Reduced descriptions of light impurities in hydrogen coronal plasmas and their validities.

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In recent years the impurity dynamics in hydrogen plasmas has been developed intensively in the context of nuclear fusion and astrophysical problems (see review papers [1-3]). The impurity radiation can change significantly the dispersion properties of plasmas. It is able to excite and damp thermal instabilities. Moreover, impurities are able to damp drift instabilities [4,5] and influence on L – H transitions. For full impurity descriptions one needs a cumbersome set of equations describing every ionization state separately. For this reason, some reduced models for impurity ionization states are under development. (See, for instance, [6-8]). As shown in [6,7], the heavy (Z>20) impurity distribution over ionization states may be approximated for coronal plasmas by Gaussian functions if the electron temperature is not too high. Light impurities with Z<10 exist in coronal plasmas in the form of few neighboring ionization states with significantly different properties [8]. The ratios between the ionization and recombination rates for ions with charges nearest to the charge of the most representative ions are significantly larger than for the more distant ionization states. Thus, the impurity may be described in terms of the two or three most representative ions, at least for slow dynamic processes. The two and three ionization state models were proposed in [8-9]. However, the validity of them is not clear for fast processes if the characteristic time of the process is of the same order of magnitude or shorter than the impurity relaxation time. The purpose of the present work is to test the validity of the models for fast processes and modify them accordingly. We assume below that the plasma is uniform and ignore any spatial effects. To the two - ion approximation, one can get from the discontinuity equation together with the normalization condition $y_z + y_{z+1} = 1$

$$\frac{d\,y_z}{d\,t} = n_e\left[R_{z+1}y_{z+1} - \left(J_z + R_{z+1}\right)y_z\right]. \qquad (1)$$

Here n_k is the concentration of ions with charge k, $y_z = n_z / \sum_k n_k$ is the relative concentration of the most representative ionization state, z is the charge of the most representative ion, n_e is the electron concentration, while J_z and R_z are the ionization and recombination rates respectively. It seems that we can restrict ourselves to (1). However, even though we know that the most representative ions are the ions with charges z and $z+1$, we know the values of z and $z+1$ for the case of the coronal equilibrium only. Moreover, z may change its value in a discrete way. Thus, it is more convenient to change this variable by a continuous one, $<z>$, which is the charge averaged over ionization states.

$$< z >= z y_z + (z+1)\,y_{z+1} = z + 1 - y_z \ . \qquad (2)$$

This yields:

$$\frac{d < z >}{d t} = -\nu_z (< z > - z_*). \tag{3}$$

Here $\nu_z = n_e (J_z + R_{z+1})$, and $z_* = z + 1 - \frac{R_{z+1}}{J_z + R_{z+1}}$ is the averaged equilibrium charge under the given electron temperature. However, the model cannot describe changes of z with temperature changes. For instance, let us assume the most representative ion charge for the initial temperature is equal to z and the temperature decreases strongly to a small magnitude for which $J_z << R_{z+1}$. Under this condition one gets $z_* = z$. Surely, this is wrong. It is possible to solve the problem substituting z_*, calculated from the full set of equations for the thermal equilibrium. This value depends only on the electron temperature and does not depend on the history of the process. The value z must be calculated as the integer part of $< z >$. The set of equations for the three- ion approximation takes the form:

$$\frac{d}{dt} < z >= -\nu_{<z>} \left[(< z > - z_*) + A_{<z^2>} \left(< z^2 > - z_*^{(2)} \right) \right], \tag{4}$$

$$\frac{d}{dt} < z^2 >= -\nu_{<z^2>} \left[\left(< z^2 > - z_*^{(2)} \right) + A_{<z>} (< z > - z_*) \right], \tag{5}$$

Here $\nu_{<z>} = n_e \left[z(J_{z-1} + 2R_z - R_{z+1} - 2J_z) + \frac{1}{2}(J_{z-1} + R_{z+1}) \right]$,

$\nu_{<z^2>} = n_e \left[z(R_{z+1} - J_{z-1} - 2R_z + 2J_z) + \frac{1}{2}(J_{z-1} + R_{z+1} + 2R_z + 2J_z) \right]$,

$A_{<z>} = \frac{4z^2 (J_{z-1} - 2J_z + 2R_z - R_{z+1}) - 4z(J_z + R_z) - J_{z-1} + R_{z+1}}{-2z(J_{z-1} - 2J_z + 2R_z - R_{z+1}) + J_{z-1} + 2J_z + 2R_z + R_{z+1}}$,

$A_{<z^2>} = \frac{R_{z+1} - J_{z-1} + 2J_z - 2R_z}{J_{z-1} + R_{z+1} - 2z(R_{z+1} - J_{z-1} + 2J_z - 2R_z)}$. Values z_* and $z_*^{(2)}$ are the equilibrium values of <z> and <z^2> correspondingly, calculated from the full model. The ionization state distribution is described by the following expressions:

$$y_z = 2z < z > - z^2 - < z^2 > + 1, \tag{6}$$

$$2 y_{z-1} = < z^2 > - < z > (2z + 1) + z(z + 1)). \tag{7}$$

The value z may be taken as an integer part of $\sqrt{< z^2 >}$. In order to test the models we have calculated evolutions of averaged values like $< z >$ and $< z^2 >$ with two-ion and three-ion models and compared them with the results of the full seven-ion model. First of all we have investigated the validity of models for processes with characteristic times of the same order of magnitude as the impurity relaxation time or one order smaller. The temperature perturbation has been taken in the form: $T_e = T_0 (1 + a \sin(2\pi\nu t))$. Here t is a time, $T_0 = 10$ eV, $a = 0.5$. The carbon ionization and recombination rates have been calculated including the dielectronic recombination and using the most updated atomic data [12]. The results are shown in Fig. 1 (a, b) for $\nu =$ 200 and 500 s^{-1}. Everywhere below curves marked by **A, B** and **C** correspond to the 7-ion, 3-ion and 2-ion models respectively. The electron density has been chosen as $n_e = 10^{13}$ cm^{-3}. One can see that both models describe satisfactorily the impurity behavior. Then, the very fast temperature change has been examined,

$T_e(t) = (T_{fin} - T_{in}) \cdot \frac{2}{\pi} \cdot \arctan\left(\frac{t}{\tau}\right) + T_{in}$. Here T_{in} is the initial temperature, T_{fin} is the final one. The value $\tau = 10^{-6}$ s has been chosen. This time is significantly shorter than the impurity relaxation time. The result for $T_{in} = 3eV$, $T_{fin} = 50$ eV and for equilibrium values, <z> and <z^2> as initial conditions is shown in Fig. 2. The models describe the impurity behavior successfully because the ionization rate decreases significantly with increasing *z*. If the temperature rises the impurity distribution over ionization states keeps its initial shape (Fig. 3). If we took into account only photo recombination we would have the same situation for the fast temperature decrease. The dielectronic recombination changes the situation significantly. The total recombination rate is not a monotonic function of z, and the initial two-ion (or three-ion) distribution is transformed significantly (Fig. 4). However, even in this case we can evaluate qualitatively the integral values like <z> and <z^2> (Fig. 5). In order to improve the two-ion model we have used instead of z_* the modified function

$$z_*^m = \begin{cases} z-1, & if \quad z_* < z-1, \\ z_*, & if \quad z-1 < z_* < z+2, \\ z+2, & if \quad z_* > z+2 \end{cases}$$

The corresponding modification of the tree-ion model does not improve results. Thus, the two and three-ion models describe the carbon behavior successfully, for processes with the characteristic times of the same order of magnitude as the carbon relaxation time, as well as for very fast processes with rising temperature. The description of very fast processes with decreasing temperature is not so successful, however it is possible to get a qualitative description of some averaged parameters like <z> and <z^2>.

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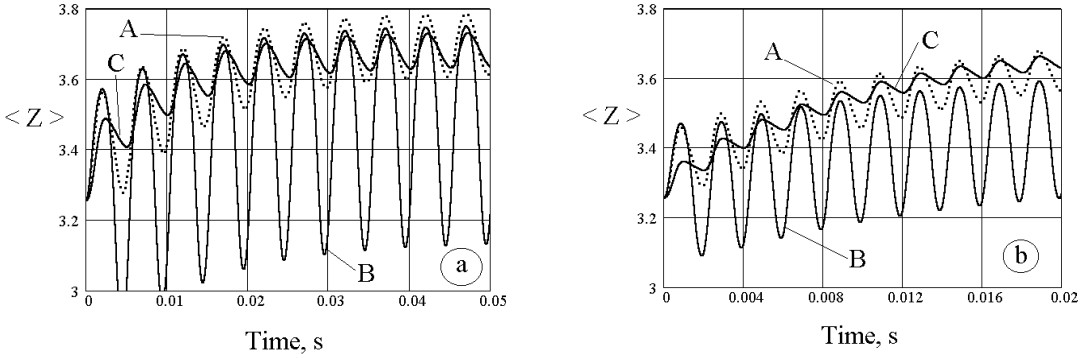

Fig. 1. Averaged charge evolution for the temperature oscillations: a) oscillation frequency $\nu = 200\ s^{-1}$, b) $\nu = 500\ s^{-1}$.

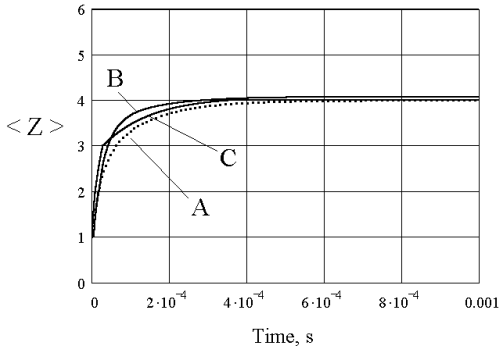

Fig. 2 Averaged charged evolution for the short time temperature jump from 3 to 50 eV.

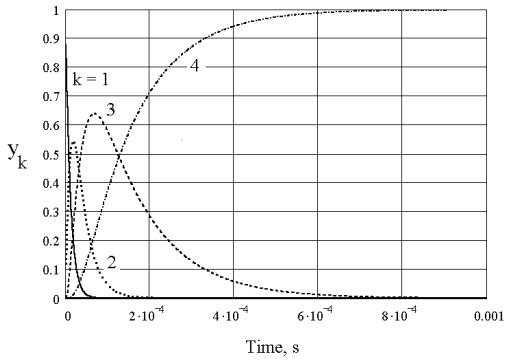

Fig. 3 Evolution of relative concentration for the short time temperature jump from 3 to 50 eV.

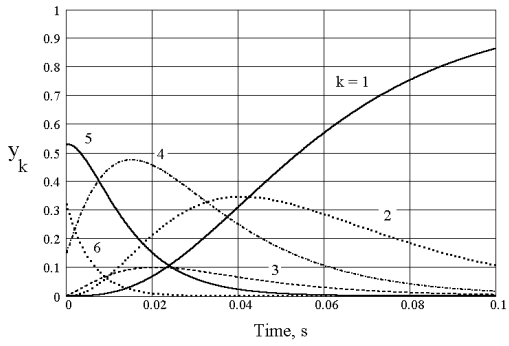

Fig. 4. The same as in Fig. 3 for the short time temperature fall down from 100 to 2 eV.

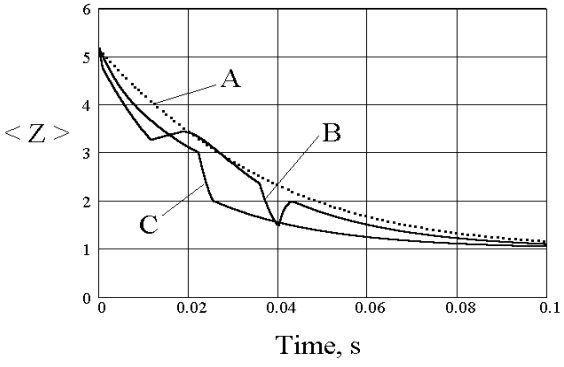

Fig. 5. Average charge evolution for the short time temperature fall down from 100 to 2 eV.