

## Surface Flute Modes Propagating in Plasma Filled Cylindrical Metal Waveguides with Noncircular Cross-Section

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### Introduction

Due to needs of modern radiophysics and plasma electronics dispersion properties of the plasma filled waveguide of different geometry are actively studied during the last time. Such type waveguides are utilized as the main component of generators and amplifiers of high frequency electromagnetic waves and guiders of intense charged particle beams [1]. There are many different modes can propagate in the indicated waveguiding structures. Study of their properties can be useful for elaboration of various radiotechnical and plasma electronical devices. Another possible application of these eigenmodes is sustaining of microwave gas discharges. Last time surface waves are widely used for plasma production and processing of solids during microwave gas discharges [2]. For instance such gas discharges are examined for the wall conditioning in fusion devices [3]. The utilization of superconducting coils at these devices makes traditional methods based on creation of direct current glow discharges for wall conditioning as inapplicable. Thus theoretical study of surface flute modes (SFM) which can propagate along direction of small azimuthal angle in cylindrical metal waveguide with non-circular cross-section seems to be useful in order to research their possible application as alternative method for wall conditioning in fusion devices during microwave gas discharge.

That is why this report is devoted to description of the SFM electrodynamical properties. They are studied analytically and numerically using Maxwell equations for description of the SFM fields and cold plasma model by the aid of successive approximation method.

### Results of analytical study

Let's consider cylindrical metal waveguide which is completely filled by magnetoactive plasma. An external uniform magnetic field  $\vec{H}_0$  is oriented along  $z$  axis. Radius of metal waveguide is described by the following expression:  $R_2 = R_1(1 + h \cos N\varphi)$  Dependence of the SFM fields on coordinates and time is chosen in the following form:  $f_m(r)\exp(im\varphi - i\omega t)$ . Here  $h$  is small parameter,  $\omega$  is the wave frequency,  $m$  is azimuthal wave number. We have supposed that  $\frac{\partial}{\partial z} = 0$ . If there is a narrow axial slot in the metal waveguide (it means that its angular size  $\varphi_0 \ll 1$ ) then using Maxwell equations and two boundary conditions: the SFM fields are of finite value at the axis of the waveguide; and tangential component of the SFM electric field is equal to zero at the metal surface one can find the following dispersion equation:

$$\frac{m\varepsilon_2}{kR_2\varepsilon_1\Psi_0^2} + \frac{I'_m(kR_2\Psi_0)}{\Psi_0 I_m(kR_2\Psi_0)} = \frac{-\varphi_0(J'_m(kR_2) + iN'_m(kR_2))}{2\pi(J_m(kR_2) + iN_m(kR_2))} \quad (1)$$

where  $\Psi_0^2 = \frac{\epsilon_2^2 - \epsilon_1^2}{\epsilon_1}$ ,  $I_m(x)$  is modified Bessel function,  $J_m(x)$  and  $N_m(x)$  are Bessel and Neiman functions, respectively, sign prime means the derivative with respect to argument,  $a$  is the  $k = \omega/c$ ,  $\epsilon_i$  is plasma permeability tensor in the approximation of cold plasma. Solution of the eq.(1) has been found in the form:  $\omega = \omega_0 + \Delta\omega - i\gamma$ , where  $|\Delta\omega| \ll \omega_0$ ,  $\gamma \ll \omega_0$ ,  $\omega_0$  is the solution of eq.(1) in the zero approximation ( $\varphi_0 = 0$ ),  $\Delta\omega_0$  and  $\gamma$  are real and imagine parts of the first order approximation for the  $\omega_0$ , respectively. In the limiting case one can find the following analytical expressions:

$$\Delta\omega \approx \frac{\varphi_0}{\pi} \Omega_e; \gamma \approx -\varphi_0 \left( \frac{kR_2}{2} \right)^{|2m|} \frac{m!^2}{\pi^2 |m|} 2\Omega_e, \text{ if } kR_2 \ll 1; \tag{2}$$

$$\Delta\omega \approx \frac{\varphi_0}{4\pi ka} \Omega_e; \gamma \approx -\frac{\varphi_0}{2\pi} \Omega_e, \text{ if } kR_2 \gg m,$$

here  $\Omega_e$  is the Langmuir frequency.

In the case of the waveguide with non-circular cross-section one can find the following analytical expressions for correction to the  $\omega_0$ . If the inequality  $kR_2\Psi_0 \gg N, m$  is valid then correction is as follows:

$$\Delta\omega \approx -\frac{h^2}{2} \omega_0 \left( 1 + \frac{|\omega_e|}{\omega_0} 2N \right). \tag{3}$$

But in the opposite limiting case  $kR_2\Psi_0 \ll N, m$  one can find another solution:

$$\Delta\omega \approx \frac{-h^2\omega_0}{2} \begin{cases} \frac{4N\omega_e^2}{kR_1\Omega_e} & \text{when } \omega_0 \rightarrow 0 \\ \frac{2N\omega_0}{|\omega_e|} & \text{when } \omega_0 \sim |\omega_e| \end{cases}. \tag{4}$$

### Results of numerical study

Numerical analysis of the eq.(1) allows to obtain the results represented at figures 1 and 2, where  $\beta = \frac{\Omega_e^2}{\omega_e^2} = 3$ . Lines marked by squares, crosses, triangles and diamonds have been obtained for the cases  $m=1, 2, 3$  and  $4$ , respectively.

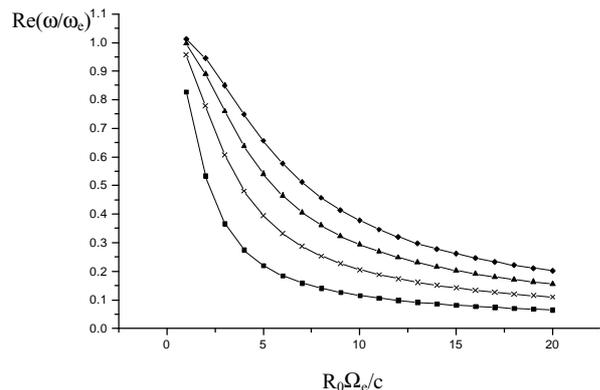


Fig. 1. Real part of SFM frequency normalized by  $\omega_0$  as function of average waveguide radius.

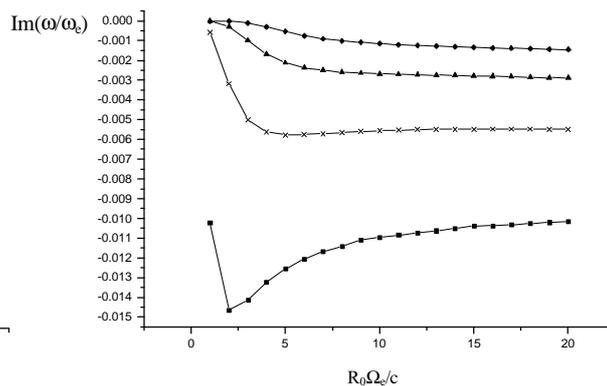


Fig. 2. Imaginary part of SFM frequency normalized by  $\omega_0$  as function of average waveguide radius.

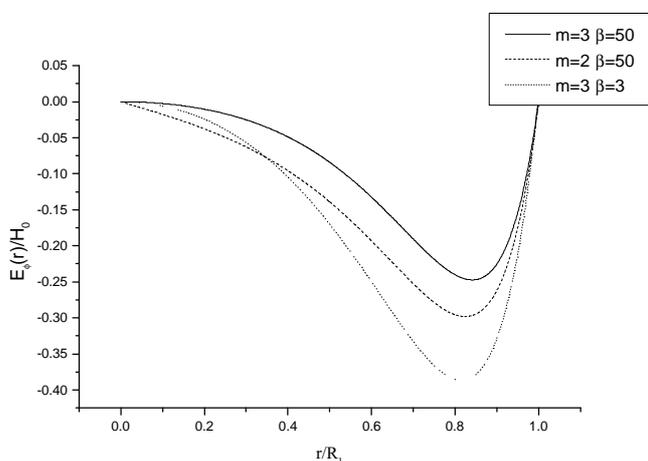


Fig. 3. Spatial structure of SFM  $E_\phi$

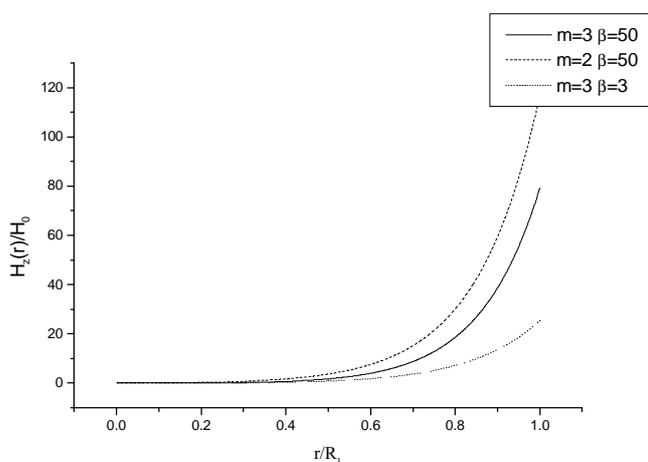


Fig. 4. Spatial structure of SFM magnetic field.

One can see that the SFM with small values of azimuthal wavenumber damp more stronger than that with relatively great wavenumbers.

Spatial structure of the SFM are represented at the figures 3 and 4 for the case  $R_1 = 7 \frac{c}{\Omega_e}$ .

One can see that decrease of the  $m$  and increase of the  $\beta$  leads to increasing of the SFM magnetic field  $H_z$  unlike the case of the  $E_\phi$  which has maximum in the region  $r \approx 0.8R_1$ . Dependence of the radial component of the SFM electric field on the waveguide radius is similar to such dependence of their magnetic component, but it value is less by order than the  $H_z$ . In the case when  $h \neq 0$  the spatial structure of the SFM fields depends strongly on the azimuthal angle and quantity of the waveguide ripples  $N$  and azimuthal wavenumber. At the Fig. 5 one can see spatial distribution of the SFM magnetic field for the case  $m=3$ .

Here we use axis  $x = \frac{r}{R_1} \cos \phi$ ,

$$y = \frac{r}{R_1} \sin \phi .$$

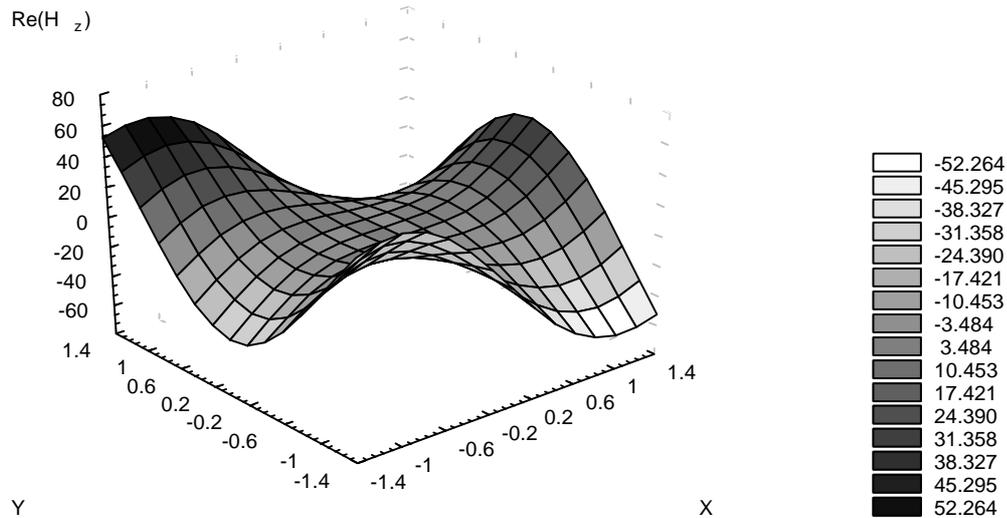


Fig.5. Spatial distribution of the SFM magnetic field for the case  $m=3$ .

## Conclusion

Dispersion properties of the SFM have been studied for the case of magnetoactive plasma filled metal waveguide with cross-section which is described by the equation  $R_2(\varphi) = R_1(1 + h \sin N\varphi)$ . Eigenfrequencies and spatial distribution of the SFM fields have been found and analyzed both analytically and numerically. The SFM damping caused by emission of their energy through axially directed narrow slot has been studied as well. It is found that these X-polarized modes are fast waves: their magnetic component is much greater than both electric components; their wave power flow is directed mainly along small azimuth angle. The SFM propagation can be considered as alternative method for wall conditioning in fusion devices during microwave gas discharge.

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## References

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