

Evidence for role of magnetic entropy in stationary Ohmic tokamak discharges

H. Weisen and E. Minardi*

*Centre de Recherches en Physique des Plasmas
Association EURATOM - Confédération Suisse*

**Istituto di Fisica del Plasma "P. Caldirola"
Associazione EURATOM-ENEA-CNR, Milano, Italy*

INTRODUCTION

Profiles from a large variety of stationary Ohmic plasmas in the TCV tokamak ($B_T < 1.5T$, $R_0 = 0.88m$, $a < 0.25m$, $\kappa < 2.7$) are compared to theoretical predictions based on the assumption that the magnetic entropy is stationary in a tokamak considered as an open system interacting with the Ohmic transformer. For each value of $\langle j \rangle / (q_0 j_0)$, where $\langle j \rangle$ is the cross sectional averaged current density, and q_0 , the theory predicts a rigid current profile in the confinement zone ($q > 1$). Integration of the force balance equation leads to a restriction of the range of possible pressure profiles.

We observe that the sawtooth inversion radius and the electron temperature in the confinement region, related to the predicted rigid current profile using the neoclassical Ohm's law, depend solely on the parameter $\langle j \rangle / (q_0 j_0)$, in agreement with the predictions. The stiffness of the temperature profile, which follows from the rigid current profile, implies a correlation between density and pressure profiles, which is also observed in the experiment.

This agreement indicates that, contrary to widespread expectance, statistical concepts such as the magnetic entropy, can be of relevance to systems driven far from thermodynamical equilibrium, such as tokamak plasmas. The theory, as it stands, does however not provide a complete description of the behaviour of the pressure and density profiles. It makes no prediction as to the scaling of pressure profiles with $\langle j \rangle / (q_0 j_0)$, although such a scaling is also observed in the experiments, indicating that additional physics will need to be incorporated.

THEORY SUMMARY

Magnetic entropy is defined as a measure of probability of current density configuration in a suitably constrained space of magnetic equilibria [1,2].

$$S \propto - \int \dot{j}^2 dV + \frac{\mu^2 c}{4\pi} \int \dot{j} \cdot \vec{A} dV, \vec{A} \text{ being the vector potential and } \dot{j} \text{ the current density.}$$

In a plasma interacting with an Ohmic transformer the time variation of S is:

$$\frac{dS}{dt} \propto \int_V \left[\frac{\vec{E}}{2} \cdot \nabla^2 \dot{j} + \vec{E} \cdot \dot{j} \right] dV$$

μ is a free parameter with the meaning of a Lagrange multiplier and \vec{E} the induced electric field. In view of the arbitrariness of the integration volume V , the requirement of stationary entropy provides an equation for the steady-state toroidal current density profile:

$$\nabla^2 \dot{j} + \mu^2 \dot{j} = 0$$

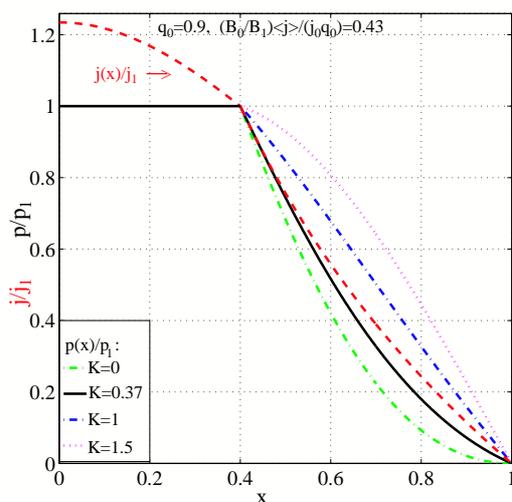


Fig.1 Theoretical current profile (dashed) and examples of pressure profiles with different values of convexity

Since the sawtooth core is not stationary, we apply the above equation to the confinement zone only. For given $j=j_1$ at $r_1=r(q=1)$, it yields a rigid current profile in cylindrical geometry, for $r>r_1$ (fig.1):

$$j(r) = j_1 [J_0(\mu a) Y_0(\mu r) - Y_0(\mu a) J_0(\mu r)] / D,$$

$$\text{where } D = J_0(\mu a) Y_0(\mu r_1) - Y_0(\mu a) J_0(\mu r_1).$$

Assuming a parabolic q profile for $r<r_1$, $j_1 = j_0 q_0 (B_1/B_0)$, where B_1 and B_0 are the toroidal field on the $q=1$ surface and the axis, the theory relates q_0 , q_a , and $\rho_1=r_1/a$. Combined with the Grad-Shafranov equation and an energy principle, the theory also restricts the range of possible pressure profiles [3]. For each q_0 , r_1 , there is a family of possible $p(r)$ parameterised by K ($0<K<1$) and differing in their degree of concavity.

COMPARISON WITH TCV OHMIC DATABASE

The 280 discharges investigated include almost the entire range of sawtooth Ohmic plasmas that can be produced in TCV, with $1<\kappa_a<2.54$, $-0.5<\delta_a<0.7$, $2<q_{95}<7$, $100\text{kA}<I_p<1.02\text{MA}$, $1.2\cdot 10^{19}\text{m}^{-3}<\bar{n}_e<1.2\cdot 10^{20}\text{m}^{-3}$ and $0.1<v_{75}^*<10$. The confinement properties of these discharges have been presented previously [4]. Since the theory is derived for circular cross sections only, we need to find a cylindrical equivalent to q_a . Theory suggests that the appropriate parameter is $B_0 \langle j \rangle / (q_0 j_0 B_1) \equiv \langle j \rangle / (q_0 j_0)$ as introduced in a previous study on sawtooth inversion radii and profile peaking factors [5]. For each sample in the database we construct a theoretical cylindrical model with the same value of $\langle j \rangle / (q_0 j_0)$ and (less importantly) the same value of β_p . The radial coordinate $x=r/a$ is identified with $(V/V_{tot})^{1/2}$. The parameter μ was set to $0.1/a$ and does not in effect play the role of a free parameter because for $\mu a < 1$, the predictions are in practice equivalent.

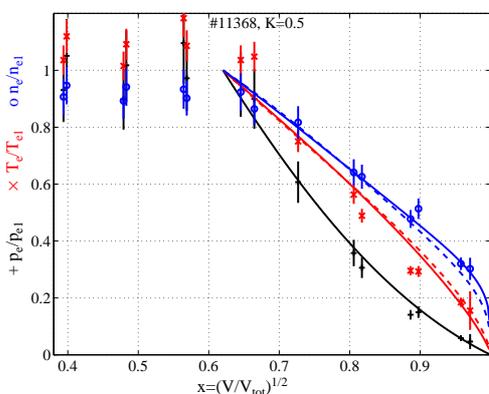


Fig.2 Measured profiles of electron temperature, pressure and density, together with theoretical profiles in the confinement zone. (dashed: Spitzer conductivity assumed)

Using the neoclassical Ohm's law, the predicted, Ohmically relaxed current profile can be related to a temperature profile, shown in red together with the pressure (black) and density profiles (blue) for an example of a TCV discharge in fig.2. These profiles are from a pulsed Thomson scattering diagnostic which is not synchronised with the sawtooth cycle. All profiles are normalised to their values at the inversion radius. The fitted line for the pressure profile corresponds in this example to a theoretical pressure profile with a convexity parameter $K=0.5$. The resulting line for the density profile is obtained by combining the theoretical electron temperature and pressure profiles.

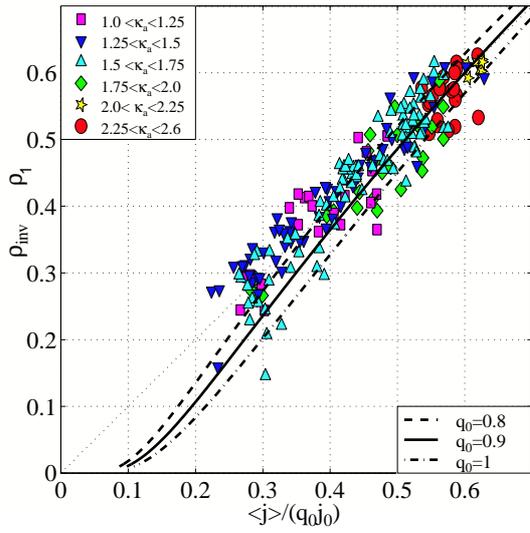


Fig.3 Measured normalised sawtooth inversion radii together with predicted positions for $q=1$ radii for three assumed values of q_0 . Symbols relate to elongation.

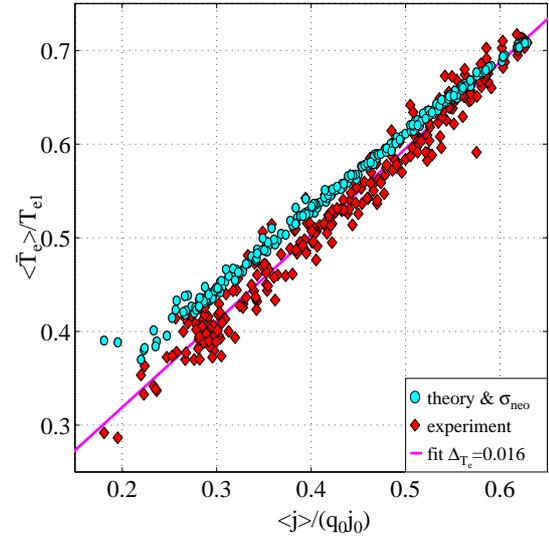


Fig.4 Scaling with $\langle j \rangle / q_0 j_0$ of electron temperature profile shape factors in confinement zone.

Predictions for $\rho_1(q_0, \langle j \rangle / (q_0 j_0))$ agree well with the observed inversion radius from X-ray tomography [5], for the generally accepted values for q_0 in sawtoothing plasmas ($0.8 < q_0 < 1$), as seen fig. 3. For the remainder of the paper we assume $q_0 = 0.9$. A systematic comparison of profile shapes in the confinement zone is made by defining “clipped profiles” such as \bar{T}_e / T_{e1} , where $\bar{T}_e = \max(T_e, T_{e1})$ which in effect disregards the reheating inside the inversion radius after the core has been flattened by a sawtooth crash (fig.4). The quantity $\langle \bar{T}_e \rangle / T_{e1}$ has the advantage of being sensitive only to the profile shape for $r > r_I$ and does not depend on the time at which the Thomson scattering measurements are taken, because plasma parameters near the inversion radius vary relatively little during the sawtooth cycle.

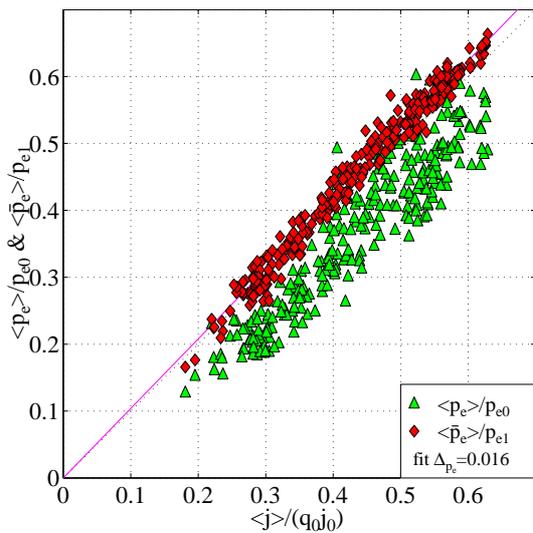


Fig.5 Scaling of experimental pressure profile factors with $\langle j \rangle / q_0 j_0$.

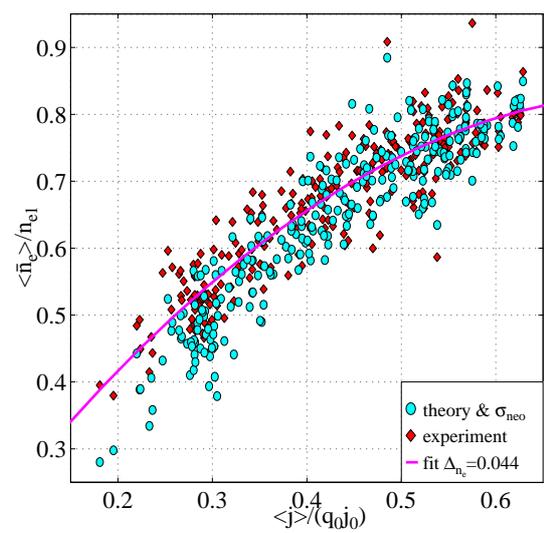


Fig.6 Scaling of experimental and expected density profile factors.

Fig.4 shows that the measured shape of the temperature profile (diamonds) in the confinement zone is in good agreement with the one expected from the theoretical current profile and Ohm's law (dots). The scaling with $\langle j \rangle / (q_0 j_0)$ of the experimental pressure profile factors $\langle \bar{p}_e \rangle / p_{e1}$ (diamonds) and $\langle p_e \rangle / p_{e0}$ (triangles) are shown in fig. 5. The latter exhibit more scatter because of sampling at random times during the sawtooth cycle. Fig.6 shows that experimental density profile factors (diamonds) also scale with $\langle j \rangle / (q_0 j_0)$. The density profile widths expected from the theoretical current profile and the matching theoretical pressure profiles are shown as dots in fig.6.

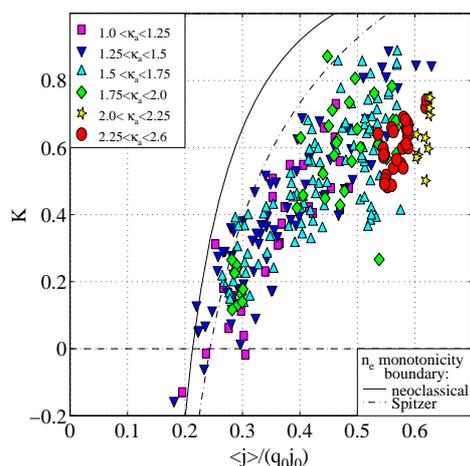


Fig.7 Convexity parameter K corresponding to experimental pressure profiles in fig.5.

The values of the theoretical convexity parameter corresponding to the pressure profiles in fig.5 (diamonds) are plotted in fig.7. There is a systematic trend, with K increasing from near zero at the lowest values of $\langle j \rangle / (q_0 j_0)$ to near 0.6 at the highest. This dependence is not predicted by the theory, which merely constrains K to be in the interval $[0,1]$. Pressure profiles with K larger than the curves plotted in fig.7 would correspond to hollow density profiles. The absence of high values of K at low values of $\langle j \rangle / (q_0 j_0)$ may be explainable if the plasma lacks a mechanism to set up the required outward particle drift over part of the cross section. In any case, a more complete description is likely to require the integration of additional physics concerning particle and/or energy transport.

In figs. 4-6 we show the experimental average scaling of shape factors as solid lines. It is interesting to note that departures from the average correlate for density and pressure, while temperature profile deviations remain uncorrelated with the former two and can be attributed to measurement errors. Such behaviour is expected from stiff temperature profiles (fig.8), such as predicted by the theory presented [3].

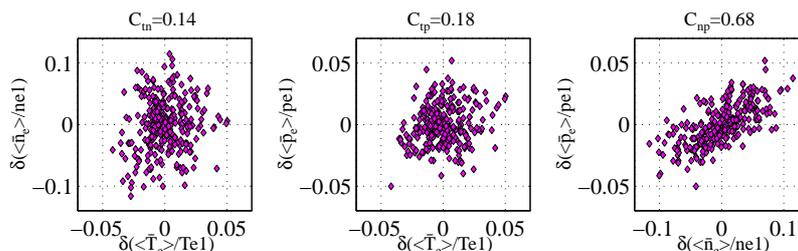


Fig.8 Cross correlations of profile shapes in ohmic database.

Acknowledgement: This work was partly supported by the Swiss National Science Foundation. The support of the entire TCV team is gratefully acknowledged.

References

- [1] E. Minardi, J. Plasma Physics **48** (1992) 281
- [2] E. Minardi, J. Plasma Physics **62** (1999), 319
- [3] E. Minardi and H. Weisen, submitted to NF (2000), CRPP report LRP 669/00.
- [4] H. Weisen et al, Plasma Phys. Contr. Fusion **39** (1997) B135
- [5] H. Weisen et al, Plasma Phys. Contr. Fusion **40** (1998) 1803