27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** *(2000) 296 -299*

Eigen kinetic Alfven waves localized between two Alfven resonances

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Abstract. Eigen electromagnetic oscillations localized in the region where hot plasma density profile reaches its maximum (minimum) between two local Alfven resonances are studied. Conditions under which these kinetic Alfven waves can propagate are found. Eigen modes and eigen frequencies are derived analytically.

1. *Introduction*

Conversion and absorption of electromagnetic waves within Alfven resonance (AR) region are intensively studied beginning from the papers [1-4] mainly because of their efficient application to plasma production and heating in fusion devices. Detailed overviews of theoretical research into AR are given, e.g., in [5,6]. Radio frequency (RF) power absorption within the AR region is known to be inversely proportional to the plasma density gradient in the case of linear density profile. That is why the case when plasma density profile has maximum (or minimum) within Alfven resonance region,

$$\varepsilon_1(r) = N_z^2\left[1 \pm \left(\frac{r - r_0}{a}\right)^2 + \delta\right], \tag{1}$$

is of extreme interest [7,8]. This kind of profile arises, in particular, during electron cyclotron resonance heating. Numerical studying of the kinetic Alfven waves excitation in the usual case of the density maximum at the axis, $r_0 = 0$, is referred to in [5]. Here $N_z = ck_z/\omega$ is the axial refractive index, k_z is axial wave vector, steady uniform magnetic field is oriented along the z-axis, $\vec{B}_0 \| \vec{e}_z$, in cylindrical coordinates.

Numerical analysis [8] shows essential difference in magnitudes of the power absorption within AR region for the cases of minimum and maximum of density profile. Explanation for this difference is given on the basis of analyzing the possibility of kinetic Alfven waves propagation.

2. *Basic equation*

The following equation for the amplitude E_r of the radial component of the wave electric field, $\tilde{E}_r = E_r(r)\exp[i(k_z z + m\vartheta - \omega t)]$, can be derived from Maxwell equations with taking into account the electron inertia and finite ion Larmor radius,

$$\left[\varepsilon_1 - N_z^2 + \left(\varepsilon_T + \frac{\varepsilon_1}{\varepsilon_3}\frac{c^2}{\omega^2}\right)\frac{d^2}{dr^2}\right]E_r = -i\varepsilon_2 E_\vartheta - \frac{cm}{\omega r}B_z. \tag{2}$$

In (2), $\varepsilon_{1,2,3}$ are the permittivity tensor components of collisionless magnetised plasma, the factor ε_T provides account for the finite ion Larmor radius [9],

$$\varepsilon_T = \sum_i \frac{3\omega_{pi}^2(r)v_{Ti}^2(r)}{\left(\omega^2 - \omega_{ci}^{(0)2}\right)\left(\omega^2 - 4\omega_{ci}^{(0)2}\right)}. \tag{3}$$

In the frequency range under the consideration, $\omega < \omega_{ci}$, ε_T is positive value and it is equal by the order of magnitude, $\varepsilon_T \sim N_z^2\rho_{Li}^2$, here $\rho_{Li} = v_{Ti}/\omega_{ci}$ is ion Larmor radius ($v_{Ti} = \sqrt{T_i/m_i}$ and T_i are the thermal velocity and temperature of ions, respectively). The other addendum $\varepsilon_1 c^2/(\varepsilon_3\omega^2)$ in the factor nearby the second derivative is responsible for account for the electron inertia. This addendum is negative one and it is equal by the order of magnitude, $\varepsilon_1 c^2/(\varepsilon_3\omega^2) \sim N_z^2 \rho_{Li}^2 T_e/T_i$. Then let us denote,

$$\varepsilon_T + \frac{\varepsilon_1}{\varepsilon_3}\frac{c^2}{\omega^2} \equiv \pm N_z^2 \rho^2, \tag{4}$$

we choose the upper sign if electrons are cold and just finite ion Larmor radius provides conditions for kinetic Alfven waves propagation. The lower sign in right hand side of (5) corresponds to the case of hot electrons in which just account for the electron inertia causes the kinetic Alfven waves propagation.

Following the authors of [1] we utilize here the approach of "narrow slab". The latter assumes slow variation of plasma density within the AR region. Slow variation of the fields in all directions excepting the radial one is also supposed.

Although both azimuthal electric field E_ϑ and axial magnetic field B_z of the wave have a singularity in the cold plasma, $E_\vartheta, B_z \propto \ln\left|\varepsilon_1 - N_z^2\right|$, within the AR region, $r = r_A$, the combination $(i\varepsilon_2 E_\vartheta + (cm/\omega r)B_z)$ in the right hand side of (2) is known to vary slowly in the vicinity of AR. That is why this combination uses to be considered as a constant, that is associated with a pumping wave and can be determined by solving the Maxwell equations outside the AR region. As far as we consider here eigen modes localized nearby the AR region then we put this combination to zero.

3. ***Analytical solutions***

Equation (2) with zero in the right hand side has solution in the form of standing waves localized in the vicinity of **maximum** at the density profile if electrons are **cold**. Equation (2) has the following form in this case,

$$\left[\delta - \left(\frac{r - r_0}{a}\right)^2 + \rho^2 \frac{d^2}{dr^2}\right] E_r = 0. \tag{5}$$

Solution of equation (6) can be expressed through Hermite Polynomials $H_n\left(\frac{r - r_0}{\sqrt{\rho a}}\right)$ [10],

$$E_r = \exp\left[-\frac{(r - r_0)^2}{2\rho a}\right] H_n\left(\frac{r - r_0}{\sqrt{\rho a}}\right). \tag{6}$$

This solution is valid if deviation δ of ε_1 from N_z^2 has perfect value,

$$\delta = (1+2n)\rho/a, \quad n = 0,1,2... \tag{7}$$

Here the radial wave number *n* can be natural and zero, it corresponds to the number of nodes of the appropriate eigen mode. This relation determines the following expression for eigen frequency of kinetic Alfven waves,

$$\omega = |k_z| v_A \left[1 + \frac{\rho}{a} 0{,}5(1 + 2n)\right]. \tag{8}$$

To find the type of waves in uniform plasma to which the waves under the consideration correspond, let us study the solution of Maxwell equations in WKB approach, $\tilde{E}_r \propto \exp\left(i\int k_r dr\right)$, $k_r a >> 1$. Then one derives the following expression for the radial refractive index squared, $N_r^2 = c^2 k_r^2 / \omega^2$ (see fig. 1, where the following values of plasma parameters are chosen: $N_z = 9$, $a = 3$ cm, $\omega\rho / c = 0{,}1$),

$$N_{r\pm}^2 = \frac{\varepsilon_1 - N_z^2}{2k_z^2\rho^2}\left[1 \pm \sqrt{1 - 4\frac{k_z^2\rho^2}{\varepsilon_1 - N_z^2}\left(N_\perp^2 - \frac{c^2 m^2}{\omega^2 r_0^2}\right)}\right]. \tag{9}$$

Here $N_\perp^2$ is radial refractive index squared for axially symmetric magneto hydrodynamic waves in cold plasma,

$$N_\perp^2 = \left[\left(\varepsilon_1 - N_z^2\right)^2 - \varepsilon_2^2\right] \Big/ \left(\varepsilon_1 - N_z^2\right), \tag{10}$$

$N_{r-}^2 \to N_\perp^2$ if $\rho^2 \to 0$. The other branch N_{r+}^2 of the dependence (9) corresponds to small-scale kinetic Alfven waves, $N_{r+}^2 \to \left(\varepsilon_1 - N_z^2\right) / \left(k_z^2 \rho^2\right)$ if $\rho^2 \to 0$.

If electrons are **hot** then equation (2) has the solution in the form of standing waves (6) localized in the vicinity of **minimum** at the density profile. The eigen frequency of kinetic Alfven waves can be written in the form (8) with negative integer radial wave number *n* in this case.

Radial dependence of the refractive index squared is given in fig. 2 for the case of minimum at the density profile. One can easy estimate the width Δr of the conversion region (N_r^2 is not shown in this radial segment because N_r^2 is not real in this region) either from (6) or (9) as follows,

$$\Delta r \sim (\rho a)^{1/2}. \tag{11}$$

It coincides with the characteristic radial scale on which radial electric field of kinetic Alfven wave varies. Then we can point out that the condition of applicability of the "narrow slab" approach, $\rho << a$, can be met easily. Note also that the characteristic parameter $k_r^2\rho^2$ is also small within the AR region, $k_r^2\rho^2 \sim \rho/a << 1$, that justifies plasma description with the permittivity tensor.

Conclusions

Kinetic Alfven waves are studied in the case when they are localized in the vicinity of maximum (minimum) on radial profile of the hot plasma density between two local ARs.

Distribution (see expression (6)) of electromagnetic fields and eigen frequency (see expression (8)) are found. Eigen frequency is higher than the frequency of Alfven continuum in the point of maximum and vice versa if minimum takes place at the density profile.

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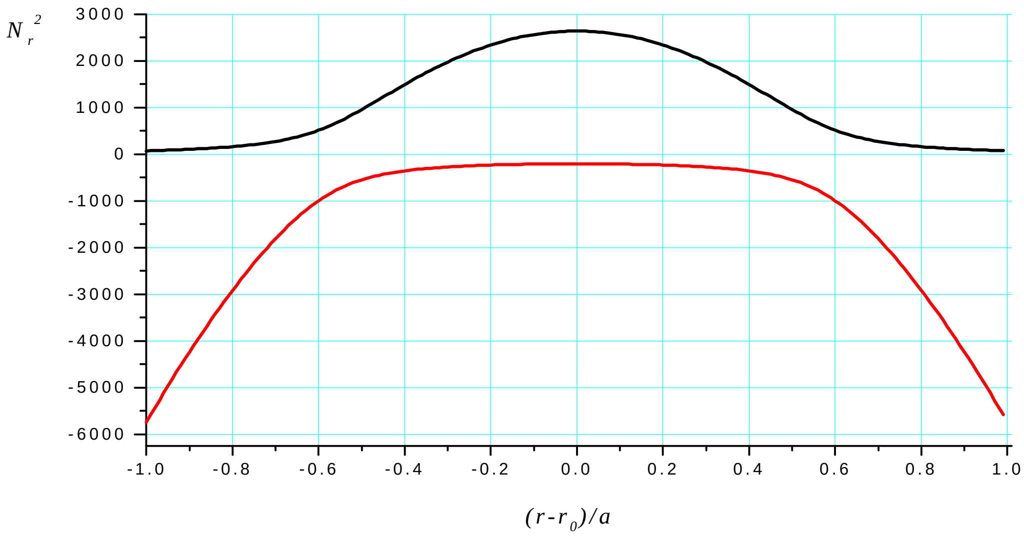

Fig.1. Radial dependence of radial refractive index squared (*m* = *0*) in the case of maximum at the density profile and cold electrons.

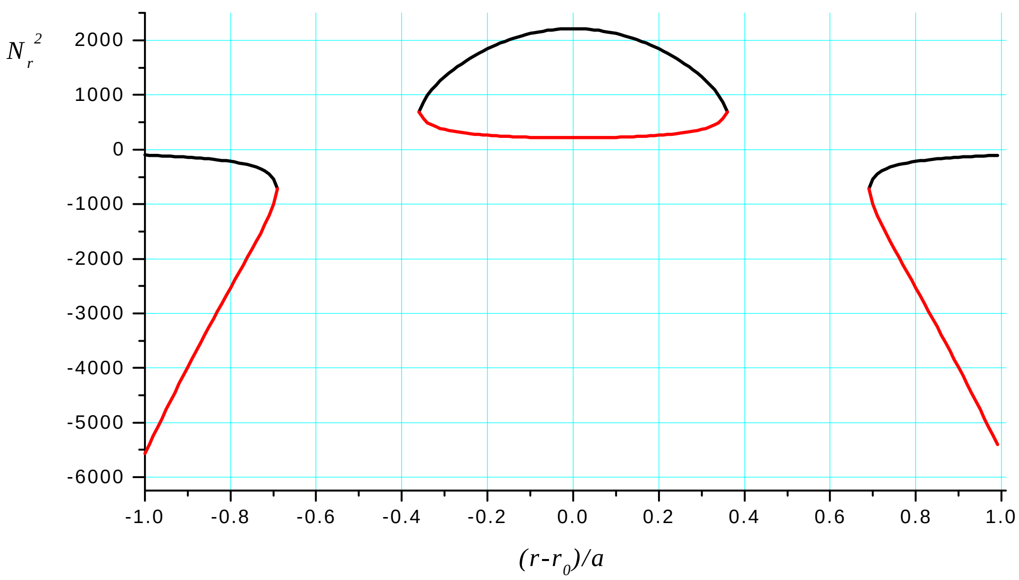

Fig.2. Radial dependence of radial refractive index squared (*m* = *0*) in the case of minimum at the density profile and hot electrons.