

Influence of Radial Electron Density Profile on the Axial Structure of Gas Discharges Sustained by Potential Surface Waves

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Gas discharges sustained by high-frequency travelling surface wave (SW) has been studied intensively till now due to their numerous technological applications. The SW that sustains the discharge is the eigen wave of discharge structure, therefore axial distribution of discharge parameters are mainly determined by the properties of the wave that sustains the discharge. Until recently, the SW propagation and, therefore, axial gas discharge structure, were studied with the assumption of uniform radial plasma density profile. However, in real systems plasma density is always heterogeneous in radial direction and the conditions of plasma or hybrid resonances may take place at the periphery of discharge vessels [1]. It is necessary to note, that transverse electromagnetic waves are slightly damping waves in weakly collisional discharge plasma. But efficiency of energy transfer can be increased substantially in the regions of hybrid resonances, where electromagnetic waves transforms into plasma waves [2]. The purpose of this report is to determine properties of the nonsymmetric potential SW (wavelength, spatial damping rate and SW radial field distribution) in magnetized plasma column with radially heterogeneous plasma electron density, and, finally, to determine plasma density axial structure in the discharges sustained by such waves in diffusion controlled regime.

The SW that sustains the discharge, propagates along slightly axial heterogeneous three medium waveguide structure, that consists of radially heterogeneous plasma cylinder with radius R_p , glass tube with external radius R_g , and waveguide metal wall with radius R_m ($R_p < R_g < R_m$). External steady magnetic field B_0 is directed along the axis of waveguide system. Plasma is considered in hydrodynamic approximation as slightly absorbing cold media. It is characterized by the constant in discharge volume effective electron collision frequency of momentum transfer ν , that is much less than wave generator frequency ω . Electron density radial profile $n(r)$ was chosen in Bessel-like form given by $n(r) = n(0) J_0(\mu r R_p^{-1})$. Heterogeneity parameter μ varies from $\mu=0$ (homogeneous case) to $\mu=2.405$ (perfect ambipolar diffusion profile). SW propagation is governed by the Poisson equation. In the considered case, when plasma density, SW wavelength and it's amplitude vary slightly along the discharge column at the distances of wave length order, the solution of the Poisson equation in cylindrical coordinate system (r, φ, z) for SW potential Ψ can be found in WKB form:

$$\Psi(r, \varphi, z, t) = \Psi(r) \exp \left(i \left[\int_{z_0}^z k_3(z') dz' + m\varphi - \omega t \right] \right), \quad (1)$$

where k_3 and m are axial and azimuthal wavenumbers, respectively.

Applying expression (1) to Poisson equation one can obtain second order differential equation for SW potential in the region of radially heterogeneous plasma [2]. This equation can be analytically solved in case of radially homogeneous plasma. When plasma density profile depends on radial coordinate r , this equation, for arbitrary discharge parameters (plasma density radial profile, external magnetic field value, geometrical parameters of discharge structure), can be solved only with the help of numerical methods. In the glass tube and vacuum region Poisson

equation possesses the solution that constitutes from modified Bessel functions of the first and second kinds. Applying ordinary linear boundary conditions for SW field, that consists in continuity of appropriate SW field components at plasma–glass ($r=R_p$) and glass–vacuum ($r=R_g$) interfaces and their vanishing on the metal wall ($r=R_m$), one can obtain the dispersion equation and SW field constants as the functions of $E_z(R_p)$. In spite of the low value of effective electron collision frequency ($\nu \ll \omega$) it is necessary to keep imaginary addenda in the expressions for magnetized plasma permittivity tensor. This imaginary addenda gives the possibility to carry out the numerical integration of the second ordinary differential equation in the region when upper hybrid resonance occur. Therefore, the complex dispersion equation is obtained, the real part of its complex solution for wavenumber gives wavelength, and imaginary part gives SW damping rate.

The results of the numerical solution of the obtained dispersion equation for symmetric ($m=0$) and dipolar ($m=-1$) waves are represented on Figs.1-4. These waves were chosen due to the fact, that they are widely used in gas discharge sustaining [3].

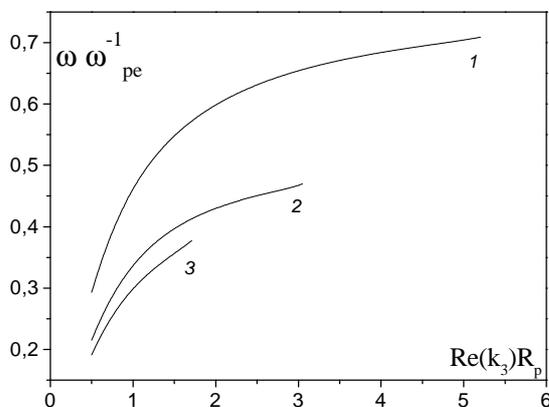


Figure 1.

Dimensionless frequency $\omega\omega_{pe}^{-1}$ of symmetric ($m=0$) wave as a function of dimensionless wavenumber $Re(k_3R)$ at $\omega_{ce}\omega^{-1}=0.9$, $R_gR_p^{-1}=1.05$, $R_mR_p^{-1}=2.5$, $\epsilon_d=4.5$, $\nu\omega^{-1}=0.016$ (ω_{ce} is electron cyclotron frequency). Curves, marked by the numbers 1, 2, 3 refer to the parameter $\mu=0, 2.1, 2.4$ values, respectively.

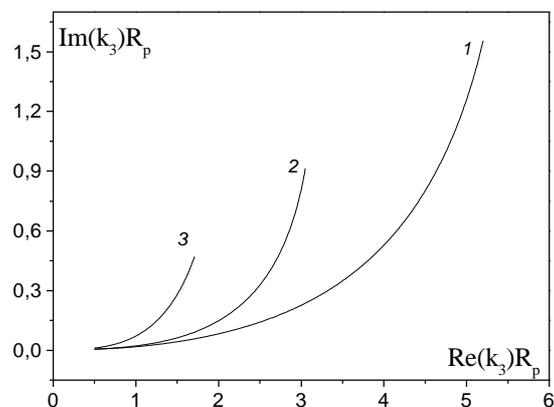


Figure 2.

Spatial damping rate $Im(k_3R_p)$ of symmetric ($m=0$) wave as a function of dimensionless wavenumber $Re(k_3R)$ at $\omega_{ce}\omega^{-1}=0.9$, $R_gR_p^{-1}=1.05$, $R_mR_p^{-1}=2.5$, $\epsilon_d=4.5$, $\nu\omega^{-1}=0.016$ (ω_{ce} is electron cyclotron frequency). Curves, marked by the numbers 1, 2, 3 refer to the parameter $\mu=0, 2.1, 2.4$ values, respectively.

Dispersion properties of symmetric wave ($m=0$) are represented in the Figs.1,2. It shows that SW dispersion characteristics essentially depend on the heterogeneity parameter μ value. Increase of parameter μ value from $\mu=0$ up to $\mu=2.405$ leads to decrease of SW phase velocity (Fig.1). Damping rate of symmetric SW essentially increases with the increase of value of parameter μ (Fig.2). It is necessary to note, that SW damping rate grows sharply in the case when the condition of upper hybrid resonance is fulfilled at the periphery of plasma column (curve 3 on Fig.2). The dependence of dimensionless frequency and dimensionless spatial damping rate of dipolar ($m=-1$) wave as a function of dimensionless wavenumber are presented on Figs.3,4. The dependence of dipolar SW dispersion properties on the heterogeneity parameter value is similar to symmetric one. It is worth to note such peculiarity of damping rate for dipolar waves, as the existence of its minimum value in the region of moderate $Re(k_3R_p)$ values. When radial density profile is of a perfect ambipolar diffusion type, dipolar SW becomes strongly damped

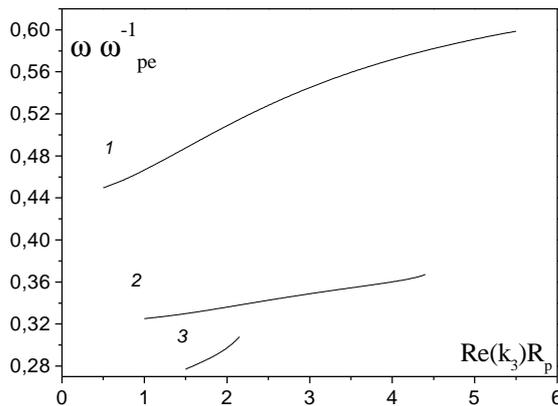


Figure 3.

Dimensionless frequency $\omega \omega_{pe}^{-1}$ of dipolar ($m=-1$) wave as a function of dimensionless wavenumber $Re(k_3 R)$ at $\omega_{ce} \omega^{-1}=0.9$, $R_g R_p^{-1}=1.05$, $R_m R_p^{-1}=2.5$, $\epsilon_d=4.5$, $\nu \omega^{-1}=0.016$ (ω_{ce} is electron cyclotron frequency). Curves, marked by the numbers 1, 2, 3 refer to the parameter $\mu=0, 2.1, 2.4$ values, respectively.

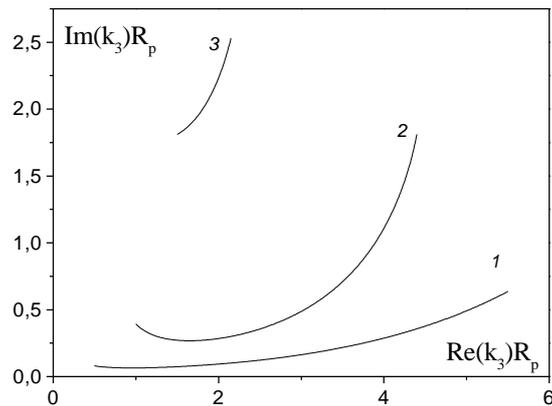


Figure 4.

Spatial damping rate $Im(k_3 R_p)$ of dipolar ($m=-1$) wave as a function of dimensionless wavenumber $Re(k_3 R)$ at $\omega_{ce} \omega^{-1}=0.9$, $R_g R_p^{-1}=1.05$, $R_m R_p^{-1}=2.5$, $\epsilon_d=4.5$, $\nu \omega^{-1}=0.016$ (ω_{ce} is electron cyclotron frequency). Curves, marked by the numbers 1, 2, 3 refer to the parameter $\mu=0, 2.1, 2.4$ values, respectively.

wave. It is also essential to note that collisional damping rate substantially depends on the azimuthal wavenumber value (the higher is azimuthal wavenumber m , the higher is SW spatial damping rate).

It is necessary to note, that one must always check two validity conditions of potential wave existence ($V_{ph} \ll c$ and $Im(k_3) \ll Re(k_3)$ where V_{ph} is the wave phase velocity). Simultaneous fulfillment of these conditions leads to the substantial decrease of the SW existence region, especially for nonsymmetric waves. Thus, for example, SW with azimuthal wavenumber $m=-2$ strongly damps even at low plasma density gradients. So, this wave can be used for discharge sustaining only in nonpotential region [4].

SW radial field structure was studied as well. Variation of heterogeneity parameter μ value affects substantially the SW radial field structure. The increase of μ value leads to the increase of SW field values. It is necessary to note that when the conditions of upper hybrid resonance are fulfilled at the periphery of plasma column, the radial SW electric field component ($E_r(r)$) grows sharply at that region. Such resonant saltus value depends on the $\nu \omega^{-1}$ ratio: the higher is ratio, the lower is saltus.

The axial profile of dimensionless density $N = \omega_{pe}^{-2} \omega^{-2}$ can be theoretically determined on the assumption of energy balance equation of gas discharge stationary state in diffusion controlled regime [5,6]. When mean power that maintains an electron in the discharge θ and electron effective collision frequency for momentum transfer ν are constant in discharge volume, one can obtain equation that governs plasma density axial distribution in the form:

$$\frac{dN}{d\xi} = - \frac{2N\alpha}{\frac{\nu}{\omega} \left\{ 1 - \frac{N}{\alpha} \frac{d\alpha}{dN} \right\}}, \quad (2)$$

where $\alpha = Im(k_3 R_p)$ is the dimensionless attenuation coefficient and $\xi = \nu z (\omega R_p)^{-1}$ is the dimensionless length.

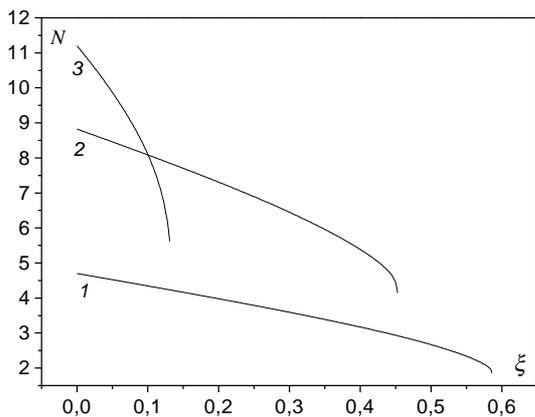


Figure 5.

Dimensionless plasma density axial profile in the discharge sustained by symmetric ($m=0$) surface potential wave at $\omega/\omega^l=0.9$, $R_g R_p^{-1}=1.05$, $R_m R_p^{-1}=2.5$, $\epsilon_d=4.5$, $\nu\omega^{-1}=0.016$ (ω_{ce} is electron cyclotron frequency). Curves, marked by the numbers 1, 2, 3 refer to the parameter $\mu=0, 2.1, 2.4$ values, respectively.

Plasma density axial distribution in the discharges sustained by symmetric and dipolar modes can be determined by numerical solution of differential equation (2). The results of numerical investigation of plasma column axial structure sustained by symmetric SW are presented in Fig.5. Gradual increase of heterogeneity parameter μ leads to maximum possible plasma density growth and to the decrease of discharge length. So, plasma density axial gradients increase while plasma density radial profile comes closer to perfect ambipolar diffusion profile. When upper hybrid resonance takes place, the maximum possible plasma density sharply grows and discharge length greatly decreases due to effective energy transfer from SW to plasma electrons (curve 3 on Fig.5). Due to large damping rate values the dipolar SW is greatly damped even at low radial plasma density gradients, so this nonsymmetric potential mode (as nonsymmetric potential modes with higher

azimuthal wavenumbers) cannot be used for long gas discharge sustaining.

The influence of given plasma density radial profile $n(r)$ on high frequency nonsymmetric potential surface wave characteristics (wavelength, spatial damping decrement and SW field radial distribution) and on the axial electron density distribution of SW sustained gas discharges under the diffusion regime of charge particle losses has been studied in the present report. The computations have shown that SW wavelength, spatial damping rate, SW radial field distribution and, also, plasma density axial distribution in SW sustained discharge substantially depends on the radial plasma density profile.

Acknowledgment

This work was supported, in part, by Science and Technology Center in Ukraine (STCU, Project #1112).

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