

Kinetic description of a rotating tokamak plasma in a steady state of Turbulent Equipartition.

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The description of particle dynamics via the invariants of motion allows to obtain the resulting distribution at a moment $t = t_{st}$ without consideration of intermediate turbulent evolution during the period $0 < t < t_{st}$. If the time t_{st} is larger than the characteristic time τ_{mix} of a mixing of the particles, then the resulting asymptotic distribution f_{st} corresponds to a global TEP (Turbulent EquiPartition) distribution f_{TEP} of plasma with a flat spatial distribution of invariants. If the the time τ_{cons} of violation of the invariants is not too small in comparison with the mixing time, then the resulting asymptotic distribution

$$f_{st} \sim (f_{TEP} + \frac{\tau_{mix}}{\tau_{cons}} f_{Maxw}) / (1 + \frac{\tau_{mix}}{\tau_{cons}}), \quad (1)$$

may be close to the f_{TEP} even if $t_{st} \gg \tau_{cons}$ [1]. The experimentally observed quick response (within $10 \div 30ms$) of a plasma on external perturbation and the experimental observations of canonical profiles of plasma density [2,3], temperature [4], radial electric field [5], plasma rotation and turbulence [6] speak in favor of the establishment of a global TEP distribution.

In a general case, the TEP distribution is unstable, which provides generation of turbulence up to a steady level $W(\vec{r})$ during relaxation of f_{TEP} to a marginally stable distribution. This gives a possibility to estimate the turbulent diffusion coefficient $D_{turf}(W(r), q(r), \vec{U}(r))$, which depends from the magnetic surface and the profile of the plasma rotation also [7,8]. Then, the time of the turbulent mixing at given radial position r can be estimated as $\tau_{mix}(r) \sim \delta^2 r / D_{turf}(r)$, which gives the possibility to take into account the violation of the invariants during particle diffusion over the distance δr . If the region of turbulence mixing suppression, where $\tau_{mix}(r) \leq \tau_{cons}$ is localized at some radial position r_0 with intensive plasma rotation, then different global TEP distributions may appear, one for hot plasma, in the central region $r < r_0$ and another in the outer region $r > r_0$ [9].

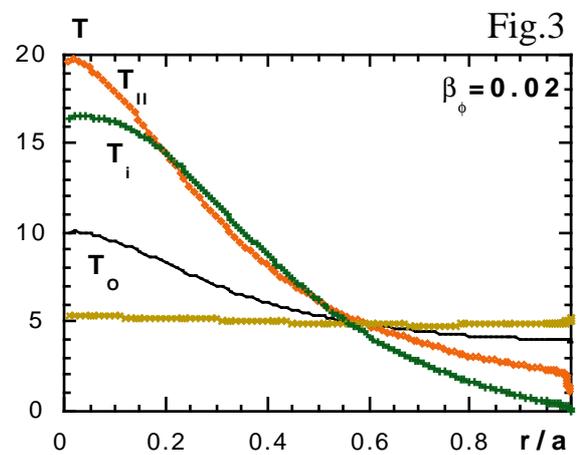
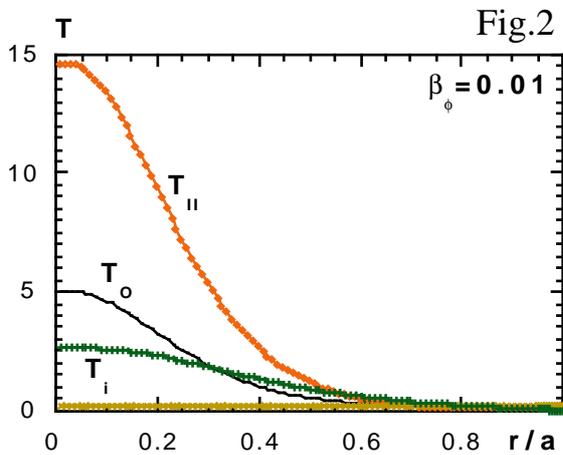
Thus, the study of the global TEP distribution and corresponding radial profiles of the turbulence level, the plasma rotation and the radial electric field is important even in a qualitative consideration. Here we consider the *global* TEP distribution of the electrons in the tokamak for fixed magnetic geometry which is described by the profiles of the safety factor $q(r)$ and by the helicity profile $(\vec{A}\vec{B})(r)$. In particular, we consider the magnetic configuration with the properties of a single-null divertor plasma in DIII-D. The usual equation of the hydrodynamic plasma equilibrium (Grad-Shafranov Equation) can be obtained after averaging over all individual TEP distributions of the electrons and ions. Thus, knowledge of the TEP distribution of the electrons gives the possibility to determine the profile of the ion temperature as well.

In particular, in the ideal (collisionless) case it is possible to obtain the TEP distribution of the electrons from their adiabatic invariants. Also we have taken into account: 1) the conservation of the total number of the electrons in the same frozen-in field tube between any two

Lagrange particles; 2) the conservation of the frozen-in field flux itself $dF = \Omega_e dS$ and 3) the conservation of the particle flux along the frozen-in field tube. This gives possibility to restore the detailed distribution function $f_{TEP}(\vec{r})$ of any group of the electrons with given values of the invariants (Ivonin I.A., Pavlenko V.P., Persson H., 1998, 1999). For instance, the best power fitting of the density, averaged over all groups of the electrons, gives, depending from the β_ϕ value, $\langle n(r) \rangle \sim 1/q(r)^{0.5 \pm 0.6}$ in accordance with the usual experimental scaling.

Each electron is a passing one in the internal tokamak region and has the possibility to be trapped in the outside region during the turbulent mixing. For the *banana* electrons from a particular group, the *general* longitudinal invariant is half the mechanical one. Its value in the TEP distribution $J_{||} = J_{||}(r, E_b(r)) = const$ does not depend from the magnetic surface r . This gives the kinetic energy $E_b(r)$ profile (and the temperature $T_b(r)$ after averaging over all groups of the electrons) of the banana electrons. Then, the knowledge of the $E_b(r)$ and $\langle n_b(r) \rangle$ profiles gives possibility to determine the profile of the toroidal drift velocity V_{de} of the particular group of the banana electrons. Similar equation of the electron *force equilibrium* can be used to determine the profile of the kinetic energy of the *passing* electrons [10,11]. The current velocity \vec{V}_c of the particular group of passing electrons with the same values of invariants can be obtained from the kinetic momentum equation and from the condition of the absence of total *regular* radial flow of the electrons. The solution of these equations describes the relative possibility to carry the current. Namely, the only deeply passing electrons can carry large current, while the nearly trapped passing electrons drift in the toroidal direction like the banana electrons.

To determine the radial profiles of plasma rotation \vec{u} we have used the drift equations for the radial equilibrium of the trapped and passing ions in the hydrodynamic approximation together with the force balance of the passing ions along the magnetic field direction.



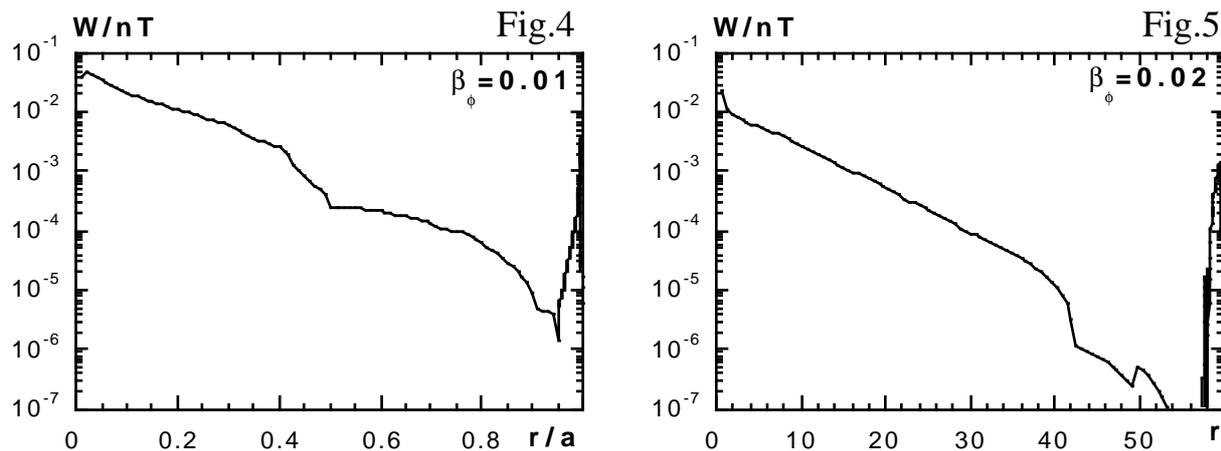
Figs.2,3. Numerical profiles of the temperatures for the values $\beta_\phi = 0.01$ and $\beta_\phi = 0.02$.

Then, the averaged toroidal component of the Maxwell equation for the toroidal current allows to determine self-consistently the TEP distribution for given magnetic field geometry.

In Figs.2-3 we plot the profiles of the effective (longitudinal) electron temperature for the values $\beta_\phi = 1\%$ (typical experimental value) and for $\beta_\phi = 2\%$. We have plot also the profiles of the total electron temperature T_0 . One can see that a larger value of β_ϕ results in the formation of sharp boundary of the temperature profile near the separatrix similar to that in the H-mode.

The numerical simulations of the TEP distribution function were made by the Monte-Carlo method, by the iteration of the TEP distribution. The initial distribution function was the

Maxwellian, with constant profiles of density and temperature. Then, for each particle from this distribution, the individual TEP distribution of the density and the energies (the total and in the longitudinal degree of freedom) were calculated according to the description above. Thus, after the first iterations, the new distribution function consisting of the sum of the individual electron TEPs was determined. Then we calculated the new distribution function, inhomogeneous in the tokamak volume, and used it for the next group of electrons instead of the initial uniform Maxwellian distribution. The convergence to the global TEP was found to be fast (1000 particles are sufficient).



Figs.4,5. The numerical profiles of normalized bulk energy W/nT_0 of the ion-sound turbulence for the values $\beta_\phi = 0.01$ and $\beta_\phi = 0.02$.

It is important that the TEP distribution function differs from a flux-surface-local Maxwell distribution, because it has different longitudinal and perpendicular temperatures. Thus, TEP may relax due to kinetic instabilities to the marginally stable distribution. Some energy may be transferred to the turbulence in this relaxation process, which is required for the TEP generation. This gives us possibility to estimate the turbulence level $W(r)$ and, thus, the rate of the turbulent mixing. From Figs.2-3 one can expect that the turbulence should be larger for the smaller β_ϕ value due to larger deviation of the effective (longitudinal) electron and the ion temperatures. In Figs.4-5 we plot the turbulence profiles W/nT_0 of the ion-sound turbulence for the particular values $\beta_\phi = 1\%$ and $\beta_\phi = 2\%$. The estimations of the turbulent mixing time τ_{mix} give the reasonable values to apply the global TEP approach at least in the central part of the tokamak ($r/a < 0.8$).

It is interesting to note that a negative radial electric field can be generated near the separatrix by the inhomogeneous level of the turbulence similar to Miller force generation. Indeed, the passing electrons may produce the turbulence during the turbulent movement toward the separatrix and toward the central region of tokamak. This can be considered as the radial turbulent force that pushes the deeply passing electrons from the separatrix and from the central region. This force generates a charge separation, and the resulting radial electric field $-\phi(r)'_r$ should compensate the turbulent force. In Fig.6 we plot the numerical calculation of this radial electric field which is negative near the separatrix and positive in the central region. In the case of small value $\beta_\phi = 0.01$ the radial electric field has no negative part near the separatrix.

In our consideration we consider the simplest possible geometry of the magnetic field with nonshifted, circular magnetic surfaces and a single-null divertor point. But the case of an arbitrary magnetic field geometry can be taken into account easily in a particular model [11].

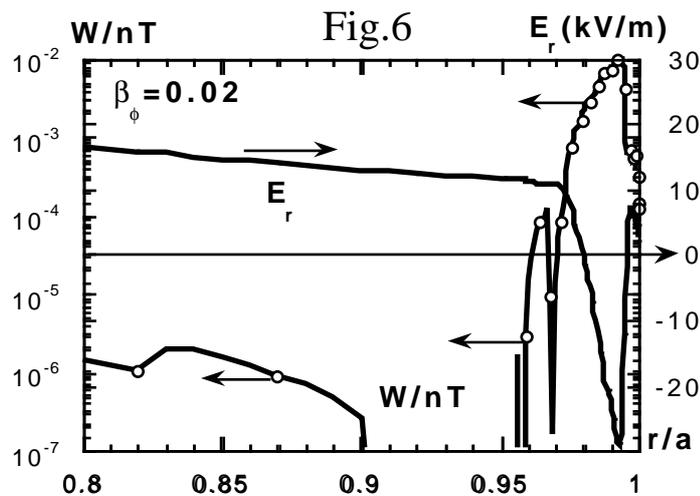


Fig.6. The numerical profile of normalized bulk turbulent energy W/nT_0 and the profile of the radial electric field E_r for the value $\beta_\phi = 0.02$.

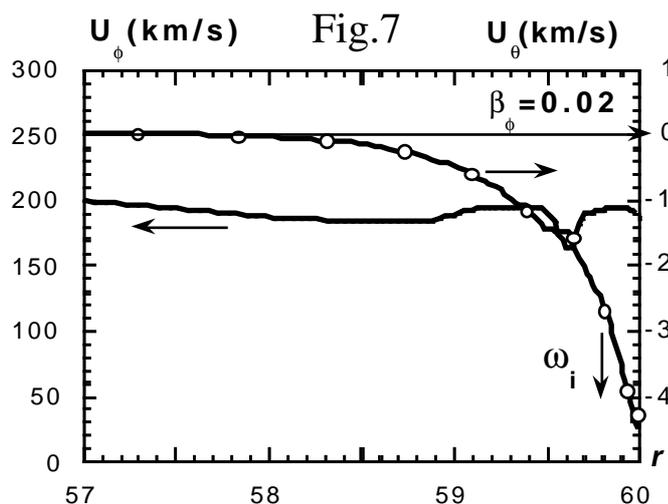


Fig.7. The numerical profiles of the toroidal and poloidal plasma rotations near the separatrix for the value $\beta_\phi = 0.02$.

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