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Propagation and Damping of Long MHD Waves In a Strongly Non-uniform Plasma

E. Lazzaro[1], M.Lontano[1], D.Ryutov[3], P. Platania[2]

[1]*Istituto di Fisica del Plasma -CNR -Via R.Cozzi 53, 20125 Milan, Italy*
[2]*Dipartimento di Fisica Universita' degli Studi,Via Celoria 16 20133 , Milan , Italy*
[3]*Lawrence Livermore National Laboratory, Livermore CA 94551, U.S.A.*

1- INTRODUCTION

In the solar photosphere the magnetic field is concentrated into narrow flux tubes (~100 km, B up to 1 kG) (*convective collapse*, [1]) and in general in the solar atmosphere the magnetic structures can be classified in two main categories: i) structures with small filling factor (pores, knots, spiculae, ...) with a typical space scale distance of magnetic flux tubes d >> a, the typical scale width of the flux tubes ; ii) structures with large filling factor (faculae, sunspots, both umbrae and penumbrae, prominences,...) and typical space scale $d \geq a$. Consequently for propagation of e.m. waves of long wavelength λ,such a medium can be modelled as strongly inhomogeneous and practically random. The problem of the propagation of long waves in a medium with small-scale nonuniformities has a long history and a broad range of applications: i) elastic waves in polycrystals [2]; ii) sound in bubbly liquid [3]; iii) sound in dusty (snowy, rainy) air; iv) MHD waves in a plasma with magnetic filaments [4]. A basic approach to treat this type of problem is the multiple scattering technique [5] but in the present work, focussed on the propagation and absorption of long MHD waves in a medium with random magnetic filaments, we adopt an alternative technique of "early averaging". In the next section the model and procedure is presented, and in the third section the results obtained are discussed.

2-Strongly inhomogeneous magnetized plasmas

The stationary unperturbed state in a one-dimensional case is defined by the constant sum of magnetic and gas-kinetic pressure:

$$P_0(x) = p_{m0}(x) + p_{g0}(x) = P_0 = \text{const.} \quad (1)$$

where $p_{g0}(x) = n_0(x)\cdot T_0(x)$ and $p_{m0}(x) = B_0^2(x)/8\pi$

It is fundamental to note that while P_0 is constant, $p_{g0}(x)$ and $p_{m0}(x)$ may present fluctuations of 100% over the small scale a (i.e., dimension of a magnetic filament).
We consider "slow" MHD perturbations of the system with :
i) slow compression at $\omega = 2\pi s/\lambda << s/a$, where $s \approx \sqrt{p_g/\rho}$ is a sound speed;
ii) long wave perturbations, with $\lambda/2\pi >> a$, at t=0 and subsequent free evolution of the system.
It follows that for slow quasistatic modes δP is almost uniform, but since the gas and magnetic field have different compressibilities, δn and δB may vary by 100 % over a.
The strong gradients of $\delta n, \delta T$ and δB affect the propagation and dissipation of long waves.
The 1-D model is obtained from the single fluid MHD equations for the mass density $\rho = M\cdot n$,the velocity U, the magnetic field B and the temperature T, including energy dissipation due to the finite thermal conductivity:

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{\partial U}{\partial x}; \quad \rho\frac{dU}{dt} = -\frac{\partial P}{\partial x}; \quad \frac{1}{B}\frac{dB}{dt} = -\frac{\partial U}{\partial x}; \quad \frac{1}{\gamma-1}\frac{1}{T}\frac{dT}{dt} - \frac{1}{\rho}\frac{d\rho}{dt} = -\frac{1}{c_V(\gamma-1)}\frac{1}{\rho T}\frac{\partial}{\partial x}\left(\chi\frac{\partial T}{\partial x}\right)$$

If it is assumed that the plasma electrical conductivity is sufficiently high to freeze the magnetic field into the plasma we have $\frac{\delta p_m}{p_m} = 2\frac{\delta\rho}{\rho}$ and if the thermal conductivity is low

enough the wave motion is *adiabatic* ($\omega >> 2\pi\cdot\chi/a^2 >> 2\pi\cdot\chi/\lambda^2$), therefore $\dfrac{\delta p_g}{p_g} = \gamma\dfrac{\delta\rho}{\rho}$.
Consequently the density and the *adiabatic* temperature response to the total pressure perturbation δP is given by: $\dfrac{\delta\rho}{\rho} = \dfrac{\delta P}{P}\dfrac{1}{\gamma + (2-\gamma)p_m/P}$ and $\dfrac{\delta T}{T} = \dfrac{\delta P}{P}\dfrac{\gamma - 1}{\gamma + (2-\gamma)p_m/P}$, since p_m varies on the small scale $a << \lambda/2\pi$ by the order of one. These steep temperature gradients cause an enhanced dissipation:

$$q_T = \frac{\chi}{T}\left\langle(\nabla T)^2\right\rangle = \frac{\chi T}{2}\frac{(\gamma-1)^2}{\gamma^2}\frac{\delta P_0^2}{\left[P + \left(\frac{2}{\gamma}-1\right)p_m\right]^2}\left\{k^2 + \left\langle\left(\frac{2}{\gamma}-1\right)^2(\nabla p_m)^2\left[P + \left(\frac{2}{\gamma}-1\right)p_m\right]^{-2}\right\rangle\right\}$$

where the first term of the dissipated heat in the brackets $\{...\}$ scales $\propto(\lambda/2\pi)^{-2}$ while the second scales $\propto a^{-2}$

3-Magnetoacoustic waves in two dimensional stratified plasmas

Consider quasi isothermal wave motion in the regime $2\pi\cdot\chi/a^2 >> \omega >> 2\pi\cdot\chi/\lambda^2$; then δT is a smooth function of the spatial coordinates and the relevant perturbed equations are (for a time dependence $\exp(-i\omega t)$ and a magnetic field oriented as $\mathbf{B}_0(x) = \hat{\mathbf{e}}_z\cdot B_0(x)$) :

$$-\omega^2\rho_0\xi = -\nabla\delta p_g - \frac{1}{4\pi}\mathbf{B}_0\times\nabla\times\delta\mathbf{B} - \frac{1}{4\pi}\delta\mathbf{B}\times\nabla\times\mathbf{B}_0$$

$$\delta\mathbf{B} = \nabla\times\xi\times\mathbf{B}_0;\quad \delta\rho = -(\xi\cdot\nabla)\rho_0 - \rho_0\nabla\cdot\xi \qquad (2)$$

$$\frac{\delta p}{p_0} - \gamma\frac{\delta\rho}{\rho_0} + \xi\cdot\left(\frac{\nabla\delta p}{p_0} - \gamma\frac{\nabla\delta\rho}{\rho_0}\right) = 0$$

For the sake of argument we assume uniformity along y-axis andharmonic dependence along the z-direction [$\exp(iqz)$], obtaining coupled equations for the pressure perturbation δP and the Lagrangian fluid displacement $\xi_x$along x:

$$\left(\omega^2\rho_0 - \frac{q^2B_0^2}{4\pi}\right)\cdot\xi_x = \frac{\partial\delta P}{\partial x} \qquad \frac{\partial\xi_x}{\partial x} = -\delta P\cdot\left(\frac{B_0^2}{4\pi} + \gamma p_{g0}\left(1 - \frac{q^2\gamma p_{g0}}{\omega^2\rho_0}\right)^{-1}\right)^{-1} \qquad (3)$$

Consider slow ($\omega = 2\pi s/\lambda << s/a$) , long wavelength ($\lambda/2\pi >> a$) perturbations quasistatic modes with δP and ξ_x smooth functions of x, with small amplitude fluctuations $\xi_x = \langle\xi_x\rangle + \tilde{\xi}_x$ $\delta P = \langle\delta P\rangle + \delta\tilde{P}$. The perturbations $\delta p, \delta\rho, \delta B_z, \xi_z$ can vary considerably due to different compressibility of the gas and of the magnetic field but the physical quantities adjust their variations in such a way as to keep the perturbation of the total pressure smooth.

A suitable *averaging procedure* is now defined as $\langle f(x)\rangle \equiv \dfrac{1}{L}\displaystyle\int_{x-\frac{L}{2}}^{x+\frac{L}{2}} f(x')dx'$, over the *intermediate* length $\boldsymbol{L}$, $a << L << \lambda$ and applied to eqs. (3), and solutions are sought of the kind $\exp(ikx)$. A dispersion relation for the slow perturbations is obtained in the form:

$$\left(u^2 - \frac{\langle\rho_0 v_a^2\rangle}{\langle\rho_0\rangle}\right)\cdot(u^2 - 1)\frac{\langle\rho_0\rangle}{2\gamma P_0}\int\frac{(\gamma u_A^2 + 2)F(u_a^2)du_a^2}{[u^2(u_a^2 + 1) - u_a^2]} = \tan^2\vartheta \qquad (4)$$

where $u^2 = v^2/\left(v_s^2 \cos^2 \vartheta\right)$, $u_a^2 = v_a^2/v_s^2$, $v_s(x) = \sqrt{\dfrac{\gamma p_{g0}(x)}{\rho_0(x)}}$, $v_a(x) = \sqrt{\dfrac{B_0^2(x)}{4\pi\rho_0(x)}}$, $v^2 = \dfrac{\omega^2}{k^2 + q^2}$ and $\tan^2 \vartheta = k^2/q^2$. The unperturbed state is determined by constant p_{g0}, ρ_0, B_0, P_0 and by two independent parameters, v_s^2 and v_a^2, for which we introduce a normalised distribution function $F\left(u_a^2\right)$ that determines the properties of the waves described by the averaged equation (4). The first two brackets represent the customary slow and fast magneto-acoustic waves. The factor appearing as an integral gives the important new results. For a model distribution of the type $F\left(u_a^2\right) = \alpha_1 \cdot \delta\left(u_a^2 - u_{a,1}^2\right) + \alpha_2 \cdot \delta\left(u_a^2 - u_{a,2}^2\right)$ with $\alpha_1 + \alpha_2 = 1$, representing a plasma with (only) two Alfve'n speeds the plasma becomes a "two phase" medium sustaining a *third* mode of wave propagation, a variant of the slow branch, as shown in Fig.1 for $\alpha_1 = \alpha_2 = 0.5$ and in Fig.2 for $\alpha_1 = 0.1$, $\alpha_2 = 0.9$. Furthermore it must be noticed that the third factor in eq.(4) contains a Landau-type integral that may produce collisionless damping of the long wavelength perturbations for distributions of finite width such as the model $F\left(u_a^2\right) = (1-\varepsilon) \cdot \delta\left(u_a^2 - v_{a,0}^2\right) + \varepsilon \cdot f\left(u_a^2\right)$ where the δ–function describes the background plasma and $f\left(u_a^2\right)$ the normalised distribution of the non-uniformities. In Fig.3 and Fig.4 the real and imaginary part of the damped root are shown.

4-Conclusions

An analysis of MHD waves propagation in a finely structured medium (with $a << 2\pi$) has been presented. .Enhanced viscous and collisionless damping have been determined. In the case of a *two-phase* plasma a new (*slow)* mode appears. The results are of interest for the physics of the MHD wave propagation in the solar atmosphere and for possible relation with a *filamentary structure* of magnetised plasmas observed in some tokamaks .

References

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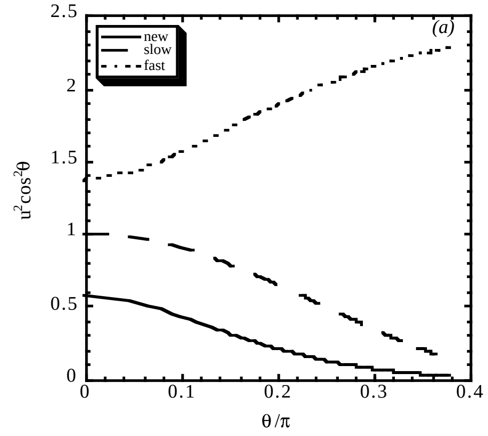

Fig.1

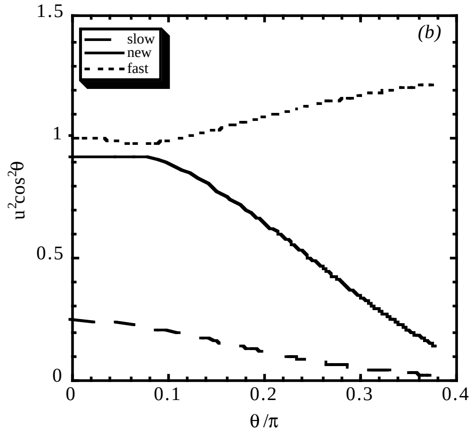

Fig.2

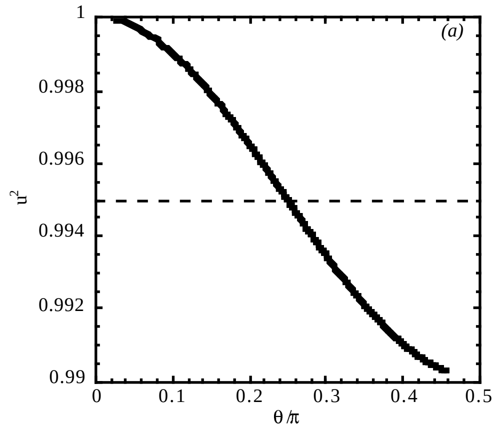

Fig.3

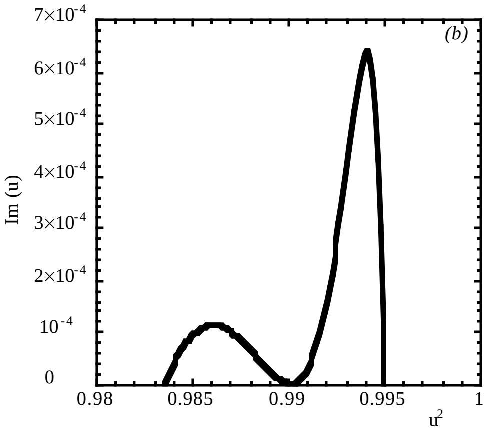

Fig.4