

## The Role of Nonlinear Effects in LH Wave - Plasma Interaction

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Lower hybrid wave (LHW) is launched by a grill whose area constitutes only a very small fraction ( $< 0.01$ ) of the plasma surface. The power density in the region actually filled with RF field much exceeds the power density averaged over the magnetic surface. The dynamics of particles in this RF region can be therefore strongly nonlinear, and the description of the interaction using the quasilinear approximation (based on averaging the RF effect over the magnetic surfaces) can be non-adequate. Indeed, we have found [1-3] a strong deviation between the numerical simulation and the quasilinear description of this interaction. The present paper discusses this phenomenon more properly.

A simplified geometry of the interaction of LHW with particles is presented in Fig. 1. Here, the region  $\Gamma$  represents a cross-section of the LHW conus with a chosen magnetic surface. We assume that trajectories of electrons cover these magnetic surfaces. A majority of our results is based on the 1D slab model of the interaction. We assume the interaction of particles with one monochromatic electrostatic wave  $E_z = E_0 \sin(k_{\parallel} z - \omega t)$  with  $E_0 \neq 0$  for  $0 < z < L$ .

Fig. 1:

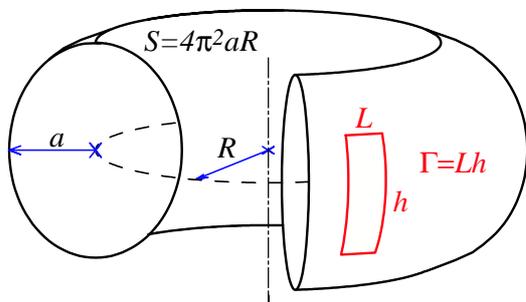
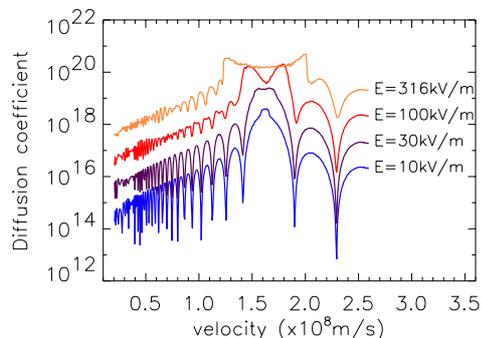


Fig. 2:

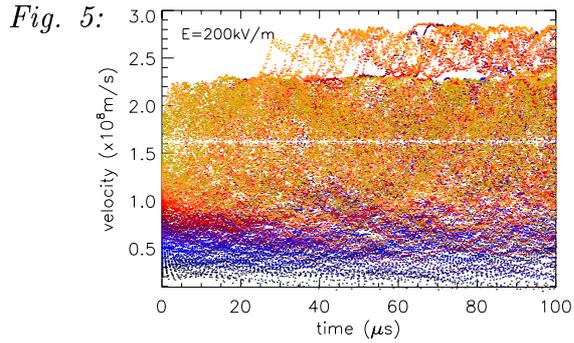
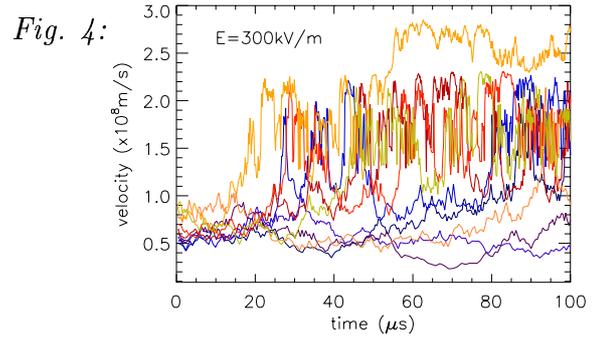
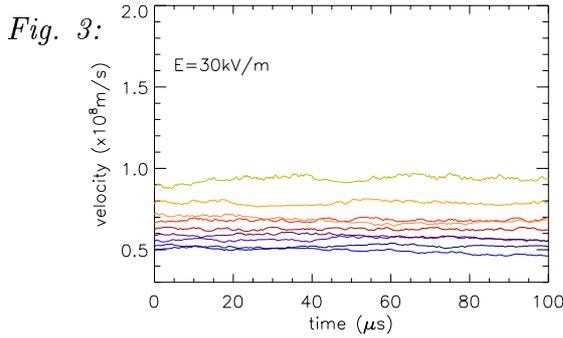


This model offers the calculation of particle's trajectory with as large precision as possible. In the RF region, namely, trajectories (in coordinates and velocities) can be expressed by means of the elliptic integrals (which can be numerically determined with an extreme precision); in the field-free region, the velocity is an integral of the motion and the trajectory is formed by a part of the same circle. There is therefore no numerical integration at all. Whereas both sections form integrable parts, their combination is already non-integrable; this is then the reason of stochasticity.

Assuming the loss of the correlation after one particle's circulation along the the toroid (here, we choose this trajectory in a form of a circle), we can determine the numerical diffusion coefficient  $D^*(v)$ , as  $D^*(v) = \langle (v - v_0)^2 \rangle / \tau$ , where  $\tau = 2\pi R/v_0$  is the toroidal orbit time and the coefficient  $h/2\pi a$  takes into account the particles that do not visit the area  $\Gamma$  (more details are given in [1]). The typical dependence of the numerical diffusion coefficient  $D^*(v)$  on the electric field amplitude  $E_0$  is presented in Fig. 2. Here, we choose  $N_{\parallel} = 1.85$ , and the frequency  $f = 3.7 GHz$ . The corresponding phase velocity is  $v_{ph} = 1.62 \times 10^8 m/s$ . We see that the numerical diffusion coefficient  $D^*$

is non-negligible far below the phase velocity and also far outside the trapping region (e.g. for  $E_0 = 1kV/cm$ , the trapping region  $\Delta v_{trapp}$  is  $\Delta v_{trapp} = 4.6 \times 10^7 m/s$ ).

To have better insight into the mechanism, it is of some interest to follow the dependence of the velocity of individual trajectories (corresponding to different initial conditions) on time for different amplitudes of the electric field  $E_0$  (Figs. 3, 4). We see that for the lower amplitude, there is no interesting change of the velocity for initial values  $v_0 \leq v_{ph}/2$ . For  $E_0 = 300kV/m$ , the acceleration of particles is dramatical even for  $v_0 \approx v_{ph}/4$ . Fig. 5 shows the same situation, but initially, the particles are equidistantly distributed between  $0.5 \times 10^8 < v_0 < 1.5 \times 10^8$  and the initial phase of the wave is random.



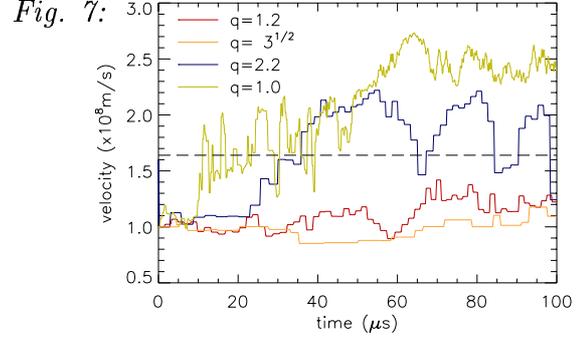
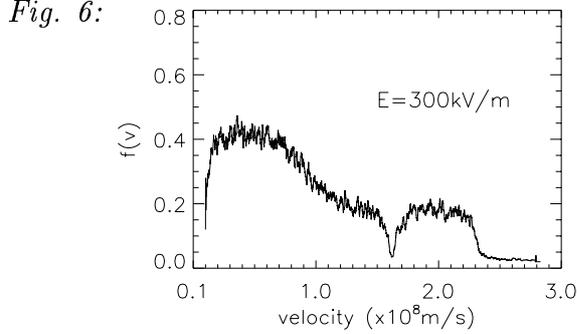
The figures show two important features. The first one is the obvious effect of some sort of attraction of velocity trajectories into the region of the phase velocity (both for  $v_0$  lower and higher than  $v_{ph}$ ). The second one consists in the fact that particles do not remain in the trapped particles region, but are almost immediately expelled.

Let us first discuss the first feature. For wave amplitudes in the range of kV/cm, the region where particles are accelerated or decelerated (further, the strong interaction region) is sufficiently larger than the maximum distance between separatrices of the wave. (These boundaries are, of course, not exactly impermeable). In such a way, a mixing of velocities of particles with different initial conditions can be expected. In our case, we have for  $v_{ph} = 1.6 \times 10^8 m/s$  the relative broadness  $\Delta v/v_{ph} = 0.88$ , while for the quasilinear case, the typical broadness of the spectrum is usually taken as  $\Delta N_{\parallel}/N_{\parallel} = (v_{max} - v_{min})/v_{ph} = 0.1$ . Consequently, the interaction region of the quasilinear case is much smaller than that of our nonlinear model.

The second feature, the fast exodus of particles from the region close to the phase velocity, will obviously result in a change of the velocity distribution function. Due to the mentioned exodus, a bump-in-tail distribution will appear (Fig. 6 - initial velocity distribution is constant in the velocity region  $1 \times 10^7 < v < 1.5 \times 10^8$ ). Moreover, since this distribution is unstable, it is to be expected that due to bump-in-tail instability, a spectrum of Langmuir electrostatic waves will be excited.

Till now we were interested in the interaction of particles in the slab model. To be closer to the reality, we consider the real form of the cut of the LHW cone with the magnetic surface (in a rectangular presentation - Fig. 1). Considering the toroidal and poloidal dimension as 0.3 m, 0.3 m, we calculate the velocity changes of a particle with

the same initial conditions and wave amplitude, but situated on the magnetic surfaces with different values of the safety factor  $q$ . (Fig. 7). This change, namely, influences the multiperiodicity of the particle's crossing the RF region. Consequently, on some magnetic surfaces, where particles cross the conus very often (e.g. for  $q = m/n$  with small integers  $m, n$ ), the interaction (and acceleration) of particles will be most impressive.



It is worthwhile to look for the threshold of the stochasticity acceleration and its relation to the loss of correlation. To obtain an analytical estimate of the threshold for the stochastic acceleration of particles, usual procedure of overlapping of resonances can be applied. For this, it is necessary to Fourier analyze our model of RF field. Using our previous expression for RF field,

$$E(z, t) = E_0 \cos(k_{\parallel} z - \omega t); \quad -L_0/2 > z < +L_0/2, E(z, t) = 0, \quad z \in \Gamma, \quad (1)$$

we can expand this form into the Fourier series.

In this region, the Chirikov overlapping criterion reads:

$$\sqrt{\frac{2q_e}{m_e(k_{\parallel} + n_0 \frac{2\pi}{L})} \frac{E_{0,th}}{2} \frac{2}{\pi n_0} \sin(\pi \frac{L_0}{L})} = \frac{\omega}{(k_{\parallel} + n_0 \frac{2\pi}{L})^2} \frac{2\pi}{L}. \quad (2)$$

Here,  $q_e = |e|$ , where  $e$  is the charge the electron,  $m_e$  is the mass of the electron and  $E_{n,th}$ ,  $E_{n+1,th}$  are amplitudes of neighbouring harmonics,  $E_n$  for  $n_0$  closest to  $n \frac{L_0}{L} = integer$  and  $(k_{\parallel} + n_0 \frac{2\pi}{L}) = 2k_{\parallel}$ , which express the requirement that the stochasticity threshold will appear for  $v = v_{ph}/2$ .

For our parameters, the required  $E_{0,th} \approx 1kV/cm$ , which very roughly corresponds to our numerical results. Nevertheless, outside of the region with  $E_n \approx 0$ , the overlapping criterion ceases its sense, because, for this amplitude, a lot of harmonics interact simultaneously (as has been mentioned already in [7]). From this point of view, the RF slab represents rather a kind of a scattering centrum with a complicated scattering mechanisms.

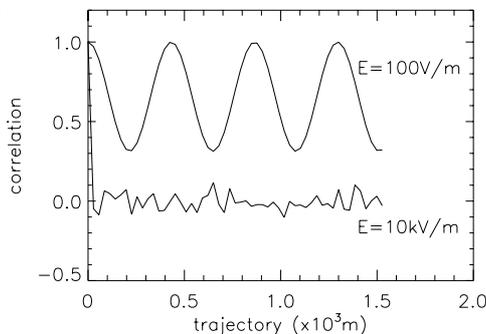
The intrinsic stochasticity regime has to be concomitantly bounded with the loss of the correlation. Fig. 8 brings spatial correlation functions of a phase which is here defined as  $\psi = (\omega t_i - k z_i)$  just in the moment when a particle enters the RF region (here, the subscript "i" means the ith enter into the RF field). The spatial correlation function is defined as

$$C(\Delta l) = \lim_{z \rightarrow \infty} \frac{1}{z} \int_0^z dz \psi(z) \psi(z + \Delta l) \quad (3)$$

We considered two values of  $E_0$  ( $v_0 = 1 \times 10^8$ ) - for  $E_0 = 0.1kV/m$  the interaction is still regular, but for  $E_0 = 10kV/m$ , the interaction is stochastic. According to Fig. 8, and also

according to our estimate of the threshold of the stochastic acceleration, this global effect of the acceleration has its own threshold, which is obviously higher than the threshold of the stochastization of phases.

Fig. 8:



### Summary

- In the case of a strong RF field, particles with velocities larger than a certain threshold velocity  $v_{th}$  are accelerated into the region of the phase velocity and later on diffuse in a broad region of velocity space from  $v = v_{th}$  up to a certain  $v_{max} > v_{ph}$ .
- The threshold velocity  $v_{th}$  only very roughly corresponds to the overlapping criterion for velocities. The threshold for the loss of the correlation in phases is considerably lower.
- The Langmuir waves generated by the bump-in-tail distribution around the phase velocity will have the tendency to smooth out this bump. The possibility of forming an additive absorption mechanism will be discussed in our prepared work.
- The effect of collisions will tend to change the velocity distribution function back to the Maxwell distribution. A stationary distribution, as a result of these two opposite tendencies, will then determine the RF power absorption. Since the nonlinear interaction region is much larger than the quasilinear one, it is possible to expect for our case much larger absorption rate.
- Generally, for the case of large amplitudes, it seems that the representation of the interaction on a base of resonant interaction loses its sense.

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