27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** (2000) 1124 -1127

New Alfvén continuum gaps and global waves induced by toroidal flow

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1 Introduction

Toroidal rotations of the order of the sound speed can easily be obtained by the injection of neutral beams for heating tokamak plasmas. On the other hand, the plasma can also rotate in the poloidal direction due to e.g. divertor action. Although the shear in these flows causes the plasma to become more turbulent, the interplay of the induced electric field can reduce the turbulence again. As a result the transport decreases, so that the plasma may trap itself into an improved confinement regime. Shear flow can also stabilize the ballooning modes so that higher values of β may be achieved. For a better understanding of the global dynamics in tokamaks in the presence of both toroidal and poloidal flow, the complete linear spectrum of MHD waves and instabilities has to be computed in genuine toroidal geometry, such as has been done in the past for static equilibria. Flow modifies this spectrum completely, e.g. Doppler shifts enter and new instabilities arise. However, the very first question to be addressed is how the structure of this spectrum changes when the background equilibrium is changed from static to stationary. To analyze these stationary flow patterns and their perturbations, a new package of fully toroidal equilibrium and spectral codes has been developed.

The new optimized spectral codes include both poloidal and toroidal flow, incorporating the five arbitrary flux functions that fix the equilibrium. Straight field line coordinates are exploited since they remain advantageous in the presence of flow. The MHD spectral equations are solved by the new Jacobi–Davidson eigenvalue solver that has been exploited in the study of resistive modes in tokamaks, as described elsewhere [1]. This solver greatly facilitates the study of the structure of the spectrum by producing collections of complex eigenvalues with unprecedented accuracy. The spectral code is fully resistive and applied to ideal stationary background flows.

In Sec. 2 we discuss the equilibria of toroidally rotating plasma. The solutions of equilibria with poloidal flow are described elsewhere [2, 3]. In Sec. 3 we investigate Alfvén gaps in the low frequency part of the spectrum. The emphasis will be on the effect of toroidal rotation. The appearance of gap modes inside these gaps are dealt with in Sec. 4.

2 Equilibrium

We consider an axisymmetric tokamak equilibrium with toroidal rotation. The normal component of the force balance reduces to a Grad–Shafranov equation for the poloidal flux ψ [4],

$$Rj_\varphi = R^2 \boldsymbol{\nabla} \cdot \left(\frac{1}{R^2} \boldsymbol{\nabla} \psi \right) = -I \frac{dI}{d\psi} - R^2 \frac{\partial \Pi}{\partial \psi}, \tag{1}$$

involving the toroidal field: $I(\psi) = RB_\varphi$. The rotation enters through the tangential component

$$\frac{1}{\rho}\frac{\partial \Pi}{\partial R^2} = \tfrac{1}{2}\Omega^2, \qquad \Omega(\psi) = V_\varphi/R. \tag{2}$$

The pressure $p = \Pi(\psi; R) = \rho T$ is no longer a flux function. We investigate two cases: $T = T(\psi)$ (Sec. 3) and $\rho = \rho(\psi)$ (Sec. 4).

3 Low frequency Alfvén gaps

For a cylindrical plasma the continuous Alfvén and slow frequency bands of modes localized on flux surfaces are determined algebraically. The Doppler shifted Alfvén frequency is

$$\Omega_A^\pm = n\Omega \pm \omega_A, \qquad \omega_A = \frac{B_\varphi}{\sqrt{\rho}}\frac{1}{qR}(m + nq), \tag{3}$$

for the separate poloidal and toroidal mode numbers m and n of the linear perturbations. In tokamaks, the continua couple at their cylindrical cross-over points $m + nq = -(m' + nq)$ due to a breaking of the poloidal symmetry. This gives rise to $\Delta m = |m - m'|$ gaps in the Alfvén spectrum. In this paper, we present some results on the low frequency $\Delta m = 0$ Alfvén gaps. An extensive study of these gaps is presented in [4, 5].

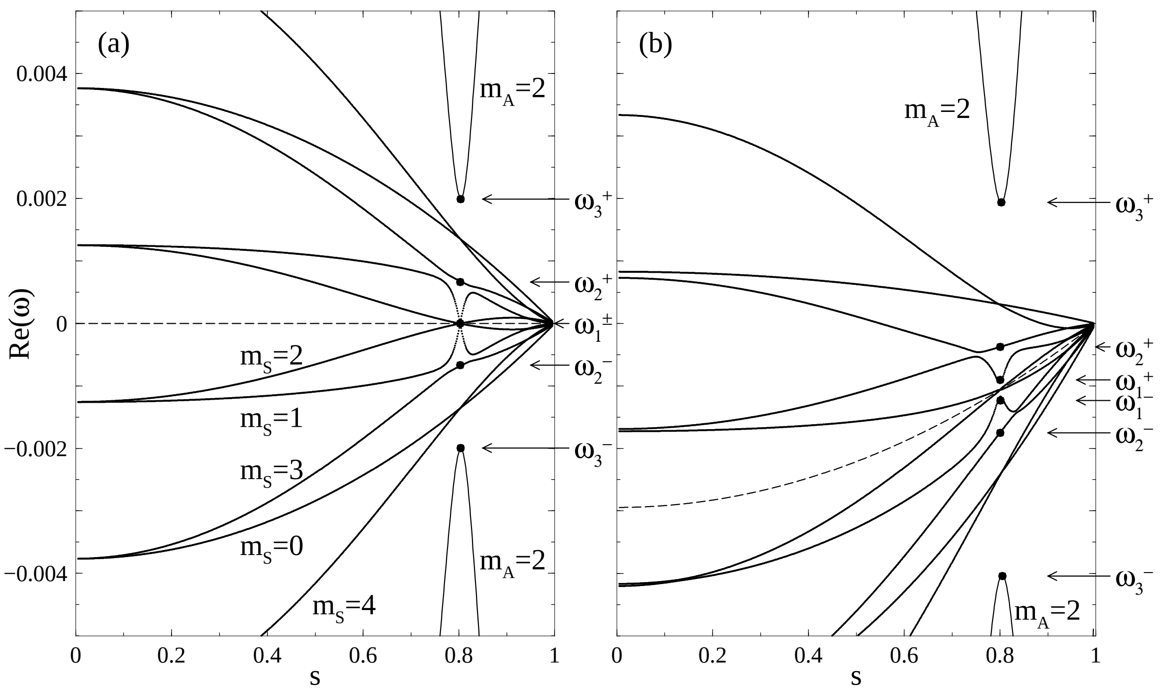

Figure 1: *(a) The $n = -1$ continua plotted against $s = \sqrt{\psi/\psi_1}$ for a static equilibrium. The $\Delta m = 0$ Alfvén gap is ranging from ω_3^- to ω_3^+ at the rational $q = 2$ surface $s \approx 0.8$. The main poloidal harmonic of the slow (m_S) and Alfvén (m_A) continuum branches is indicated. (b) The $\Delta m = 0$ gap is shifted downwards and enlarged by the toroidal rotation [$\Omega(\psi) = \sqrt{T(\psi)}/R_0$, but otherwise the same parameters as Fig. 1(a)]. A new gap ranging from ω_1^- to ω_1^+ appears. The Doppler shift $n\Omega$ is indicated by a dashed line; $\epsilon = 0.1$, $\gamma = 5/3$.*

Fig. 1(a) shows the radial dependence of the low frequency continuous spectrum for a circular cross-section tokamak. The safety factor increases monotonically from $q = 1.5$ on axis to $q = 2.89$ on the plasma edge. Near the rational $m + nq = 0$ surface, the $m = 2$ Alfvén branch enters the slow frequency domain. There, a $\Delta m = 0$ Alfvén gap is formed that appears as a cross-over of the $m = 2$ Alfvén branch with itself. This gap is produced by the geodesic curvature (magnetic field strength varies within flux surfaces), together with finite compressibility and pressure. Coupling to the two sideband slow modes is also involved, so that the gap size is essentially determined by a three mode coupling scheme. From the same coupling mechanism it follows that the neighboring $m = 1$ and $m = 3$ slow modes do not produce an ordinary $\Delta m = 2$ gap. Instead, due to the intervention of the $m = 2$ Alfvén mode, one coupled slow branch is pulled to $\omega_1^{\pm} = 0$.

Toroidal flow strongly modifies the low frequency part of the spectrum, see Fig. 1(b). The $\Delta m = 0$ gap is enlarged by the centrifugal and Coriolis effects and is also Doppler shifted. The marginal frequencies $\omega_1^{\pm}$ are split and form a new purely flow-driven gap located around the Doppler shift. For the derivation of the continuum equations as well as the analytical determination of these frequencies, see [4].

4 Flow-induced gap global waves

Now, we will demonstrate that flow-induced global waves can appear in the $\Delta m = 0$ Alfvén gap. In Fig. 2 the spectrum is shown for a rigid rotating plasma with $\epsilon = 0.25$ and circular cross-section. The density is a flux function: $\rho = 1 - 0.85s^2$. The safety factor increases monoton-

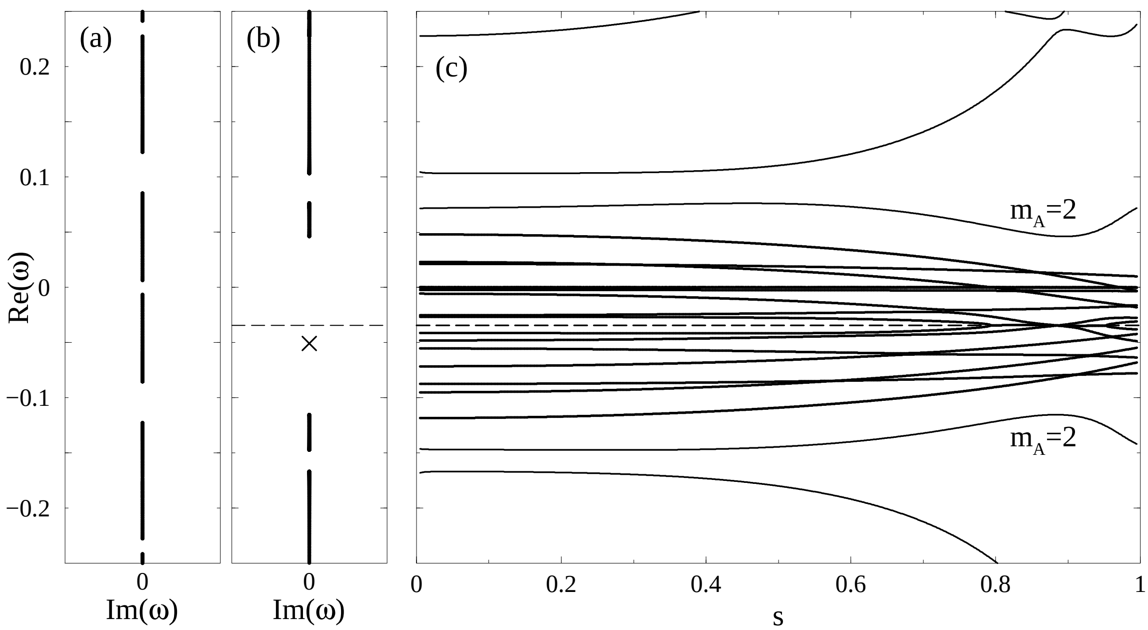

Figure 2: *(a) For a static equilibrium, a small $\Delta m = 0$ gap around $\omega = 0$ is produced. (b) Toroidal rotation [$\Omega \approx 0.034V_A/a$, but otherwise the same parameters as Fig. 2(a)] significantly enlarges this gap and a flow-driven mode (cross) appears below the Doppler shift $n\Omega$ (dashed line). (c) The gap in the $n = -1$ continuous spectrum is located at $s = \sqrt{\psi/\psi_1} \approx 0.89$;* $\epsilon = 0.25$, $\beta = 1\%$, $\gamma = 5/3$.

ically from $q = 1.4$ to $q = 2.46$ and $q = 2$ is located at $s \approx 0.89$. By omitting the slow continua, the small $\Delta m = 0$ Alfvén gap around $\omega = 0$ for a static plasma in Fig. 2(a) becomes visible. This gap is significantly enlarged by the toroidal rotation as shown in Fig. 2(b). The gap appears as an avoided crossing of the two $m = 2$ Alfvén continuum branches and is radially located at the $q = 2$ surface as shown in Fig. 2(c).

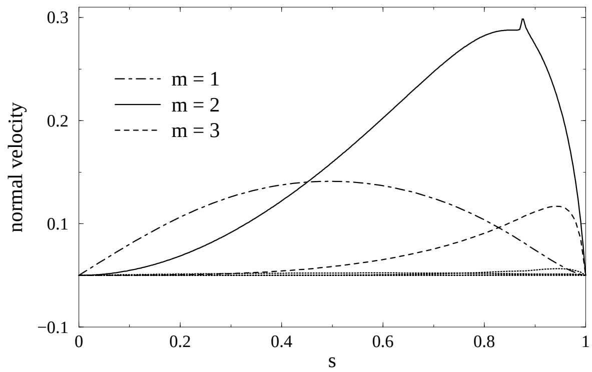

Figure 3: *Mode structure of the perturbed normal velocity component of the flow-driven gap mode in Fig. 2(b).*

Inside the broad gap of Fig. 2(b) a global mode with a frequency $\mathrm{Re}(\omega) \approx -0.05$ appears. From the asymmetric location of the mode with respect to the Doppler shift it follows that the Coriolis effect is responsible, otherwise a mode symmetric above the Doppler shift should occur. The modal content of the normal velocity of this wave is shown in Fig. 3. The near singular behavior at $s \approx 0.87$ is due to the resonant damping by the interaction with the continua inside the $\Delta m = 0$ gap. Possible excitation of these waves by particles should be investigated. The destabilization by the fast α-particles is not likely, since the frequency of this wave is low.

5 Conclusions

- The Alfvén spectrum splits at the mode rational $m+nq = 0$ surface, so that low frequency $\Delta m = 0$ Alfvén gaps appear around the Doppler shift. The centrifugal and Coriolis effects enlarge these gaps.
- With toroidal rotation, the marginal frequency is not only Doppler shifted but also split, giving rise to another gap inside the $\Delta m = 0$ gap.
- Flow-induced global waves are found in the $\Delta m = 0$ Alfvén gaps.
- This global mode may be useful for the waves excited in MHD spectroscopy [6]. Since these waves are in the low frequency regime, they will have important implications for stability.

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