

Poloidal Plasma Rotation by the Ponderomotive Force of Intense Electron Cyclotron Waves

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1 Introduction

There is a great deal of interest in producing sheared radial electric fields in tokamak plasmas and its associated poloidal plasma rotation, due to the role they play in the transition to improved confinement regimes. In many cases the rotation arises spontaneously as the plasma heating exceeds a certain threshold and the resulting transport barrier appears near the plasma edge, as in the H mode. However, it would be desirable to be able to control when and where the transport barrier occurs in order to manage thermal confinement at our convenience. In this respect there have been several works trying to provide methods to produce sheared plasma rotation by external means, like for instance RF power [1]. Here we present an alternative method for driving plasma rotation, based on the ponderomotive (PM) force associated with a large amplitude RF wave. The new feature of this approach lies in the fact that momentum is not imparted directly to the plasma by the PM force, but rather through a spin-up instability similar to Stringer spin-up [2]. Thus, the asymmetric nature of the RF wave injection produces a field-aligned return flow to maintain mass continuity, which turns out to be rotationally unstable under the influence of field line curvature.

The dynamics we consider evolves in a slow time scale, of the order of $\sim \delta^2 \nu_{ii}$ where $\delta = \rho_i/a$ is the ratio of ion gyroradius to minor radius, and ν_{ii} the ion collision frequency. The field aligned flows may arise in two different ways, depending on the direction of the PM force: (1) For a poloidal/toroidal PM force $F_{\theta/\zeta}$ the resulting drift $F_{\theta/\zeta} \times B$ directly produces a poloidally asymmetric radial flow; and (2) a radial PM force F_r in presence of collisional dissipation can give rise to a poloidal drift $v_\theta = F_r \times B$, which produces a frictional force $F_{fr} = -mn\nu v_\theta$, originating in turn the asymmetric radial drift $F_{fr} \times B$. In the next section we make the dynamical analysis of the rotation and in Sec.3 we discuss the nature of the PM force in each one of the two cases mentioned, generated by electron cyclotron (EC) waves. The use of this type of waves allows to have a more localized rotation region which in turn increases the shear flow that produces the transport barrier.

2 Plasma Dynamics

We use a two-fluid model for the plasma and consider axisymmetric toroidal geometry with the magnetic field given by $\mathbf{B} = \nabla\psi \times \nabla\zeta + I\nabla\zeta$. The relevant equations are continuity and momentum balance equations for each species $a = e, i$,

$$\frac{\partial n_a}{\partial t} = -\nabla \cdot (n_a \mathbf{u}_a) \quad (1)$$

$$m_a n_a \left(\frac{\partial \mathbf{u}_a}{\partial t} + \mathbf{u}_a \cdot \nabla \mathbf{u}_a \right) = -\nabla p_a - \nabla \cdot \pi_a - q_a n_a (\nabla \phi - \frac{1}{c} \mathbf{u}_a \times \mathbf{B}) - \sum_b m_a n_a \nu_{ab} (\mathbf{u}_a - \mathbf{u}_b) + \mathbf{F}_{ap} \quad (2)$$

where only the electrostatic field is included, and we consider a collision term with particles of species b (which includes neutrals) and a momentum source given by the ponderomotive force \mathbf{F}_{ap} . In terms of the expansion parameter δ , the lowest order terms of these equations, denoted by capital letters, yield $\Phi = \Phi(\psi)$ and $P_a = P_a(\psi)$ and the following expressions for the first order velocity \mathbf{U}_a :

$$U_{a\zeta} = -\frac{c}{B_\theta} \left(\Phi'(\psi) + \frac{1}{q_a n_a} P_a'(\psi) \right) + \lambda_a B_\zeta \quad (3)$$

$$U_{a\theta} = \lambda_a B_\theta \quad (4)$$

where λ_a is a function to be determined; as it can be noticed it measures the poloidal velocity. We will assume $\lambda_a = \lambda_a(\psi)$ which is valid if n_a is also a surface quantity to lowest order. We will be concerned with determining the time evolution of λ_a and for that purpose we need the higher order contributions of Eqs.(1-2). Thus, we write $\mathbf{v}_a = \mathbf{u}_a - \mathbf{U}_a$ and subtract the lowest order equations mentioned before from the full equations to find the evolution of λ_i , which directly related to the plasma flow. Instead of using the three components of the momentum equation we only need the parallel component and the toroidal component, and then average over a magnetic surface, in order to eliminate the pressure term. When we add the equations for ions and for electrons, neglecting electron inertia, we arrive at a single equation for each component, which, together with the flux-surface averaged Ampere-Maxwell equation, are,

$$\frac{\partial}{\partial t} \langle \nabla \Phi \cdot \nabla \psi \rangle = 4\pi \langle \mathbf{j} \cdot \nabla \psi \rangle \quad (5)$$

$$m_i n_i \left(\frac{\partial}{\partial t} \langle \mathbf{U}_i \cdot \mathbf{B} \rangle + \langle \mathbf{B} \cdot (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i \rangle \right) = -\langle \mathbf{B} \cdot \nabla \cdot \pi_i \rangle - m_i n_i \langle \nu_{in} \mathbf{B} \cdot \mathbf{U}_i \rangle + \langle \mathbf{B} \cdot \mathbf{F}_p \rangle \quad (6)$$

$$m_i n_i \left(\frac{\partial}{\partial t} \langle U_{i\zeta} R \rangle + \langle R \hat{\zeta} \cdot (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i \rangle \right) = -\frac{1}{c} \langle \mathbf{j} \cdot \nabla \psi \rangle - m_i n_i \langle \nu_{in} R U_{i\zeta} \rangle + \langle R F_{p\zeta} \rangle \quad (7)$$

As mentioned above, the time evolution of the velocity is of second order in δ , so the other terms have to be obtained to this order. The remaining collisions here are those with neutrals, which are assumed to have negligible velocity. The convective terms can be written as,

$$\langle \mathbf{B} \cdot (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i \rangle = \frac{d}{d\psi} \langle \mathbf{B} \cdot \mathbf{U}_i \mathbf{v}_i \cdot \nabla \psi \rangle \quad (8)$$

$$\langle R \hat{\zeta} \cdot (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i \rangle = \frac{d}{d\psi} \langle R U_{i\zeta} \mathbf{v}_i \cdot \nabla \psi \rangle \quad (9)$$

and thus contains the radial component of the velocity \mathbf{v} , which has to be evaluated to order δ^2 . This provides the drive for the spin-up when it is poloidally asymmetric. In the case we are considering here the radial flow is produced by the PM force. We compute this velocity for two different cases:

(1) PM force has a (poloidally asymmetric) component on the magnetic surface. Then, applying the operator $\nabla \psi \cdot \mathbf{B} \times$ to Eq.(2) for ions to second order, one gets

$$\mathbf{v}_i^{(2)} \cdot \nabla \psi = \nabla \psi \cdot \frac{\mathbf{F}_p^{(2)} \times \mathbf{B}}{B^2} + T^{(2)} \quad (10)$$

where the term $T^{(2)}$ includes the other terms that are not of relevance in this analysis. Thus, a relatively weak PM force would drive the required radial flow.

(2) The more interesting case when there is radial component of the PM force. Then we can obtain the velocity $\mathbf{v}_i^{(2)}$ easier, from a combination of the total momentum equation

and the generalized Ohm's law (which result from adding and subtracting the Eqs.(2) for electrons and ions)

$$\mathbf{v}^{(2)} \cdot \nabla \psi = \nabla \psi \cdot c \frac{\mathbf{B}}{B^2} \times \mathbf{j}^{(1)} \eta + T = -\eta c^2 \frac{\mathbf{B}}{B^4} \times (\mathbf{B} \times \mathbf{F}_p^{(1)}) \cdot \nabla \psi + T = \frac{\eta c^2}{B^2} \mathbf{F}_p^{(2)} \cdot \nabla \psi + T \quad (11)$$

T again represents the remaining terms. Then we see that the frictional drag, either from neutrals or from Coulomb collisions, can produce the radial drive, but in this case the radial PM force has to be of first order; so a more powerful wave is required. In next section we estimate its amplitude.

Once the radial velocity is known, we turn back to the evolution equations for the velocity \mathbf{U} . In principle, we still need to compute the viscosity term in Eq.(6), but to reduce the calculations we just write it as a linear function of the velocity, which is valid for the parallel viscosity (perpendicular and gyroviscosity may be neglected [3]) with low speeds: $\langle \mathbf{B} \cdot \nabla \cdot \pi_i \rangle = \mu U_\theta + k$; in axisymmetry U_ζ does not appear. We may now substitute Eqs.(3-4) in Eqs.(6-7) in order to find two coupled equations for $\lambda_i(\psi, t)$ and $\Phi'(\psi, t)$,

$$a_1 \frac{\partial \Phi'}{\partial t} + a_2 \frac{\partial \lambda}{\partial t} = a_3 \Phi' + a_4 \lambda + c_1 \quad (12)$$

$$b_1 \frac{\partial \Phi'}{\partial t} + b_2 \frac{\partial \lambda}{\partial t} = b_3 \Phi' + b_4 \lambda + c_2 \quad (13)$$

with,

$$\begin{aligned} a_1 &= m_i n_i J \left\langle \frac{R}{B_\theta} \right\rangle, & a_2 &= -m_i n_i I, & a_3 &= -m_i n_i \left(\frac{d}{d\psi} \left\langle \frac{R}{B_\theta} \mathbf{v}_i^{(2)} \cdot \nabla \psi \right\rangle + \left\langle \frac{\nu_{in} R}{B_\theta} \right\rangle \right) \\ a_4 &= m_i n_i \left(\frac{d}{d\psi} \left\langle R B_\zeta \mathbf{v}_i^{(2)} \cdot \nabla \psi \right\rangle + \left\langle \nu_{in} R B_\zeta \right\rangle \right), & b_1 &= m_i n_i \left\langle \frac{B_\zeta}{B\theta} \right\rangle, & b_2 &= -m_i n_i \langle B^2 \rangle \\ b_3 &= -m_i n_i \left(\frac{d}{d\psi} \left\langle \frac{B_\zeta}{B_\theta} \mathbf{v}_i^{(2)} \cdot \nabla \psi \right\rangle + \left\langle \frac{\nu_{in} B_\zeta}{B_\theta} \right\rangle \right), & c_1 &= \langle R F_{p\zeta} \rangle, & c_2 &= k + \langle \mathbf{B} \cdot \mathbf{F}_p \rangle, \\ b_4 &= m_i n_i \left(\frac{d}{d\psi} \langle B^2 \mathbf{v}_i^{(2)} \cdot \nabla \psi \rangle + \langle \nu_{in} B^2 \rangle \right) + \mu B_\theta & J &= 1 - v_{A\theta}^2 / c^2 \end{aligned}$$

The solution to this equations might have growing solutions, depending on the sign of the coefficients a 's and b 's. It will be of the type $e^{\gamma_{\pm} t}$ where γ_{\pm} are the eigenvalues of the characteristic equation, which has a complicated form. We notice, however, that the coefficients a_3, a_4, b_3, b_4 , which contain the radial flow are responsible for the exponential behavior. There should be a poloidal dependence of $\mathbf{v}_i^{(2)} \cdot \nabla \psi$ and of the right sign, for these terms to contribute to a spin-up. This is of course competing with other terms producing dissipation, and specific cases will have to be considered in order to determine the relative contribution of each.

3 PM force from EC waves

We now turn the attention to the source of the PM force. The complete expression of the PM force has been derived in [4] to be,

$$F_{pj} = -\frac{1}{2\omega} \text{Im} \left[\frac{\partial E_k^*}{\partial x_j} \sigma_{kl} E_l - \frac{\partial}{\partial x_k} (\sigma_{kl} E_l E_j^*) \right] - \frac{2\pi}{\omega^2} \text{Re} \frac{\partial}{\partial x_k} \left[\sigma_{kl} E_l \sigma_{jm}^* \frac{E_m^*}{v} \right] \quad (14)$$

where $v = \omega_{pa}^2 / \omega^2$. Since the force is proportional to the wave field amplitude gradient it is reasonable to look for waves that are absorbed within a short distance, to maximize

the magnitude of F_p . We therefore consider EC waves as viable candidates for this goal. For case (1) above, a PM force on the flux surface (preferably in the poloidal direction for larger $v_i^{(2)}$ in Eq.(10)) could be generated by launching the wave in tangential direction close to a resonant surface so that the amplitude decreases as the wave is absorbed along its propagation direction. The technique would be the same as the one used in recent current drive experiments [5]. A very large PM force would not be expected with this method since the absorption length in poloidal direction is not small, but on the other hand a very large F_p is not needed since it has to be of second order, as seen in Eq.(10).

The more interesting case is (2) which we will consider in more detail. A radial PM force may be generated by launching the EC wave radially inwards, perpendicularly to the magnetic field, and as it reaches the resonant surface it is absorbed within a distance $\Delta_R \approx R\beta_T^2$ [6], where β_T is the thermal speed relative to c . Using the conductivity for a cold plasma the resulting radial ponderomotive force is,

$$F_{pr} = \frac{1}{8\pi} \frac{v \text{Im}k_r}{(1 - \varpi)^2} \left((1 + \varpi)|E|^2 + 4\sqrt{\varpi} \text{Im}(E_r E_\theta^*) \right) - F \quad (15)$$

where $\varpi = \omega_{Ba}^2/\omega^2$, F is a function of the radial derivatives of the plasma parameters not given here. The imaginary radial wavenumber k_r measures the wave absorption and depends on the wave mode used. The most convenient would be the X-mode at the second harmonic; this mode has a longitudinal E -component that originates an efficient radial PM force. The magnitude of F_{pr} could then be computed from this mode's value [6] of $\text{Im}k_r = k_o 18\sqrt{\pi} q z^{5/2} \exp(-z^2)/15(3-q)^2$, with $q = v/\varpi$ and $z = (2\omega_{Ba} - \omega)/\omega_{Ba}\beta_T^2$. Ray tracing computations have shown that wave propagation remains radial for a small angular range [6]. Since Eq.(11) requires a force of order δ , the wave amplitude can be estimated i.e. for DIII-D parameters: $\delta \sim 10^{-2}$, $E \sim [\delta|\nabla P|16\pi\Delta_R/(1-\epsilon)(1-e^{-\tau})^{1/2}] \sim 10 \text{KV/cm}$, (τ - optical depth, ϵ - dielectric constant) which is large but attainable with few gyrotrons acting together.

4 Conclusions

We have proposed two methods for using EC waves to produce localized poloidal rotation, through the radial PM force resulting from their absorption. A required poloidal asymmetry is intrinsic to the wave injection process. The approach based on a radial PM force has not been considered before, and seems to be more attractive, since wave launching is quite easy; it would have to be done from the HFS (inboard). Antenna phasing would not be needed. In addition, it may be convenient for controlling the resulting transport barrier position by adjusting the location of the resonant surface. Relatively high powers are necessary $P \sim 5 \text{KW/cm}^2$.

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References

1. J.A. Tataronis and V. Petržílka, Phys. Plasmas **3**, 4434 (1996).
2. T.E. Stringer, Phys. Rev. Lett. **22**, 1770 (1969); A.B. Hassam and J.F. Drake, Phys. Fluids B **5**, 4022 (1993).
3. J.W. Connor, S.C. Cowley, R.J. Hastie and L.R. Pan, Plasma Phys. Control. Fusion **29**, 919 (1987).
4. R. Klima, V.A. Petržílka, Czech. Journal Phys. B **30**, 1002 (1980).
5. O. Sauter et al., Phys. Rev. Lett. **84**, 3322 (2000).
6. V.V. Alikhaev, A.G. Litvak, E.V. Suvorov and A.A. Fraiman, in *High-Frequency Plasma Heating*. A.G. Litvak (Ed.) (AIP, New York, 1992)