the standard EMHD reconnection eigensystem because two length scales are involved, making a numerical treatment necessary in general. We solve the differential system with $\lambda_e/d_e = 20$ (corresponding to $\omega_{pe}/\omega_{ge} = 4.35$) and we compare this solution with the solution obtained in the incompressible case [7]. We find that in the constant-ψ regime discussed in Ref. [2] the eigenvalue is reduced while the width of the eigenmodes is essentially the same in the compressible and in the incompressible case. This behavior changes outside the region in which the constant-ψ approximation holds as well as where the boundary layer approach does not apply.

First, we consider the eigenvalue $\gamma(d_e, k)$, as obtained numerically both in the compressible and incompressible case. In both situations, a single-maximum surface is obtained, where the maximum corresponds to the most unstable mode as a function of k and d_e. In the strongly magnetized case a general reduction of the mode growth rate occurs over all the parameter space. In Fig. 2a the dispersion relation $\gamma(d_e, k)$ at fixed d_e shows this reduction. The ratio of the compressible and of the incompressible rate shows that the

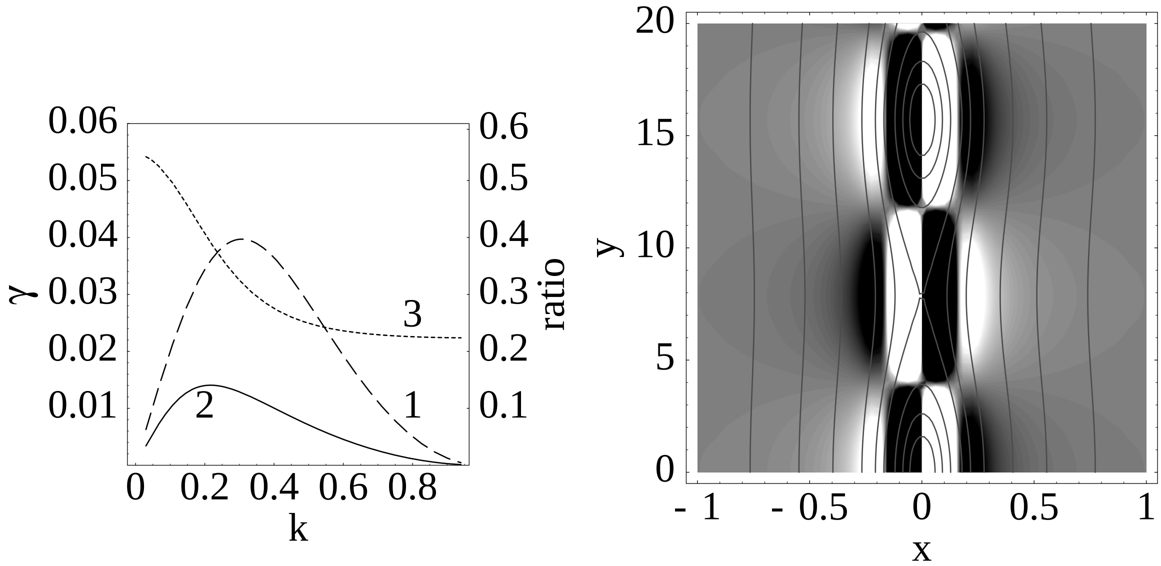

Figure 2: *(a) Stabilization of the growth rate (*$d_e = 0.2$*) (1) incompressible (2) compressible (3) ratio. (b) Structure of the density perturbations with* $d_e = 0.2$ *and* $k = 0.4$ *and magnetic island.*

stabilizing effect of electron compression is more important in the low-k regime and become weaker toward the constant-ψ region. The compressible width is always larger than the incompressible one, as expected because of the electron skin depth renormalization in Eq. (1), aside for the constant-ψ regime where the same values are obtained as found analytically. In Fig. 2b the spatial structure of the density perturbation (shading) in the poloidal plane is shown and compared with the island structure of the magnetic field for $d_e = 0.2$ and $k = 0.4$. The charge separation effects are relevant only in a region close to the null line of the sheared magnetic field and have the same width of the magnetic field perturbations. As is clearly seen from Fig. 2b, the density perturbations have a bipolar structure around the X and O points, with alternate polarities. A secondary spatial oscillation in the density distribution, with polarity opposite the main one, is present in the external tail of the density perturbation.

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