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The application of an invariant method for the solution of the Vlasov equation at a plasma edge

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A new computer time saving method is presented for the calculation of the charge separation and electric field at the plasma edge of a tokamak. This problem has been recently presented in one spatial dimension using an adiabatic equation for the electrons, and a kinetic Vlasov equation for the ions with three velocity dimensions[1]. These equations were solved using a method of fractional steps for the solution of the kinetic equation[2], which has four phase-space dimensionality. However, if we take advantage of the existence of exact invariants of the solution present in the Vlasov equation, we can express the distribution function in terms of the invariant, which allows to reduce the dimensionality of the phase-space of the Valsov equation[2]. The invariant only appears as a label of the Vlasov equation, and can be coarsely discretized. We apply this method of invariant for the previously presented 1D solution[1]. We are able to reproduce accurately the same results with a gain of a factor of 10 in the computation time using only 10 invariants instead of 50 points in the corresponding velocity space.

Time is normalized to ω_{pi}^{-1}. Velocity is normalized to the acoustic speed c_s, and the length to $c_s\ \omega_{pi}^{-1}$. We approximate the edge of the toroidal plasma by a slab in which the direction y represents the radial direction. The periodic toroidal z and the periodic poloidal x directions are assumed homogenous. The constant magnetic field B is located in the (x, z) plane and makes an angle θ with the x axis (θ is close to $\pi / 2$ since the magnetic field is almost in the toroidal direction z). With this notation, the 1D Vlasov equation is written for the main ion species:

$$\frac{\partial f_i}{\partial t}+\nu_y\frac{\partial f_i}{\partial y}+\nu_y\omega_{ci}\sin\theta\frac{\partial f_i}{\partial \nu_x}+\left(E_y-\nu_x\omega_{ci}\sin\theta+\nu_z\omega_{ci}\cos\theta\right)\frac{\partial f_i}{\partial \nu_y}-\nu_y\omega_{ci}\cos\theta\frac{\partial f_i}{\partial \nu_z}=0 \quad (1)$$

A similar equations holds for the impurity ions distribution function f_I, with a factor m_i / m_I added in front of the acceleration terms, where m_i is the mass of the main ion species, and m_I the mass of the impurity ions. Electrons are described by an adiabatic low $n_e = n(y)\exp(\varphi)$, where n_e is the electron density, $n(y)$ a profile to be specified, and φ is the potential normalized to T_e / e, calculated from Poisson equation:

$$\Delta\varphi = -(n_i + n_I - n_e)\,; \quad \text{with } n_{i,I} = \int f_{i,I}\, d\vec{v} \quad (2)$$

The characteristics of Eq. (1) are given by:

$$\frac{dy}{dt}=\nu_y \quad ; \quad \frac{d\nu_x}{dt}=\nu_y\omega_{ci}\sin\theta \ , \quad (3)$$

$$\frac{d\nu_y}{dt}=E_y-\nu_x\omega_{ci}\sin\theta+\nu_z\omega_{ci}\cos\theta \quad ; \quad \frac{d\nu_z}{dt}=-\nu_y\omega_{ci}\cos\theta \quad (4)$$

From the first of Eqs. (3) and the second of Eqs. (4), we derive the exact invariant:

$$I=\nu_z+y\omega_{ci}\cos\theta \quad (5)$$

We go from the phase-space (y,ν_x,ν_y,ν_z) to the new phase-space (y,ν_x,ν_y,I). We calculate the different derivatives of F_i for instance

$$\frac{\partial f_i}{\partial t}=\frac{\partial F_i}{\partial t}+\frac{\partial F_i}{\partial I}\frac{\partial I}{\partial t} \quad (6)$$

we obtain for $F_i\ (y,\nu_x,\nu_y,I)$:

$$\frac{\partial F_i}{\partial t}+\nu_y\frac{\partial F_i}{\partial y}+\left(E_y-\nu_x\omega_{ci}\sin\theta+\left(I-y\omega_{ci}\cos\theta\right)\omega_{ci}\cos\theta\right)\frac{\partial F_i}{\partial \nu_y}+\nu_y\omega_{ci}\sin\theta\frac{\partial F_i}{\partial \nu_x}=0 \quad (7)$$

The initial Maxwellian distribution for the main impurity ions, used in the fully kinetic code, is given by:

$$f_i(y,\nu_x,\nu_y,\nu_z)=n_i(y)\frac{e^{-\left(\nu_x^2+\nu_y^2\right)/2T_{io}}}{2\pi\, T_{io}}\frac{e^{-\nu_z^2/2\ T_i(y)}}{\left(2\pi\, T_i(y)\right)^{1/2}} \quad (8)$$

becomes

$$F_i\left(y, \nu_x, \nu_y, I\right) = n_i(y) \frac{e^{-\left(\nu_x^2 + \nu_y^2\right)/2T_{io}}}{2\pi\, T_{io}} \frac{e^{-(I - y\omega_{ci}\cos\theta)^2/2\,T_i(y)}}{\left(2\pi\, T_i(y)\right)^{1/2}} \tag{9}$$

I is now acting as a label to the distribution function. Usually taking $(I_{\max} - y\omega_{ci}\cos\theta)$ three to four times $T_i(y)$ is sufficient. Note also that the Maxwellian can be cut off more abruptly using the invariant, since no diffusion can take place in I-space. In the present calculation, we take $|\nu_{z\max}| = 3$, or equivalently $(I - y\, ø\, c_{ci} \cos Ł) = 3$ ($\theta = 89°$), and $(-4 < \nu_x, \nu_y < 4)$.Ten values of I, are taken in Eq. (9). Our expectation is that for an angle θ close to 90°, a small number of invariants should be sufficient to reproduce the correct results. This could not be envisaged for the ν_z variable, since numerical differentiation cannot tolerate a coarse representation of the distribution function. The results which have been presented in Ref. [1] with $N_y \times N_{íx} \times N_{íy} \times N_{íz} = 160 \times 50 \times 50 \times 50$, have been accurately reproduced with $N_y \times N_{íx} \times N_{íy} = 160 \times 50 \times 50$ grid points and only ten invariants I. The same time step $\Delta t = 0.1$ was used. We use the same initial conditions as in Ref. [1].

$$n(y) = 0.5\left(1 + \tanh(y/15)\right) \; ; \; T_i(y) = T_{io}\left(0.2 + 0.4(1 + \tanh(y/10))\right) \tag{10}$$

in the domain $-80 < y < 80$, $T_e = T_{io} = 1$, $m_i / m_e = 1840$, $m_I / m_i = 10$, $\omega_{ci} / \omega_{pi} = 0.1$.

The ratio of the ion gyro-radius to the Debye length is given by:

$$\frac{\rho_i}{\lambda_{DE}} = \frac{\nu_{ti} / \omega_{ci}}{\lambda_{DE}} = \frac{\sqrt{T_{io} / T_e}}{\omega_{ci} / \omega_{pi}} = 10 \tag{11}$$

For a fraction of 5% impurity ions, $n_I = 0.05\, n(y)$, $n_i = 0.95\, n(y)$, the simulation was extended to much longer time than in Ref. [1]. We show in the figures the potential, electric field, charge and density profile calculated during a half period between $t = 5860$ and $t = 5885$, showing essentially the same physics discussed in Ref. [1]. The period of oscillation due to the main ioin species gyration is nicely conserved around 55. The potential shows a shape with a growing bump reaching a peak in front of the gradient. The electric field is growing along the gradient and reaches its peak around the bottom of the gradient. The full curve in the density plot is for the electrons, the broken curve for the main ion species, and the impurity ions is multiplied by a factor of 10 and represented by the dot-dash curve. Extending these results to two-spatial dimensions, including a drift kinetic equation to describe the electrons, is underway.

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References

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