

## Neoclassical poloidal rotation in presence of RF waves in tokamaks

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### 1. Introduction

In the past years, one of the critical achievements in magnetically confined plasmas was the recognition of the role of the  $\vec{E} \times \vec{B}$  flow, or of its shear, in reducing the plasma turbulence and consequently in improving the plasma confinement [1]. The  $\vec{E} \times \vec{B}$  flow is driven by the radial electric field which itself depends not only on the pressure gradient but also on the poloidal and toroidal velocities of the plasma. Therefore, any mechanism acting on the plasma rotation could in principle be helpful to attain an improved mode.

A self-consistent picture of the plasma rotation can be obtained from the neoclassical theory [2-5]. The aim of this work is to extend the neoclassical analysis to non-ohmic plasmas in situations where the auxiliary heating is strong enough to significantly distort the distribution function away from the Maxwellian equilibrium. The analysis, based on a moment approach [4], is specifically devoted to ICRH.

### 2. Solving the kinetic equation

We assume that the plasma can be described by the Vlasov-Landau kinetic equation in which the Lorentz force term due to the resonant external electromagnetic waves can be reduced to a quasi-linear contribution. The standard procedure for solving the ohmic version of this equation assumes that the full distribution function is not too different from the Maxwellian distribution  $f_M^\alpha$ , so that  $f^\alpha = f_{eq}^\alpha (1 + \chi_C^\alpha)$  with  $f_{eq}^\alpha = f_M^\alpha$  provided  $\chi_C^\alpha \ll 1$ , where superscript  $\alpha$  indicates the species of particles. Now taking the quasi-linear operator into account, the reference distribution function itself differs from the Maxwellian  $f_{eq}^\alpha = f_M^\alpha (1 + \chi_Q^\alpha)$  with  $\chi_Q^\alpha \ll 1$ . The full distribution function in the non-ohmic case thus reduces to  $f^\alpha = f_M^\alpha (1 + \chi_Q^\alpha)(1 + \chi_C^\alpha) \equiv f_M^\alpha (1 + \chi^\alpha)$ . In the strong magnetic field limit, the kinetic equation can be gyrophase averaged and reduced to the drift kinetic equation (DKE) which includes a quasi-linear RF term :

$$\frac{\partial f^\alpha}{\partial t} + (\vec{v}_\parallel + \vec{v}_d) \cdot \nabla f^\alpha + \vec{E} \cdot \nabla_v f^\alpha = C(f^\alpha) + Q(f^\alpha). \quad (1)$$

In this equation, we assume that both the non-ohmic heating and the collisions are fast processes on time scale  $\tau_C$  compared to the hydrodynamical processes on time scale  $\tau_H$  with  $\delta = \tau_C / \tau_H \ll 1$ . The following study is focused on ion cyclotron resonant heating (ICRH). The electron reference distribution function is the same as that without RF heating. The reference distribution function for ions is obtained from the bounce-averaged Fokker-Planck equation in the form [6]:

$$f_{\text{eq}}^i = f_M^i(1 + \chi_Q) = f_M^i \left\{ S \sum_{\sigma=\pm 1} \exp \left[ -\frac{x^{3/2}}{(\Delta l)^2} (\sqrt{1 - \lambda B_m} - \sigma \sqrt{1 - \lambda B_{\text{res}}})^2 \right] \right\},$$

where  $S$  is determined by the normalization condition,  $B_{\text{res}}$  is evaluated at the resonant layer and  $B_m$  is the minimum of  $B$  on a flux surface and  $(\Delta l)^2 = v_{ti} v^3 / 2v_s v_c^3 = 4/[15(1 + m_t/m_i)]$  with  $m_t$  the mass of the resonant ions and  $m_i$  the mass of the background ions. Here, coordinates  $\{\lambda, x\} = \{M/(E - e_\alpha \Phi), (E - e_\alpha \Phi)/T_\alpha\}$  are used with  $(E - e_\alpha \Phi)$  the kinetic energy and  $M$  the magnetic momentum.

The equation (1) has been treated by usual mixed ordering expansion in the collisionless regime. The distribution function is denoted as usual by  $f^\alpha = f_0^{\alpha(0)} + f_1^{\alpha(0)} + f_0^{\alpha(1)} + f_1^{\alpha(1)} + \dots$ . The collision and quasilinear RF diffusion operators are given by:

$$C(f_1^{\alpha(0)}) = \frac{2v_\alpha}{B} \sqrt{1 - \lambda B} \frac{\partial}{\partial \lambda} \lambda \sqrt{1 - \lambda B} \frac{\partial f_1^{\alpha(0)}}{\partial \lambda} + v_{\parallel} N_C f_0^{\alpha(0)},$$

$$Q(f_1^{i(0)}) = \frac{2m_\alpha}{BT} \frac{\sqrt{1 - \lambda B}}{x} \frac{\partial}{\partial \lambda} D_\perp \delta(\omega - n\Omega - k_{\parallel} v_{\parallel}) \frac{\lambda}{\sqrt{1 - \lambda B}} \frac{\partial f_1^{i(0)}}{\partial \lambda} + v_{\parallel} N_Q f_0^{i(0)}.$$

The first terms of the above two equations are pitch-angle scattering operators. Following the conventional neoclassical theory, the second terms, proportional to  $N_Q$  or  $N_C$ , do not need to be given in explicit form. The solution of the first order of the DKE, is as usual a sum of two terms  $f_1^{i(0)} = f_M(G_{(0)} + G_{(1)})$ , where  $G_{(0)}$  is related to the classical thermodynamic sources  $g_p^{i(n)}$  and  $G_{(1)}$  is proportional to a function  $\vartheta(x, \rho)$ , which is itself related to  $M_Q$  and  $M_C$ :

$$G_0 = \frac{v_{\parallel}}{R_0 B_p \Omega_\alpha} \frac{1}{\tau_\alpha} \left( \frac{m_\alpha}{T_\alpha} \right)^{1/2} I_\rho [(g_p^{\alpha(1)} - g_p^{\alpha(1)A}) + \sqrt{\frac{2}{5}} (x - \frac{5}{2}) g_p^{\alpha(3)}],$$

$$G_{(1)} = \Theta(\lambda_c - \lambda) \left\{ 2V(\lambda, x, \rho) \sqrt{x} \vartheta(x, \rho) f_M - \frac{\iota \Gamma}{R_0 B_p e_\alpha} x D(\lambda, x, \rho) \frac{\partial f_M}{\partial \rho} \right\}.$$

Here  $\Theta(\lambda_c - \lambda)$  is the Heaviside function which is non-zero for passing particles and

$$V(\lambda, x, \rho) = \sigma v_{\text{th}} \int_P B_0 d\lambda \frac{\mu(x) x \sqrt{1 - \lambda B_{\text{res}}}}{\mu(x) x \sqrt{(1 - \lambda B_{\text{res}})(1 - \lambda B_0)} + P_{\text{rf}}},$$

$$D(\lambda, x, \rho) = \sigma v_{\text{th}} \int_P B_0 d\lambda \frac{P_{\text{rf}}}{\sqrt{x(1 - \lambda B_{\text{res}})} [\mu(x) x \sqrt{(1 - \lambda B_{\text{res}})(1 - \lambda B_0)} + P_{\text{rf}}]},$$

where  $P_{\text{rf}} = \beta m D_{\perp 0} / T v_{\alpha 0}$  is the heating dependence.  $v_\alpha = v_{\alpha 0} \mu(x)$ . We notice that now  $G_{(1)}$  contains a new contribution  $D$ , which is dependent, through  $P_{\text{rf}}$ , on the heating mechanism. The term  $\vartheta(x, \rho)$  is then expanded in Laguerre-Sonine polynomials:  $\vartheta(x, \rho) = \sum a_n L_n^{3/2}(x)$  and the coefficients  $a_n$  determined by combining the zero divergence conditions  $\nabla \cdot \vec{j} = \nabla \cdot \vec{\Gamma} = \nabla \cdot \vec{q} = 0$  and the expansion of the perturbed distribution function  $\chi$  in Laguerre-Sonine polynomials. This leads to a set of relations between the coefficients  $a_n$  and the poloidal fluxes  $\omega_{2n+1}$ :

$$a_n = \mu_{n0} \omega_{n0} + \sum_{m=1,3,5} \mu_{nm} \omega_m, \quad (2)$$

where the coefficients  $\mu_{nm}$ , which have very complicated expressions, depend on both the collisionality and the heating mechanism. We have:

$$\omega_{p0} = \frac{3}{4\sqrt{2}} \kappa_\alpha P_{rf} [b_4^p (g_\rho^{\alpha(1)} - g_\rho^{\alpha(1)A}) + b_5^p g_\rho^{\alpha(3)}] - \sqrt{3} \frac{B_0}{B} S a_y^p, \quad (3)$$

where  $g_\rho^{\alpha(p)}$  are the source terms defined in Ref[4],  $a_y^p$  are the coefficients of Laguerre-Sonine expansion of  $\chi_Q$  and  $b_n^p$  are dimensionless coefficients.

### 3. Poloidal fluxes from transport equations

From the definition of the generalized stress tensors and the perturbed distribution function obtained above, the generalized stress tensors are evaluated in terms of the  $a_n$  which are eliminated in favor of the  $\omega_{2n+1}$  by Eq. (2). Then the additional source terms  $\bar{g}_{//}^{\alpha(p)}$  are obtained as:

$$\frac{1}{B_0} \langle B \bar{g}_{//}^{\alpha(2n+1)} \rangle = -(\zeta_{n0}^\alpha + \sum_{m=1,3,5} \zeta_{nm}^\alpha \omega_m). \quad (4)$$

where

$$\begin{pmatrix} \zeta_{p1} \\ \zeta_{p3} \\ \zeta_{p5} \end{pmatrix} = \frac{4}{\sqrt{5\pi}} B_0 \tau_i v_i \begin{pmatrix} \hat{\kappa}_0^p \mu_{11} + \hat{\kappa}_1^p \mu_{31} + \hat{\kappa}_2^p \mu_{51} \\ \hat{\kappa}_0^p \mu_{13} + \hat{\kappa}_1^p \mu_{33} + \hat{\kappa}_2^p \mu_{53} \\ \hat{\kappa}_0^p \mu_{15} + \hat{\kappa}_1^p \mu_{35} + \hat{\kappa}_2^p \mu_{55} \end{pmatrix}. \quad (5)$$

$\zeta_{p0}$ , which are related to the rf heating, are derived as

$$\zeta_{p0} = \hat{\kappa}_0^p \mu_{10} \omega_{10} + \hat{\kappa}_1^p \mu_{30} \omega_{30} + \hat{\kappa}_2^p \mu_{50} \omega_{50} + S \tau_\alpha v_{\alpha 0} B_0 \hat{\kappa}_3^p + \kappa_\alpha \tau_\alpha v_\alpha B_0 P_{rf} [\hat{\kappa}_4^p (g_\rho^{\alpha(1)} - g_\rho^{\alpha(1)A}) + \hat{\kappa}_5^p g_\rho^{\alpha(3)}], \quad (6)$$

where  $\hat{\kappa}_n^p$  are dimensionless coefficients determined by performing the integrals for the generalized stress tensor. This set of equations is then used together with the parallel components of the transport equations (i.e. moments of the DKE) to obtain a closed set of linear equations for the poloidal fluxes. The poloidal rotation is finally obtained in the form:

$$\omega_1 = \frac{J_1^e / \kappa_e - (\zeta_{13} A_{32} + \zeta_{15} A_{52}) \langle g_\rho^{i(3)} \rangle - (\zeta_{13} A_{30} + \zeta_{15} A_{50} + \zeta_{10})}{\zeta_{11} A + \zeta_{13} A_{33} + \zeta_{15} A_{53}}, \quad (7)$$

where  $A = v_{thi} \Omega_e \tau_e / v_{the} \Omega_i \tau_i$ ,  $\kappa_\alpha = \nu_\rho / R_0 B_p \Omega_{\alpha 0} \tau_\alpha$  ( $\alpha = i, e$ ),  $J_1^e$  is the electron flux. Because ICRH has no direct effect on electrons,  $J_1^e$  is the same as that given in the standard neoclassical theory (e.g. Eq.(15.2.10) in [4]). The coefficients  $A_{mm}$  depend on the usual neoclassical transport coefficients ( $\tilde{\kappa}_{//}$ ,  $\tilde{\delta}_{//}$ , ... all symbols with a tilde) and  $\zeta_{nm}$ :

$$A_{30} = [(\tilde{\kappa}_{//}^i \zeta_{35} + \tilde{\delta}_{//}^i \zeta_{55})(\tilde{\delta}_{//}^i \zeta_{30} + \tilde{\epsilon}_{//}^i \zeta_{50}) - (1 + \tilde{\delta}_{//}^i \zeta_{35} + \tilde{\epsilon}_{//}^i \zeta_{55})(\tilde{\kappa}_{//}^i \zeta_{30} + \tilde{\delta}_{//}^i \zeta_{50})] / \Theta,$$

$$A_{31} = \kappa_i (1 + \tilde{\delta}_{//}^i \zeta_{35} + \tilde{\epsilon}_{//}^i \zeta_{55}) / \Theta,$$

$$A_{32} = [(\tilde{\kappa}_{//}^i \zeta_{35} + \tilde{\delta}_{//}^i \zeta_{55})(\tilde{\delta}_{//}^i \zeta_{31} + \tilde{\epsilon}_{//}^i \zeta_{51}) - (1 + \tilde{\delta}_{//}^i \zeta_{35} + \tilde{\epsilon}_{//}^i \zeta_{55})(\tilde{\kappa}_{//}^i \zeta_{31} + \tilde{\delta}_{//}^i \zeta_{51})] / \Theta,$$

$$A_{50} = [(\tilde{\kappa}_{//}^i \zeta_{30} + \tilde{\delta}_{//}^i \zeta_{50})(\tilde{\delta}_{//}^i \zeta_{33} + \tilde{\epsilon}_{//}^i \zeta_{53}) - (1 + \tilde{\kappa}_{//}^i \zeta_{33} + \tilde{\delta}_{//}^i \zeta_{53})(\tilde{\delta}_{//}^i \zeta_{30} + \tilde{\epsilon}_{//}^i \zeta_{50})] / \Theta,$$

$$A_{51} = \kappa_i (\tilde{\delta}_{//}^i \zeta_{33} + \tilde{\epsilon}_{//}^i \zeta_{53}) / \Theta,$$

$$A_{52} = [(\tilde{\kappa}_{//}^i \zeta_{31} + \tilde{\delta}_{//}^i \zeta_{51})(\tilde{\delta}_{//}^i \zeta_{33} + \tilde{\epsilon}_{//}^i \zeta_{53}) - (1 + \tilde{\kappa}_{//}^i \zeta_{33} + \tilde{\delta}_{//}^i \zeta_{53})(\tilde{\delta}_{//}^i \zeta_{31} + \tilde{\epsilon}_{//}^i \zeta_{51})] / \Theta,$$

$$\Theta = (1 + \tilde{\kappa}_{//}^i \zeta_{33} + \tilde{\delta}_{//}^i \zeta_{53})(1 + \tilde{\delta}_{//}^i \zeta_{35} + \tilde{\epsilon}_{//}^i \zeta_{55}) - (\tilde{\kappa}_{//}^i \zeta_{35} + \tilde{\delta}_{//}^i \zeta_{55})(\tilde{\delta}_{//}^i \zeta_{33} + \tilde{\epsilon}_{//}^i \zeta_{53}).$$

In the absence of RF heating in plasmas, i.e.  $P_{rf} = 0$ , the reference distribution function is Maxwellian  $f_{eq}^i = f_M$ , and we therefore have  $a_y^p = 0$ . From Eqs.(2) and (3)  $\omega_{p0} = 0$  and  $\mu_{nm}$  reduces to a diagonal matrix. From Eqs.(5) and (6) we have  $\zeta_{p0} = 0$ . Then Eq.(7) becomes

$$\omega_1 = \frac{J_1^e / \kappa_e - (\zeta_{13} A_{32} + \zeta_{15} A_{52}) \langle g_p^{i(3)} \rangle}{\zeta_{11} A + \zeta_{13} A_{33} + \zeta_{15} A_{53}},$$

which is the same as the result from conventional neoclassical theory[4]. A qualitative comparison of Eq.(7) with the pure-ohmic neoclassical poloidal flux shows:

- a) The poloidal flux has a new contribution symbolized by the terms proportional to both RF power  $P_{rf}$  and the distorted reference distribution  $S$ , which vanishes when  $P_{rf} = 0$ .
- b) The poloidal flux, which is represented by the terms proportional to  $J_1^e$  and the source terms  $g_p^{i(p)}$ , has a new dependence on  $g_p^{i(1)}$ . Without rf heating the poloidal flux only depends on  $J_1^e$  and  $g_p^{i(3)}$ .
- c) Without RF heating all terms of poloidal flux have density or temperature gradient dependence. The poloidal flux now contains a term proportional to  $S$ , which is independent of the gradients. This term is due to the deformation of the reference distribution function in presence of strong heating.

#### 4. Summary

A moment method is developed to study the transport properties of a plasma even during strong heating where the reference distribution function strongly differs from a Maxwellian. This method has been applied to the case of ICRH and the plasma rotation coefficients have been evaluated using well-controlled approximation schemes. The new features of the poloidal plasma rotation due to ICRH are discussed

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