

## A Possibility of Plasma Current Density Measurements Using Suprathermal Electron Bremsstrahlung Emission

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Soft X-ray Bremsstrahlung emission of suprathermal electrons is commonly used for electron temperature measurements in plasma experiments. A difference in the suprathermal and bulk electron distributions results in a discrepancy between the measured and real electron temperatures, considered as a diagnostic demerit. On the other hand, this demerit carries a specific information about factors disturbing the electron distribution. One of them is an electric field driving the electrons along a magnetic field. A way of the measurements of the longitudinal electric field using suprathermal electron Bremsstrahlung emission has been proposed and analyzed in the paper. The plasma current density is found from the Ohm's law.

### Bremsstrahlung emission of an anisotropic electron distribution

The general expression for the Bremsstrahlung emission intensity radiated within  $dW_\gamma$  energy range, from  $dV$  plasma volume and inside  $d\Omega_\gamma$  solid angle is as follows:

$$dI = dW_\gamma d\Omega_\gamma dV n_e \sum_i n_i \int_{v_\gamma}^\infty dv_e v_e^3 \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta \sigma(W_e, W_\gamma, \theta_0) f_e(v_e, \theta, \phi) \quad (1)$$

Here,  $n_{e,i}$  is the electron and ion densities,  $W_{e,\gamma}$  are the electron a photon energies,  $v_e$  is the electron velocity,  $v_\gamma$  is the velocity of electron having  $W_\gamma$  energy,  $\theta_0$  is the angle between the electron and photon directions,  $f_e(v_e, \theta, \phi)$  is the electron distribution function. The non-relativistic Bremsstrahlung cross-section found by Sommerfeld can be presented as:

$$\sigma = \sigma_0 \sin^2 \theta_0 + \sigma_1 \left(1 - \frac{3}{2} \sin^2 \theta_0\right) \quad (2)$$

The  $\theta_0$  angle relates to the  $\theta, \phi$  angles as  $\cos\theta_0 = \cos\theta \cos\alpha + \sin\theta \sin\alpha \cos\phi$ , where  $\alpha$  is the angle between the z-axis and the photon direction. Choosing the z-axis to be along the longitudinal electric field and assuming the distribution to be independent on the  $\phi$  angle, we come to the following expression for the Bremsstrahlung emission intensity:

$$dI = dW_\gamma d\Omega_\gamma dV \frac{8\pi n_e}{3m_e^2} \sum_i n_i \int_{v_\gamma}^\infty dW_e W_e \left[ F_0 \sigma_0 + \frac{(F_0 - 3F_2)(\sigma_0 - 3\sigma_1/2)}{2} \left(1 - \frac{3}{2} \sin^2 \alpha\right) \right] \quad (3)$$

$$F_0 = \int_0^\pi f_e \sin^2 \theta d\theta, \quad F_2 = \int_0^\pi f_e \cos^2 \theta \sin^2 \theta d\theta$$

Two angular momenta of the distribution function  $F_0$  and  $F_2$  related to the density and longitudinal kinetic energy of electrons determine the emission. For an isotropic plasma,  $F_0 = 3F_2$  and the distribution is directly reconstructed from the emission spectra,  $dI \sim dW_e W_e F_0 \sigma_0$ . In the electric field, such a simple reconstruction is possible only if the emission is collected at  $\alpha = 55^\circ$  or  $\alpha = 125^\circ$  angles to the electric field direction, i.e.  $\sin^2 \alpha = 2/3$ . For other  $\alpha$  angles, a fine reconstruction requires knowing the relationship between  $F_0$  and  $F_2$ . The available expressions for the electron distribution function [1] do not allow this relationship to be determined analytically with necessary accuracy.

### Angular momenta of the electron distribution function

The angular momenta are found from the kinetic equation. For suprathermal electrons, it has the linear collision term. In steady state, we write the equation in the following form:

$$\varepsilon w \left( 2\mu \frac{\partial f}{\partial w} + \frac{1-\mu^2}{w} \frac{\partial f}{\partial \mu} \right) = \frac{\partial}{\partial w} \left( \frac{\partial f}{\partial w} + f \right) + \frac{1}{4w} \left( 1 + z_{\text{eff}} - \frac{1}{4w} \right) \frac{\partial}{\partial \mu} \left( (1-\mu^2) \frac{\partial f}{\partial \mu} \right) \quad (4)$$

$$\mu = \text{Cos}\theta, \quad w = mv_e^2 / 2T_e, \quad \varepsilon = E / E_c, \quad E_c = 4\pi n_e e^3 \Lambda / T_e$$

Here,  $E_c$  is the critical electric field,  $T_e$  is the electron temperature and  $\Lambda$  is the Coulomb logarithm. Multiplying (4) by  $\mu^n$  and integrating it over  $-1 < \mu < 1$ , we find equations

for the momenta  $F_n(w) = \int_{-1}^1 d\mu \mu^n f$ :

$$\begin{aligned} F'_0 + F_0 + C &= 2\varepsilon w F_1 \quad (n=0) \\ (F'_n + F_n)' - \frac{n}{4w} \left( 1 + z_{\text{eff}} - \frac{1}{4w} \right) [(n+1)F_n - (n-1)F_{n-2}] &= \varepsilon (2wF'_{n+1} + (n+2)F_{n+1} - nF_{n-1}) \end{aligned} \quad (5)$$

The  $C$  constant relates to the runaway rate [2] and is  $\sim \sqrt{\varepsilon} \exp(-\frac{1}{4\varepsilon} - \sqrt{\frac{2}{\varepsilon}})$ . Let us separate the Maxwellian momenta  $F_n^M$  and their corrections  $\phi_n$ :

$$F_n = F_n^M(w) + \phi_n(w), \quad F_n^M = \begin{cases} 2Ae^{-w}/(n+1), & \text{even } n \\ 0, & \text{odd } n \end{cases} \quad (6)$$

Here,  $A$  is determined by  $\int_0^\infty dw \sqrt{w} F_0 = 1$  and equals  $1/\sqrt{\pi}$  at  $\varepsilon=0$ . We solve set (5)

expanding  $\phi_n$  in powers of  $\varepsilon$ . For odd  $n=2p+1$ , the main term in the right-hand side of (5) is  $2\varepsilon w F'_{n+1}$ . The rest terms are supposed to be small and neglected. For even  $n=2p$ , all terms are remained in the set. The equations derived from (5) are as follows:

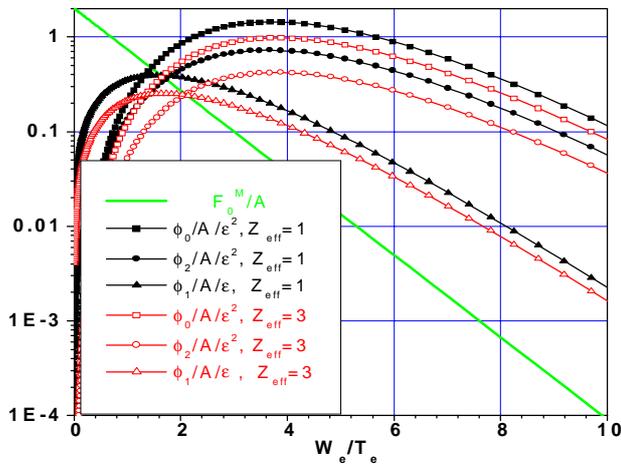
$$\begin{aligned} \phi'_0 + \phi_0 &= 2\varepsilon w \phi_1, & (\phi'_n + \phi_n)' + \left( k_n + \frac{1/4 - m_n^2}{w} \right) \frac{\phi_n}{w} &= \Phi_n \\ k_n &= -n(n+1)(1 + z_{\text{eff}})/4, & m_n^2 &= 1/4 - n(n+1)/16 \\ \Phi_{2p} &= -\frac{2p-1}{2p+1} \left( k_{2p} + \frac{1/4 - m_{2p}^2}{w} \right) \frac{\phi_{2p-2}}{w} + 2\varepsilon ((p+1)\phi_{2p+1} - p\phi_{2p-1}) + 2\varepsilon w \phi'_{2p+1} \\ \Phi_{2p+1} &= -\frac{p}{p+1} \left( k_{2p+1} + \frac{1/4 - m_{2p+1}^2}{w} \right) \frac{\phi_{2p-1}}{w} - \frac{4A}{2p+3} \varepsilon w e^{-w} \end{aligned} \quad (7)$$

The runaway constant is ignored because it yields exponentially small contribution in the region of interest. The set is solved in the following sequence:  $(n=1) \rightarrow (n=0) \rightarrow (n=3) \rightarrow (n=2)$  and so on. The solutions of (7) are expressed with the Whittaker functions  $W_{k,m}$ . Two integration constant are chosen so that to provide zero limits of the corrections when  $w \rightarrow \infty$  and  $w \rightarrow 0$ . The second requirement for odd corrections is due to the distribution function depends on  $v_{\parallel}$ , i.e.  $w \text{Cos}\theta$ . A non-zero limit of any even momenta results in diverging momenta of a higher order. The found solutions are as follows:

$$\begin{aligned} \phi_1 &= -\frac{4A\varepsilon}{3} W_{k_1, m_1} e^{-w/2} \int_0^w \frac{dx}{W_{k_1, m_1}^2} \int_{k_1, m_1}^x W_{k_1, m_1} e^{-t/2} dt, & \phi_0 &= 2\varepsilon e^{-w} \int_0^w x e^x \phi_1 dx, \\ \phi_n &= W_{k_n, m_n} e^{-w/2} \int_0^w \frac{dx}{W_{k_n, m_n}^2} \int_{k_n, m_n}^x W_{k_n, m_n} e^{t/2} \Phi_n(t) dt, & n &> 1 \end{aligned} \quad (8)$$

The normalized first three moments are plotted in comparison with the Maxwellian zero moment in Fig. 1 for two  $Z_{eff}$  values. In principal, set (9) allows the computation of  $\phi_n$  and reconstruction of the electron distribution function  $f_e(w, \theta)$ . The asymptotic expansions of (8) when  $w \rightarrow \infty$  are much easier for applications:

$$\begin{aligned} \phi_0^a &= \frac{4}{3(5+Z_{eff})} A \varepsilon^2 w^4 e^{-w} \left( 1 + \frac{17}{3(3+Z_{eff})w} \right), & \phi_1^a &= \frac{8}{3(5+Z_{eff})} A \varepsilon w^2 e^{-w} \left( 1 + \frac{17}{4(3+Z_{eff})w} \right), \\ \phi_2^a &= \frac{4(29+5Z_{eff})}{15(5+Z_{eff})(11+3Z_{eff})} A \varepsilon^2 w^4 e^{-w} \left( 1 + \frac{5093+1940Z_{eff}+111Z_{eff}^2}{9(3+Z_{eff})^2(29+5Z_{eff})w} \right), & (9) \\ \phi_3^a &= \frac{8}{5(5+Z_{eff})} A \varepsilon w^2 e^{-w} \left( 1 + \frac{68+51Z_{eff}}{4(3+Z_{eff})(4+3Z_{eff})w} \right) \end{aligned}$$



**Fig.1**

*The normalized angular moment corrections.*

### Diagnostic capabilities

The solutions obtained present a number of diagnostic possibilities based on the analysis of suprathermal electron Bremsstrahlung spectra. Some of them are discussed here.

1. The Bremsstrahlung emission collected at  $90^\circ$  to the electric field direction is determined by the sum  $(F_0 + F_2)\sigma_0 + (F_0 - 3F_2)\sigma_1/2$ . The second term is much less than the first one because  $\sigma_1/\sigma_0$  ratio approaches to zero at  $W_e \sim W_\gamma$ . It is this region which most contributes to the emission (3) from the exponentially distributed electrons.

The logarithmic slope of  $F_0+F_2$  is related to the temperature  $T_B$  determined from the Bremsstrahlung spectra:

$$-T_e d \ln(F_0 + F_2) / dW_e = T_e / T_B = 1 - G_1(Z_{eff}) \varepsilon^2 w^3 - G_2(Z_{eff}) \varepsilon^2 w^2, \quad (11)$$

$$G_1 = \frac{8(21+5Z_{eff})}{5(5+Z_{eff})(11+3Z_{eff})}, \quad G_2 = \frac{2(3377+1760Z_{eff}+219Z_{eff}^2)}{15(5+Z_{eff})(11+3Z_{eff})(3+Z_{eff})^2}$$

The second derivative of the logarithm gives the changes of  $T_B$  with  $W_e$ :

$$\frac{T_e^2}{T_B^2} \frac{dT_B}{dW_e} = 3G_1 \varepsilon^2 w^2 + 2G_2 \varepsilon^2 w \quad (12)$$

The sought electric field E is derived from either (11) or (12):

For suprathermal electrons,  $w > 3$ , the asymptotes differ from (8) less than 5%. They show that condition  $\phi_{2p} < < F_{2p}^M$  is equivalent to  $\varepsilon^2 w^4 < < 10$ . So, the applicability region of the solution obtained is:

$$3 < w < 1/\sqrt{\varepsilon} \quad (10)$$

In most plasma experiments,  $\varepsilon$  value is well less than 0.1. This means that the energy range from  $\sim 3T_e$  to  $\sim 6T_e$  can be used for the reconstruction of the electron distribution according to (8, 9) and diagnostics.

$$E = \sqrt{\frac{T_e}{T_B} \frac{(T_B - T_e)}{(G_1 + G_2/w)W_e}} \frac{4\pi n_e e^3 \Lambda}{W_e} = \frac{T_e}{T_B} \sqrt{\frac{dT_B/dW_e}{(3G_1 + 2G_2/w)}} \frac{4\pi n_e e^3 \Lambda}{W_e} \quad (13)$$

The value  $\sqrt{G_1 + G_2/w}$  depends slightly on  $Z_{eff}$ . At  $W_e = 5T_e$ , it changes from 0.77 to 0.58 with  $Z_{eff}$  increasing from 1 to 5. So, a rough estimation of  $Z_{eff}$  value is sufficient for the electron field measurements. The accuracy of  $(T_B - T_e)/T_B$  or  $dT_B/dW_e$  measurements could be more critical point for the diagnostics.

2. The equality of the square roots in (13) shows the possibility to correct the electron temperature measured from the Bremsstrahlung spectra in the near suprathermal region:

$$T_e \approx \frac{T_B}{1 + \frac{W_e}{3T_B} \frac{dT_B}{dW_e}} \quad (14)$$

3. The Ohm's law allows the calculation of the current density for the electron bulk. The first moment of the distribution function makes it possible to determine also the power  $dP_e(W_e)$  contributed to the suprathermal electrons from the electric field:

$$dP_e = eE v_{Te} n_e \phi_1 w dw, \quad (15)$$

4. If the emission is collected at  $55^\circ$  or  $125^\circ$  to the electric field direction than the only zero moment  $F_0$  determines the Bremsstrahlung spectrum. When reconstructed,  $F_0$  allows the assumption-free calculation of the first moment (5) and power  $dP_e(W_e)$  (15). For known  $F_0$ , higher momenta  $F_{n+1}$  are easily calculated from the first order differential equations (5). In particular, the longitudinal kinetic energy  $W_{||}$  of electrons can be determined as a function of their total energy  $W_e$  with:

$$W_{||} = W_e F_2 / F_0 \quad (16)$$

So, the  $55^\circ$  and  $125^\circ$  Bremsstrahlung spectra allow the measurements of the angular momenta of the suprathermal electron distribution governed by the kinetic equation (4).

### Relativistic restrictions

For  $W_e \ll m_e c^2$ , relativistic terms  $(1 - v_e \cos \theta_0 / c)^{-N}$ ,  $N=1..4$ , in the Bremsstrahlung cross-section [3] can violate the value (2). The relativistic corrections must be much less than the even moment corrections. Taking  $w=4$ , this requirement can be written as

$$T_e \cos^2 \alpha \ll 2\epsilon^2 m_e c^2. \quad (17)$$

For the  $\alpha=55^\circ$  and  $125^\circ$  spectra, (17) means that  $T_e \ll 6\epsilon^2 m_e c^2 \sim 10$  keV. For  $90^\circ$  spectra, the main relativistic restriction is  $W_e \ll m_e c^2$ . So, the proposed diagnostics can be used in plasmas with the electron temperatures up to a few keV.

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