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# The plateau regime of neoclassical transport in stellarators*

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## Introduction

The plateau regime of transport in stellarators corresponds to the so called intermediate region of particle collision frequencies for which the effects related to the 1/$\nu$ transport are yet not important and for which transport coefficients depend only weakly on the collision frequency. Here, a new formula for the transport coefficients is obtained which is based upon a solution of the drift kinetic equation. This formula holds for conditions corresponding to the plateau regime without any further restriction for the magnetic field geometry but the existance of intact flux surfaces. The transport coefficients are expressed in terms of a weighted integral of the geodesic curvature along magnetic field lines. The proposed technique is applied to the computation of transport coefficients for a number of interesting stellarator magnetic configurations, U-3M, W-7X, and the quasi-helically symmetric stellarator HSX.

## Derivation

It is well known that for sufficiently small collision frequencies $\nu$, particles with small $v_{||}$ give the main contribution to the neoclassical transport as long as the radial electric field is not to high. On the other hand, if $\nu$ is still large enough so that effective particle trapping does not occur, the plateau regime of neoclassical transport is realized (see, e.g. [1]). Theoretically this regime was extensively studied either within kinetic theory for rather simple stellarator magnetic field models (e.g. [1,2]) or within fluid theory in magnetic coordinates for more general magnetic field models (e.g. [3]). For solving the drift kinetic equation within this regime, it is convenient to consider the distribution function $f = f_M + \tilde{f}$ as a function of $\mathbf{r}$, $w$ and $v_{||}$ ($w$ being the particle energy). With this, the drift kinetic equation becomes

$$v_{||}\mathbf{h} \cdot \nabla \tilde{f} + \mathbf{V} \cdot \nabla f_M = St(\tilde{f}) \ , \tag{1}$$

where $f_M = f_M(\psi, w)$ is the Maxwellian distribution function, $\psi$ is the magnetic surface label, $\mathbf{h} = \mathbf{B}/B$, and $\mathbf{V}$ is the drift velocity. In (1), the term $-(\partial \tilde{f}/\partial v_{||}) J_\perp \mathbf{h} \cdot \nabla B/2$ ($J_\perp = v^2/B$, $v$ velocity module) has been omitted. This defines the lower limit for $\nu > v_{||}\mathbf{h} \cdot \nabla B/B$ in accordance with the usual limit for the plateau regime [1]. In the general case, the transport coefficients in the intermediate region of particle collision frequencies are monotonic functions of the collision frequency (see, e.g. [4]). However, for a sufficiently large aspect ratio of the torus it is known that the neoclassical transport coefficients in this regime practically do not depend on the collision frequency. Theoretically it was shown earlier that the plateau transport coefficients practically do not depend on details of the collision operator and do not differ for various forms of this operator [1,2].

Therefore, the simplest form of the collision operator in so called $\tau$ approximation was chosen [1]. For this approximation of $St(\tilde{f})$, the drift kinetic equation has the following form,

$$v_{||}\mathbf{h} \cdot \nabla \tilde{f} + \mathbf{V} \cdot \nabla \psi \frac{\partial f_M}{\partial \psi} = -\nu \tilde{f} \ . \tag{2}$$

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This equation can be written as

$$\frac{\partial \tilde{f}}{\partial s} + p\tilde{f} = -\frac{1}{v_{||}} P \frac{\partial f_M}{\partial \psi} \ , \tag{3}$$

with $p = \nu / v_{||}$ and

$$P = P(s) = \mathbf{V} \cdot \nabla\psi = \frac{v^2 B_0}{2\omega_{c0}} \left(1 + \lambda^2\right) \frac{|\nabla\psi| k_G}{B} \ , \tag{4}$$

with the geodesic curvature $k_G = (\mathbf{h} \times (\mathbf{h} \cdot \nabla)\mathbf{h}) \cdot \nabla\psi / |\nabla\psi|$ , the arc-length $s$ of a magnetic field line, the cyclotron frequency $\omega_{c0} = eB_0/(mc)$ at the reference magnetic field $B_0$, and $\lambda = v_{||}/v$. The single-valued solution of (3) is

$$\tilde{f} = -\frac{1}{v_{||}} \frac{\partial f_M}{\partial \psi} \int_{-\infty}^{s} P(s') e^{-p(s-s')} ds', \quad v_{||} > 0 \quad (p > 0) \ , \tag{5}$$

$$\tilde{f} = -\frac{1}{|v_{||}|} \frac{\partial f_M}{\partial \psi} \int_{s}^{\infty} P(s') e^{-|p|(s'-s)} ds', \quad v_{||} < 0 \quad (p < 0) \ , \tag{6}$$

$$\tilde{f} = -\frac{1}{\nu} P \frac{\partial f_M}{\partial \psi}, \quad v_{||} = 0 \ . \tag{7}$$

A form of the particle flux density, $F_n$, valid in all regimes is

$$\begin{aligned} F_n &= \frac{2\pi}{mS} \oint_{\psi} \frac{dS}{|\nabla\psi|} \int dv_{||} \int dw \tilde{f} \, \mathbf{V} \cdot \nabla\psi \\ &= 2\pi \left( \int_0^{L_s} \frac{ds}{B} |\nabla\psi| \right)^{-1} \int_0^{\infty} dv v^2 \int_0^{L_s} \frac{ds}{B} \int_{-1}^{1} d\lambda \, P\tilde{f} \ , \qquad (L_s \to \infty) \ . \end{aligned} \tag{8}$$

Using (4) and taking into account the solution for $\tilde{f}$ one obtains

$$F_n = -2\pi \left( \int_0^{L_s} \frac{ds}{B} |\nabla\psi| \right)^{-1} \int_0^{\infty} dv v^2 \frac{\partial f_M}{\partial \psi} \int_{-1}^{1} d\lambda \, I \ , \tag{9}$$

$$I = \frac{v^3 B_0^2}{4\omega_{c0}^2} \left(1 + \lambda^2\right)^2 \frac{1}{|\lambda|} Z(L_s) \ , \qquad (L_s \to \infty) \ , \tag{10}$$

$$Z(L_s) = \int_0^{L_s} ds \frac{|\nabla\psi| k_G}{B} \int_0^{s} ds' \frac{|\nabla\psi| k_G}{B^2} e^{p(s-s')}, \qquad v_{||} < 0 \ , \tag{11}$$

$$Z(L_s) = \int_0^{-L_s} ds \frac{|\nabla\psi| k_G}{B} \int_0^{s} ds' \frac{|\nabla\psi| k_G}{B^2} e^{p(s-s')}, \qquad v_{||} > 0 \ , \tag{12}$$

$$Z(L_s) = \frac{|v_{||}|}{\nu} \int_0^{L_s} ds \frac{(|\nabla\psi| k_G)^2}{B^3}, \qquad v_{||} \to 0 \ . \tag{13}$$

From these equations follows that $Z(L_s)$ can be computed by integrating the following differential equations over the interval $0 \div L_s$ (for $v_{||} < 0$) or $0 \div -L_s$ (for $v_{||} > 0$):

$$\frac{dZ}{ds} = \frac{|\nabla\psi| k_G}{B} Y, \qquad (Z(0) = 0) \,, \tag{14}$$

$$\frac{dY}{ds} = \frac{|\nabla\psi| k_G}{B^2} + p\, Y, \qquad (Y(0) = 0) \,, \tag{15}$$

and for $v_{||} \to 0$

$$\frac{dZ}{ds} = \frac{|v_{||}|}{\nu} \frac{(|\nabla\psi| k_G)^2}{B^3}, \qquad (Z(0) = 0) \,. \tag{16}$$

First $I$ as defined in (10) is computed for various values of $\lambda$ and then the numerical integration over $\lambda$ in (9) is performed. The integration shows to be insensitive to the lower and upper limits ($-1$ and 1) for not too large values of $\nu$.

The final result can be presented in the standard form for the plateau regime

$$F_n = -\frac{\sqrt{\pi}\rho_L^2 v_T}{8R\iota}\Lambda_p \int\limits_0^\infty dz\; z^2 e^{-z} \frac{n}{f_M}\frac{\partial f_M}{\partial \psi} < |\nabla\psi| > \,, \tag{17}$$

$$\Lambda_p = \frac{2RB_0^2\iota}{\pi} \left( \int\limits_0^{L_s} \frac{ds}{B}|\nabla\psi| \right)^{-2} \left( \int\limits_0^{L_s} \frac{ds}{B} \right) \int\limits_{-1}^{1} d\lambda \frac{\left(1+\lambda^2\right)^2}{|\lambda|} Z(L_s) \,, \tag{18}$$

$$< |\nabla\psi| > = \int\limits_0^{L_s} \frac{ds}{B}|\nabla\psi| / \int\limits_0^{L_s} \frac{ds}{B} \,, \tag{19}$$

where $\rho_L = mcv_T/(eB_0)$ is the mean Larmor radius, $z = mv^2/(2T)$, and $\iota$ is the rotational transform. The expression for the energy flux density differs from (17) by a factor $zT$ under the integral. For an axially symmetric tokamak with large aspect ratio $\Lambda_p$ equals unity. The deviation of $\Lambda_p$ from unity gives the effect of the magnetic field geometry on the transport coefficients.

**Results**

The proposed technique is numerically verified in the limit of a tokamak-like axially symmetric magnetic field. For $\iota$ in the range of 0.3÷0.9 one obtains $\Lambda_p \approx 1.005 \div 1.02$ for an aspect ratio of 50 and $\Lambda_p \approx 1.05 \div 1.07$ for an aspect ratio of 10. The technique is applied to study the transport in the plateau regime in the zero-$\beta$ limit of the quasi-helically symmetric stellarator HSX [5], the W-7X standard configuration with parameters given in [6], and a simplified magnetic field corresponding to the U-3M torsatron. The calculations are done in real space coordinates. For HSX the magnetic field (and its spatial derivatives) are computed from the the Biot-Savart law. The W-7X magnetic field is presented as a superposition of a finite number (465) of toroidal harmonic functions containing the associated Legendre functions. For U-3M a simplified magnetic field containing only one toroidal harmonic is considered.

The computational results (magnetic surfaces with no islands) are presented in Fig. 1. The results show that for the simplified U-3M configuration $\Lambda_p$ is rather close to unity. Only for the region near the boundary $\Lambda_p$ is increasing. This increase can be explained by the effect of the helical magnetic field modulation. For HSX, $\Lambda_p$ is significantly smaller than the corresponding value for U-3M. The largest decrease in $\Lambda_p$ is seen for the W-7X configuration yielding an improvement by a factor of 0.25÷0.3 compared to a tokamak with equivalent big radius and rotational transform.

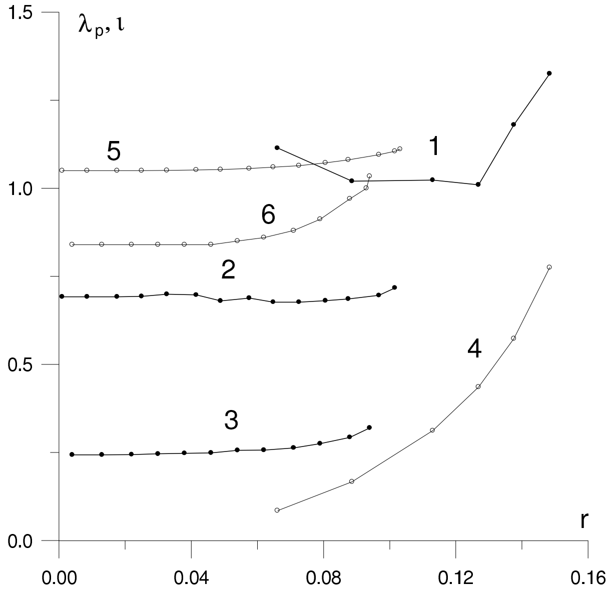


Fig. 1. The parameter $\Lambda_p$ and the rotational transform $\iota$ (in units of $2\pi$) versus the mean magnetic surface radius in units of the big torus radius. Curves 1 and 4 give $\Lambda_p$ and $\iota$ for U-3M, respectively, curves 2 and 5 for HSX, and curves 3 and 6 for W-7X.

**Summary**

A new technique is derived for calculating the transport coefficients in the plateau regime. The transport coefficients are computed through an integration along magnetic field lines in a given magnetic field. This magnetic field can be given in real space as well as in Boozer coordinates. For equivalent big torus radii and rotational transforms, the plateau transport coefficients for the simplified U-3M are of the same order (or somewhat larger) than those for a tokamak. The transport coefficients are about 30% smaller for HSX and approximately four times smaller for W-7X than those for an equivalent tokamak.

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