

TWO DIMENSIONAL THEORY OF CORRELATION REFLECTOMETRY SCATTERING DIAGNOSTICS

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Introduction.

Fluctuation reflectometry is widely used technique providing information on the plasma low frequency turbulence [1]. It is often used for monitoring the density fluctuations behavior in the tokamak discharge, in particular for indication of the transition to improved confinement. The interpretation of experimental data in this technique is based on two essential suppositions. According to the first one the reflectometer scattering is produced by long scale fluctuations, which dominate in the density fluctuation spectrum. According to the second one, which is the natural consequence of the first in 1D linear theory [2,3], the reflectometer scattering is localized in the vicinity of cut off of the probing wave. Based on this model a more sophisticated correlative technique for estimation of the turbulence radial coherence length was proposed, using simultaneously different frequencies for probing [4,5]. The radial correlative technique was applied at different tokamaks [4,5], but in contradiction with initial expectations the two scattering signals appear to loose coherence at the cut off distance much smaller than estimated turbulence scales. In some experiments this distance was smaller than the probing wavelength, which is in disagreement with the 1D theoretical model mentioned above. The above problems had stimulated the development of 2D fluctuation reflectometer theory [6,7]. It was shown that along with the cut off vicinity the small angle scattering all over the probing wave trajectory contributes a lot to the reflectometry scattering signal.

The 2D model.

In the present paper the realistic 2D linear theory of the radial correlative O-mode reflectometer is presented. In order to simplify the analyses we suppose the linear dependence of the unperturbed density on the “radial” coordinate

$$n_e(x) = n_e x/L \quad (1)$$

where $n_e = m_e \omega^2 / 4\pi e^2$, ω is the probing wave frequency. We also represent density fluctuations in the following form

$$\tilde{n}(x, y) = \int \frac{d\kappa dq}{(2\pi)^2} e^{-i\kappa(x-L) - iqy} \delta n(q, \kappa) \quad (2)$$

where y is a coordinate along the “poloidal” direction; κ and q are correspondingly “radial” and “poloidal” components of the fluctuation wave vector. In the present paper we suppose the turbulence to be statistically homogeneous, so that the following relation holds for the average cross-correlation spectrum

$$\langle \delta n(q, \kappa) \delta n^*(q', \kappa') \rangle = (2\pi)^2 |\delta n|_{q,x}^2 \delta(q - q') \delta(\kappa - \kappa') \quad (3)$$

Following the approach of [6,7] we represent fluctuation reflectometry signal as a superposition of signals, produced by density spectral harmonics

$$A_s(\omega) = \frac{2e^2 \ell^2}{mc^2} \sqrt{P_i} \int \frac{dk_y d\kappa dq}{(2\pi)^3} f(k_y) f(q - k_y) C(\kappa, q, k_y) e^{i\psi} \delta n_{q,\kappa} \quad (4)$$

where $C(\kappa, q, k_y)$ is the scattering efficiency [6,7] given for $\kappa L \gg 1$ by asymptotic expression

$$C = -2\pi\sqrt{i\pi/\beta} \exp\{i[\beta^3/12 - \beta\sigma - \delta/4\beta]\} \quad (5)$$

$$\beta = \kappa L, \quad \delta = \ell^4 q^2 (q - 2k_y)^2, \quad \sigma = -\ell^2 [k_y^2 + (q - k_y)^2], \quad \ell = (Lc^2/\omega^2)^{1/3}$$

$$f(k_y) = 2\sqrt{\pi\rho} \exp[-k_y^2 \rho^2/2] \quad (6)$$

is the antenna diagram, supposed to be Gaussian and the phase of scattered wave for the probing in direction of density gradient $((\omega\rho/c)^2 \gg 1)$ takes the form

$$\Psi \approx \frac{4L\omega}{3c} - \frac{Lc}{\omega} [k_y^2 + (q - k_y)^2] + \frac{1}{2} \left(\frac{c}{\omega}\right)^3 L [k_y^4 + (q - k_y)^4] \quad (7)$$

The Cross-Correlation Function calculation.

Carrying out the integration over the poloidal wave number of incident wave in (4) by saddle point method and using condition (3) we get the expression for the Cross-Correlation Function (CCF) of two scattering signals at frequencies ω_0 and ω in terms of the turbulence spectrum

$$\langle A_s(\omega_0) A_s^*(\omega) \rangle = 16\pi^2 \left(\frac{e^2}{mc^2}\right)^2 P_i \ell^3 \rho^2 \exp\left\{-i\frac{4L\Delta\omega}{3c}\right\} J(\omega - \omega_0) \quad (8)$$

where $\Delta\omega = \omega - \omega_0$

$$J(\omega - \omega_0) = \iint d\kappa dq \frac{\exp\left\{-\frac{q^2 \rho^2}{2} + i\Delta\omega/\omega [q^2 Lc/\omega - 2\kappa L]\right\} |\delta n_{q,\kappa}^2|}{\sqrt{\kappa^2 \rho^4 + \left[2Lc/\omega \kappa + L(cq/\omega)\right]^2}} \quad (9)$$

For the sake of simplicity we first analyze expression (9) for the gaussian turbulence spectrum

$$|\delta n_{q,\kappa}^2| = \frac{4\pi(\delta n)^2}{k^2} \exp[-(\kappa^2 + q^2)/k^2] \quad (10)$$

supposing long scale of fluctuations $kc/\omega \ll 1$ and large enough distance from antenna to the cut off $L > \omega\rho^2/2c$. In this case the main contribution to (9) is provided by the singularities of denominator which are given by condition

$$\kappa = -\frac{q^2 c}{\left[2\omega \left(1 \pm i \rho^2 \omega / 2Lc\right)\right]} \quad (11)$$

The asymptotic behavior of (8) in the case $2(\Delta\omega/\omega)L \gg k^{-1}$ when the distance between cut offs of the two probing waves is larger than the turbulence coherence length, can be obtained from (9) in the following form

$$\langle A_s(\omega_0) A_s^*(\omega) \rangle = 4\pi P_i \frac{\omega^3 \rho}{c^3 k^2} \left(\frac{\delta n}{n_c}\right)^2 \frac{\left\{-\ln\left[\frac{\Delta\omega}{4\omega} \left(\frac{1}{2} + \frac{1}{k^2 \rho^2} - 2i \frac{\Delta\omega}{\omega} \frac{Lc}{\omega \rho^2}\right)^{-1}\right] - \gamma\right\} \sqrt{\pi} + \varphi}{\sqrt{\frac{1}{2} + \frac{1}{k^2 \rho^2} - 2i \frac{\Delta\omega}{\omega} \frac{Lc}{\omega \rho^2}}} \times \exp\left\{-i\frac{4L\Delta\omega}{3c}\right\} \quad (12)$$

where $\gamma = 0.577$, $\varphi = 3.48$

It is possible to obtain the explicit expression for the autocorrelation function in the limit $kc/\omega \ll 1$ as well. It is given by (12), where $\Delta\omega/\omega$ is taken to be $\Delta\omega/\omega = 16kLe^{\gamma/2}$. The normalized CCF $K(\omega_0, \omega) = \langle A_s(\omega_0)A_s^*(\omega) \rangle / \langle A_s(\omega_0)A_s^*(\omega_0) \rangle$ has a very slow variation for $\Delta\omega/\omega > 1/2kL$. It is logarithmic up to $\Delta\omega/\omega = \omega/2Lck^2$. At higher $\Delta\omega/\omega > \omega/2Lck^2$ one get $K \sim (\Delta\omega/\omega)^{-1/2}$. The dependence $K(\Delta\omega/\omega)$ given by (12) is shown in Fig. 1 by green dotted line for $k = 0.2\omega/c$, $L = 212\pi c/\omega$, $\rho = 0.752\pi c/\omega$. Green triangles there correspond to $K(\Delta\omega/\omega)$ calculated numerically directly from equations (8), (9). As one see, (12) gives perfect approximation of K up to $K \sim 0.95$.

Dependence (12) for CCF is very different from exponential one

$$K \sim \exp\left\{-\left(\frac{\Delta\omega}{\omega}\right)^2 k^2 L^2\right\} \tag{13}$$

shown by red dotted line in Fig. 1, which one should expect if the cut off plays the dominant role in the signal generation. Such a difference is explained by the significant role of small angle scattering far from cut off in the scattering signal origination. The signal is produced all over the trajectory of the probing wave by fluctuations satisfying condition (11). Only for these fluctuations the Bragg conditions $k_{sy} = k_{iy} - q$, $k_{sx}(x_s) = k_{ix}(x_s) - \kappa$ are fulfilled simultaneously with the ray tracing condition that the scattered radiation comes back to the horn, which radiated the incident wave : $\Delta y_s = -\Delta y_i$, here Δy_i is the poloidal displacement of the incident ray trajectory on the way from the horn to the scattering point and Δy_s is the displacement of the scattered wave trajectory on the way back.

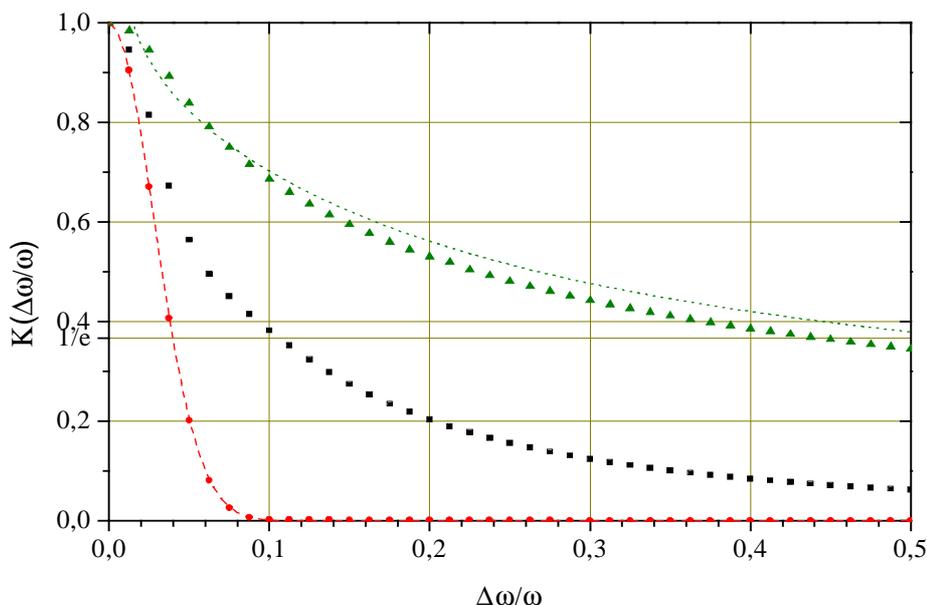


Fig.1 Dependence of normalised CCF on frequency mismatch of two channels

According to (11) for these fluctuations condition $\kappa \ll q$ holds. Their influence, is reduced if long scales are suppressed in the fluctuation spectrum, as it is shown by numerical calculation of CCF for spectrum $|\delta n_{q,\kappa}^2|/n_c^2 \sim (\kappa^2 + q^2)/k^2 \exp\{-(\kappa^2 + q^2)/k^2\}$. The corresponding results are shown by black squares in Fig. 1. In spite of much quicker decrease of the CCF for this spectrum it is still slower than given by condition (13). The expectations of the naive consideration are confirmed by the 2D theory only for spectrum

$$\frac{|\delta n_{q,\kappa}^2|}{n_c^2} \sim \frac{|\kappa + cq^2/2\omega|}{k} e^{-\frac{\kappa^2 + q^2}{k^2}}$$

where the small angle scattering is completely suppressed. (See red circles in Fig. 1).

Conclusions.

The slow decrease of the coherence, shown above for different fluctuation spectra, demonstrates big role of small angle scattering contributing to the fluctuation reflectometry all over the ray trajectory. This effect makes the localization of the diagnostics poor, at least for statistically homogeneous turbulence and in linear approximation. It is likely that for higher fluctuation level the strong small angle scattering turns into the multi-scattering leading to the fast suppression of the coherence for growing frequency difference of probing waves. This effect can be responsible for mentioned above surprising observations [4,5].

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