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Equilibria of Suydam Stable RFP Plasma with High Flow Shear

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1. Introduction

Recently RFP plasmas are improved by many experiments[1-6]. Although they are very interesting results, their stability criterion is limited by the usual Suydam limit and we could not expect the high quality of RFP plasmas compared with Tokamaks. Therefore it is still important to find more improved states in RFP. We show here that the high shear flow is effective for RFP equilibria with high β and β_p.

Suydam stable plasma equilibria of RFP with flow in a cylindrical model are already provided by variational methods with several constraints in the previous paper[7]. But the stability effect due to high flow shear has not been shown in this paper. Recently it is reported in the experiments[5] that a toroidal flow, V_z, shows a reversal part in the outer region. This fact seems to suggest us that a flow helicity also is a constraint in RFP plasmas adding to the magnetic helicity constraints of J.B. Taylor[8]. Equilibria obtained have ploidal and toroidal flows whose profiles of flows are similar to magnetic fields, that is the toroidal flow is reversed like the toroidal magnetic field. This gives a profile with high flow shear. The aim of this study is to look for RFP equilibria with high β and β_p that can be stable against Suydam condition with flow owing to an effect of high flow shear.

2. Equilibria of RFP

We can specify the flux surface function, $P(\Psi) = p + (1/2)\mathbf{V}^2$, $I(\Psi) = B_z$ and $K(\Psi) = V_z$ in the Grad-Shafranov equation by the variational method under several constraints as in the previous paper. First P is specified by the variation under the constraint of constant P':

$$P = \frac{\xi c_p}{2}\{1 - \exp[2(\Psi - \Psi_a)/\xi]\} + P_a. \qquad (2-1)$$

Second $I(\Psi)$ and $K(\Phi)$ are specified by the variation under constrains of the two constant helicities as is introduced by J.B. Taylor. It is likely in RFP that a flow helicity is another constraint adding to the magnetic helicity as we stated before. This introduction gives flow profiles with high flow shear that improve the Suydam stability limit. The variational yields the following results:

$$I(\Psi) = \lambda_1\Psi + I_a \pm I_0 \exp[(\Psi - \Psi_a)/\xi], \qquad (2-2)$$

$$K(\Phi) = \lambda_2\Phi + K_a, \qquad (2-3)$$

where $I_0 = \sqrt{\xi\alpha_1 c_p}$.

Third the relation between the magnetic stream function,Ψ, and the flow stream function,Φ, can be decided by introducing the cross helicity constraint. This constraint gives the linear relation:

$$\eta\Phi = \lambda_1\Psi + I_a, \qquad (2-4)$$

where $C_A = \lambda_1/\eta$. All surface functions in the Grad-Shafranov equation are specified as the function of Ψ. Thus we can solve the Grad-Shafranov equation.

3. Suydam Stability of RFP Equilibria

The Suydam necessary condition with flow is derived by A. Bondeson et al. The criterion is given by

$$D_{suy} > 0, \qquad (3-1)$$

where

$$D_{suy} = D_{suy1} + D_{suy2} + D_{suy3} + D_{suy4}, \qquad (3-2)$$

$$D_{suy1} = \frac{1}{4}(\frac{q'B_z}{q})^2(1-M^2), \qquad (3-3)$$

$$D_{suy2} = 2[\frac{1}{\rho}(p' + \frac{\rho_O V_\theta^2}{\rho}) + \frac{B^2}{B_\theta^2}\sqrt{\rho_o}(\frac{V_\theta}{\rho})' + \frac{q'}{\rho q}\frac{B_z^2}{B_\theta}M\sqrt{\rho_o}V_\theta], \qquad (3-4)$$

$$D_{suy3} = -\frac{4}{\rho^2(1-M^2)}(\sqrt{\rho_o}V_\theta - MB_\theta)^2, \qquad (3-5)$$

$$D_{suy4} = \frac{1}{\rho^2 B_\theta^2}\frac{1-\beta_\gamma}{(1-M^2)(M^2-\beta_\gamma)}[(\sqrt{\rho_o}V_\theta - MB_\theta)^2 + M^2(B_\theta^2 - \rho_o V_\theta^2)]^2, \quad (3-6)$$

$\beta_\gamma = \gamma p/(B^2+\gamma p)$, $\tilde{\omega} = \omega + \mathbf{k}\cdot\mathbf{V}$, $F = \mathbf{k}\cdot\mathbf{B}$, $M = \sqrt{\rho_o}\tilde{\omega}'/F'$ and the prime means the differential due to radial coordinate, ρ. The notation is the same as that of Bondeson's. There is an interesting term, D_{suy4}, in their expression of Suydam condition that turns out to be in a stable side when there is a high flow shear. This occurs when $M^2 > \beta_\gamma$ where M means a ratio of flow shear to magnetic shear. This indicates that the high flow shear suppresses the localized interchange instabilities.

Here we have to check about the two things related to $M^2 > \beta_\gamma$: The first is about the Suydam sufficient condition and the second is about $\nabla\cdot\mathbf{v}$ and γ. This is pointed out by M.S.Chu[6]. He derived a sufficient condition for localized interchange in a toroidal geometry by using the formulation according to Frieman and Rotenberg. He also derived an additive condition of rotation shear to the stability criterion in the process of his derivation: interchange modes are always unstable with $M_s^2 > \beta_\Gamma$. We checked this sufficient condition in the cylindrical geometry according to the method of M.S. Chu. We confirmed that the Bondeson's stability condition is a necessary and sufficient condition in the case of localized modes in the cylindrical geometry. Furthermore the additional condition has to be $M^2 > \beta_\gamma$. It is possible that $\tilde{\omega}$ can vanish at a rational surface for accumulating Suydam modes. But we could not always set $\nabla\cdot\mathbf{v} = 0$ in the case of flow since $\tilde{\omega}$ is a function of the radial coordinate. Also $\tilde{\omega}$ can have a finite value for the other modes than accumulating Suydam modes. Although the kinetic formulation is correct, a MHD model with $\gamma = 1$ can be used where the perturbed temperature is constant along the magnetic field lines, i.e. isothermal compression. As will be shown below, magnetic sound waves are always stable in the Suydam stable state. We can imagine to set formally $\gamma = 0$ for suppressing the magnetic sound waves. Therefore γ can be a parameter that expresses the way of compression. We check the Suydam condition for three cases of γ: $\gamma = 0, 1, 5/3$.

We have to consider the slow mode instability in the case of strong flow. The necessary condition against the magnetic sound wave is given by $D_{sound} > 0$, that we extended the condition derived by Bondeson et al. to include the poloidal flows: where

$$D_{sound} = \frac{1}{8}\frac{S''(1-\beta_\gamma)}{\beta_\gamma^2} - [2B_\theta \frac{d}{d\rho}(\frac{B_\theta}{\rho}) - (\frac{2kB_\theta}{\rho F})^2(B^2+\gamma p) + (1-\beta_\gamma)F^2] -$$

$$-\rho\frac{d}{d\rho}(\frac{\rho_o V_\theta^2}{\rho^2\beta_\gamma} - \frac{2B_\theta\sqrt{\rho_o}V_\theta}{\sqrt{\beta_\gamma}\rho^2}) - \frac{4k^2\gamma p}{\rho^2\beta_\gamma^2 F^2}(2\sqrt{\beta_\gamma}B_\theta\sqrt{\rho_o}V_\theta - \beta_\gamma\rho_o V_\theta^2) +$$

$$+\frac{k^2+m^2/\rho^2}{\rho^2\beta_\gamma^2F^2}[8\beta_\gamma B_\theta^2 + (1-\beta_\gamma)\rho_o V_\theta^2 - 4\sqrt{\beta_\gamma}B_\theta\sqrt{\rho_o}V_\theta]\rho_o V_\theta^2 +$$

$$+\frac{4\sqrt{\rho_o}V_\theta}{\beta_\gamma\rho^2}(2\sqrt{\beta_\gamma}B_\theta - \beta_\gamma\sqrt{\rho_o}V_\theta) -$$

$$-\frac{4m\sqrt{\beta_\gamma}\sqrt{\rho_o}V_\theta}{\rho^3\beta_\gamma F}(2B_\theta - \sqrt{\rho_o}V_\theta)(B_\theta + \sqrt{\rho_o}V_\theta), \quad (3-7)$$

where $S = (\mathbf{B}^2 + \gamma p)[\rho_o\tilde{\omega}^2 - \beta_\gamma(\mathbf{k}\cdot\mathbf{B})^2]$. However unstable slow magnetic sound modes could not be found at $M^2 = \beta_\gamma$.

4. Numerical Results and Discussions

First we show the equiribria with high shear flow($M \sim 0.58$) with $C_A = 0.5$, $\gamma = 1$. The plasma parameter indicates $F = -1.883, \Theta = 2.969, \beta = 0.339, \beta_p = 0.562$. Fig.2 shows that the flow profiles are very similar to that of magnetic fields.

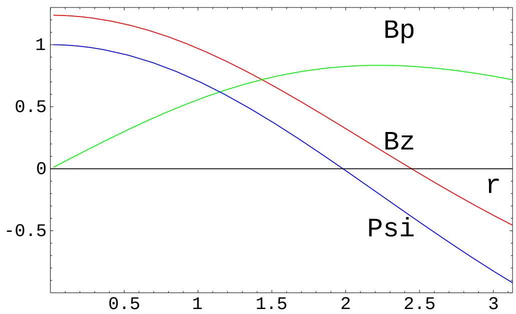

Figure 1: Ψ, B_z, B_p-ρ

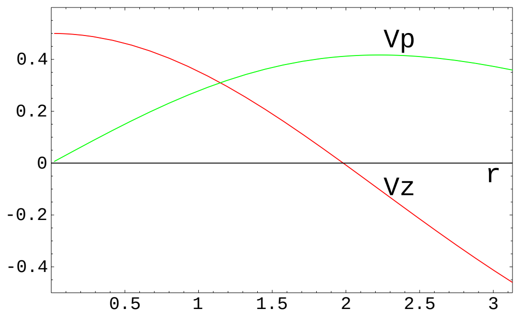

Figure 2: V_z, V_p-ρ

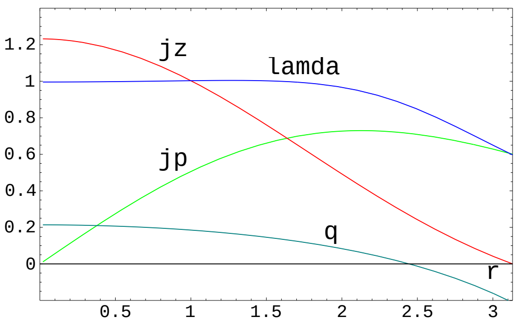

Figure 3: j_z, j_p, λ, q-ρ

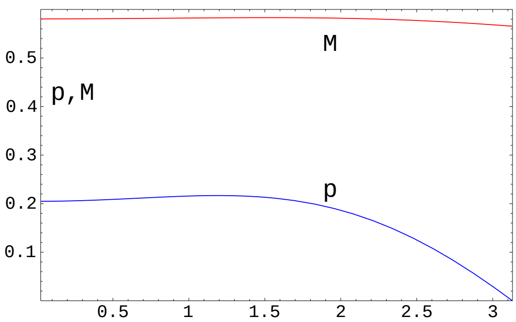

Figure 4: p, M-ρ

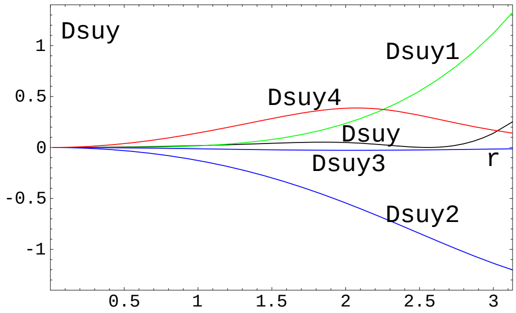

Figure 5: D_{suy}-ρ

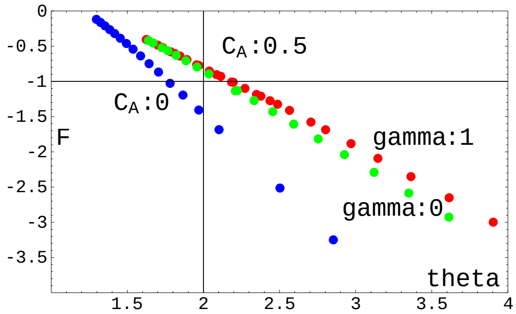

Figure 6: F-Θ

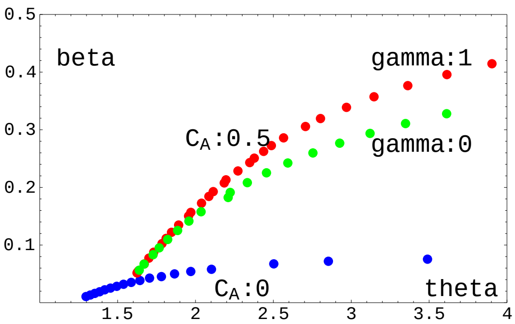

Figure 7: β-Θ

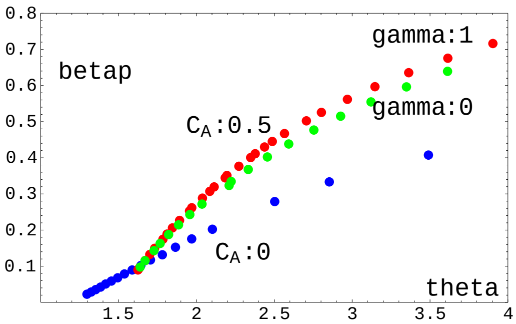

Figure 8: β_p-Θ

The toroidal flow, V_z, has a reversal part whose reversal point is at the inner side than that of B_z. The shearing ratio, M, is almost constant over the radius as is seen from Fig.4. The pressure profile is hollow.

Each term in D_{suy} is plotted in the Fig.5. We see that D_{suy1} and D_{suy4} are large positive terms. It means that D_{suy4} supports the high magnetic shear profile which has a value of large D_{suy1}. Figs.6-8 show the Suydam stable criterions where the lower regions of the plots are stable. The upper two plots are for $C_A = 0.5$ and the lower one is for $C_A = 0$ without flow. The upper one within two $C_A = 0.5$ cases shows the case of $\gamma = 1$ and the lower one for the case of $\gamma = 0$. The β limit is extremely improved as is seen from these graphs that is favorable for a future RFP.

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