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Ship-Wave Eigenmodes of Drift Type in Rotating Tokamak Plasmas

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For some time non-local effects has been ascribed to the development of the L-H (Low to High confinement) transition. When the transition occurs, a so called transport barrier is formed at the plasma edge which reduces the flux of particles and heat out of the plasma most drastically. The formation of the transport barrier affects the core plasma at a rate much faster than the diffusion time, hence it has been ruled out as the transfer mechanism. It is now thought that the fast response is due to non-local effects such as global modes connecting magnetic surfaces at the edge and in the core plasma. The global modes can be, *e.g.* drift-type waves.

Plasmas tend to relax into an equilibrium state by excitation of waves. In conventional studies of drift waves, the waves are driven by some instability which releases free energy connected with, *e.g.* the density inhomogeneity. The free energy will then dissipate through the excitation of waves and plasma relaxes. For rotating plasmas there exists another mechanism for exciting and driving waves, namely waves that are resonantly excited with respect to the particle drift velocity. Waves in this case will have a Doppler shifted frequency $\Omega = \omega - \mathbf{k}\cdot\mathbf{V}$. One kind of waves with this property are known in both fluid dynamics and plasma physics as ship waves and are excited by a fluid flow passing a solid body. In this paper we will consider having a static obstacle perturbing the plasma flow. The excited waves will then have a stationary structure and their frequency, in the rest system of the object, will be equal to zero.

The plasma flow we will consider is the poloidal component of the equilibrium $E \times B$-drift. At the L-H transition it has been observed that the E-field at the edge, consists of a radial component only. Moreover the magnetic field constitutes of a strong applied toroidal component and a weaker plasma current induced component. Together it permits us to neglect the toroidal component of the $E \times B$-plasma rotation.

The key features of waves excited in an axisymmetric tokamak plasma can be described by a set of fluid equations for the electrons and the ions supplemented by the Maxwell equations for the electromagnetic field. In the limit of low-β(the ratio of the plasma pressure to the magnetic field pressure) the magnetic field perturbations are negligible and the drift waves will become collisionless electrostatic waves of low frequency. Assuming that the electron inertia is negligible with respect to the thermal motion, electrons will be Boltzmann distributed and the quasineutrality condition can replace Maxwell's equations. Solving the fluid equations for the ions we get the ions' velocity parallel and perpendicular to the magnetic field.

$$\frac{\partial v_\parallel}{\partial t} + \hat{\boldsymbol{b}}\cdot(\mathbf{v}_0\cdot\nabla)\mathbf{v} + \hat{\boldsymbol{b}}\cdot(\mathbf{v}\cdot\nabla)\mathbf{v}_0 = -\frac{e}{m_\mathrm{i}}\hat{\boldsymbol{b}}\cdot\nabla\phi \tag{1}$$

$$\mathbf{v}_\perp = \mathbf{v}_E + \frac{\hat{\boldsymbol{b}}}{\omega_\mathrm{ci}}\times\left[\frac{\partial \mathbf{v}_E}{\partial t} + (\mathbf{v}_0\cdot\nabla)\mathbf{v}_E + (\mathbf{v}_E\cdot\nabla)\mathbf{v}_0\right] \tag{2}$$

with

$$\mathbf{v}_E = \frac{\mathrm{c}}{B}\hat{\boldsymbol{b}}\times\nabla\phi \quad \text{and} \quad \hat{\boldsymbol{b}} = \frac{\mathbf{B}}{B}. \tag{3}$$

The toroidal geometry together with the curvilinear character of the magnetic field will have a strong impact on the solutions of these equations. Using a Fourier decomposition method

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which will allow modes to couple to each other this impact can be fully studied. Drift modes are known to be long wavelength along the magnetic field line and short wavelength across it. Therefore, we consider solutions for the modes localised on the rational magnetic surface $r = r_0$, defined by $m_0 - nq(r_0) = 0$, where m_0 is the poloidal mode number and $q(r_0)$ is the safety factor at the given surface around which the mode is centred. It should be noted that $m_0 \gg 1$ for the drift modes of principal interest. Fourier expanding around a given rational surface yields

$$\begin{aligned} \phi(r,\theta) &= \exp(\mathrm{i}m_0\theta)\sum_l \phi_l(r)\exp(\mathrm{i}l\theta) \\ v_{\|}(r,\theta) &= \exp(\mathrm{i}m_0\theta)\sum_l v_l(r)\exp(\mathrm{i}l\theta) \end{aligned} \tag{4}$$

Here we have also added a radial dependence to the Fourier amplitudes to account for the velocity shear. The final equation for the electrostatic potential will under these circumstances become, as a function of the new variable $x = (r - r_0)/\rho$

$$\frac{\mathrm{d}^2U_l}{\mathrm{d}x^2} - DU_l + \sigma^2\left(x - \frac{l}{k_\theta\rho s}\right)^2 U_l = \alpha\left(U_{l+1} + U_{l-1}\right) + \frac{c_s}{R\Omega}\frac{\mathrm{d}}{\mathrm{d}x}\left(U_{l+1} - U_{l-1}\right) \tag{5}$$

Here

$$D = 1 + k_\theta^2\rho^2 + \frac{c_s\rho}{V_0 r_n} - \frac{\rho^2}{r_0^2}\{1 - \xi - \xi\xi'\} - \frac{\rho^2}{r_0 r_n}\{1 + \xi\}, \tag{6}$$

$$\sigma^2 = \left(\frac{c_s\rho s}{V_0 q_0 R}\right)^2 \quad \text{and} \quad \alpha = \frac{r_0}{2R} + \frac{c_s\rho}{V_0 R}\left(\frac{r_0}{2r_n} - 1\right) \tag{7}$$

and with

$$V_0 = v_0(r_0), \qquad \xi = r_0V_0'/V_0, \qquad \xi' = r_0V_0''/V_0'$$

$$V_0' = \left.\frac{\mathrm{d}v_0}{\mathrm{d}r}\right|_{r=r_0}, \qquad V_0'' = \left.\frac{\mathrm{d}^2v_0}{\mathrm{d}r^2}\right|_{r=r_0}, \qquad \frac{1}{r_n} = -\left.\frac{\mathrm{d}\ln n_0}{\mathrm{d}r}\right|_{r=r_0}$$

where V_0 is the local rotation velocity, r_n is the spatial scale of density inhomogeneity and $k_\theta = m_0/r_0$ is the poloidal wave vector.

Eq. (5) is a differential-difference equation and to solve it the so called strong and weak coupling approximations, developed by Horton and Tang, will be used. Which approximation to use depends on the plasma parameters. The edge plasma is characterised by strong gradients which will inhibit more than a few harmonics to couple to each other, *i.e.* we will use the weak coupling approximation. In the core plasma the gradients are more flat and many modes couple to one another. Hence we will use the strong coupling approximation for the core plasma.

The coupling of many modes makes is possible to replace the discrete set of functions $U_l(x)$ with the continuous function $U(x,l)$ by the following transformation

$$U_{l\pm1}(x) \longrightarrow U(x,l) \pm \frac{\partial U(x,l)}{\partial l} + \frac{1}{2}\frac{\partial^2 U(x,l)}{\partial l^2} \tag{8}$$

under the condition that $1 \ll \Delta_l \ll m_0$ (Δ_l is the spread of the mode numbers). This will give us a two-dimensional problem to solve that can be reduced to a one-dimensional problem by introducing a new variable $y = x - l/k_\theta\rho s$ and the final one-dimensional equation is

$$\frac{\partial^2 U}{\partial y^2} + \left(\Lambda - \zeta^2 y^2\right) U = 0 \tag{9}$$

with constants

$$\Lambda = B/F \quad \text{and} \quad \zeta^2 = -\sigma^2/F \tag{10}$$

where

$$F = 1 - \frac{\alpha}{k_\theta^2 \rho^2 s^2} - \frac{2c_s}{V_0 k_\theta^2 \rho R s}, \quad B = 2\alpha + \frac{c_s \rho}{V_0 r_n} - 1 - k_\theta^2 \rho^2 \tag{11}$$

The equation (9) has the form of the stationary Schrödinger equation and using methods developed in quantum mechanics we can derive the corresponding dispersion relations. As we have used the ship wave condition, $\omega = 0$ the dispersion relations derived will give us a measure of the spatial spread of the wave and not be the customary dispersion relations of the frequency. In the case of $\zeta^2 < 0$ the Eq. (9), in analogy with quantum mechanics will contain a well. The dispersion relation of the global modes trapped in this well will be

$$k_{\theta N}^2 \rho^2 \simeq (2N+1)^2 \frac{r_n^2}{q_0^2 R^2} \left[\frac{c_s \rho}{V_0 R}\left(2s + \frac{r_0}{2r_n}\right) + \frac{r_0}{2R}\right] \left[\left(1 + \frac{V_0 r_n}{c_s \rho}\right)^2 + (2N+1)^2 \frac{r_n^2 s^2}{q_0^2 R^2}\right]^{-1} \tag{12}$$

It is evident that the eigenvalues $k_{\theta N}$ have an accumulation point defined by the limit $N \longrightarrow \infty$,

$$k_{\theta\infty}^2 \rho^2 = \frac{1}{s^2}\left[\frac{c_s \rho}{V_0 R}\left(2s + \frac{r_0}{2r_n}\right) + \frac{r_0}{2R}\right] \tag{13}$$

For the propagating modes, *i.e.* when $\zeta^2 > 0$ and the well turns into an antiwell the dispersion relation will become

$$1 + k_\theta^2 \rho^2 - \frac{\rho c_s}{r_n |V_0|} = -\mathrm{i}(2N+1) \frac{\rho c_s}{R|V_0|} \frac{s}{q_0} \sqrt{1 + \frac{\rho c_s}{R|V_0|} \frac{1}{s^2} \frac{1}{k_\theta^2 \rho^2}\left(2s - \frac{r_0 |V_0|}{2\rho c_s} + \frac{r_0}{2r_n}\right)} \tag{14}$$

Since the right hand side of this equation is complex we can conclude that k_θ will be complex and hence that there will be a damping or a growth of the waves. Further calculations will show, if decomposing k_θ into $k_\theta = k + \mathrm{i}\Gamma$ with $|\Gamma| \ll |k|$ where $\Gamma > 0$ corresponds to damping of a wave with $k > 0$, that the real part of Eq. (14) approximately is

$$k^2 \rho^2 = \frac{\rho c_s}{r_n |V_0|} - 1 \tag{15}$$

Substituting the solution (15) into the imaginary part of Eq. (14), we find an approximate expression for Γ:

$$\frac{\Gamma}{k} = \left(N + \frac{1}{2}\right) \frac{r_n s}{q_0 R} \left(1 - \frac{r_n |V_0|}{\rho c_s}\right)^{-1} \tag{16}$$

From the expression above one can see that the damping depends on the magnetic shear, s and thus the propagating waves will experience shear convective damping. This shear damping can be counteracted by a large enough rotation velocity.

Within the edge plasma the steep derivatives only let the closest neighbouring modes couple to each other. Hence we truncate the system of equations Eq. (5) by restricting the values of l to $l = 0, \pm 1$. For the main mode this will yield

$$\frac{\mathrm{d}^2 U_0}{\mathrm{d}x^2} + (\lambda - \xi^2 x^2) U_0 = 0 \tag{17}$$

with

$$\lambda = \frac{D^2 - 2\alpha^2}{2\left(\frac{c_s}{V_0 k_\theta R}\right)^2 - D} \quad \text{and} \quad \xi^2 = \frac{\sigma^2 D}{2\left(\frac{c_s}{V_0 k_\theta R}\right)^2 - D} \tag{18}$$

The form of Eq. (17) coincides exactly with Eq. (9), so using the analysis of this equation from the previous section. In the case of global modes, *i.e.* $\xi^2 < 0$ the dispersion relation will become,

$$k_{\theta N}^2 \rho^2 = (2N+1)^2 \frac{2\rho^4 c_s^2 s^2}{R^4 V_0^2 q_0^2} \left(1 + \frac{\rho c_s}{r_n V_0}\right)^{-1} \left[\left(1 + \frac{\rho c_s}{r_n V_0}\right)^2 + (2N+1)^2 \frac{\rho^2 c_s^2 s^2}{R^2 V_0^2 q_0^2}\right]^{-1} \tag{19}$$

with the accumulation point

$$k_{\theta\infty}^2 \rho^2 = \frac{c_s^2 \rho^2}{V_0^2 R^2} \left(1 + \frac{\rho c_s}{r_n V_0}\right)^{-1} \tag{20}$$

It can be shown that there are no propagating modes in the edge plasma.

Conclusions

In this paper we have studied the dispersion properties of drift waves resonantly excited with the drift velocity of the particles, *i.e.* waves with the Doppler shifted frequency $\Omega = \omega - k_\theta V_0$. The means of excitation is a static obstacle placed a the plasma edge which will give rise to stationary waves ($\omega = 0$) of ship type. The dispersion equations are derived from a simple model of a low-β plasma with concentric, circular magnetic surfaces and a large aspect ratio. The plasma rotation is assumed to be driven by the radial electric field which is a typical feature of an H-mode plasma. The low frequency electrostatic drift oscillation are studied using the assumptions of adiabatic electrons and quasineutrality. Applying these conditions on a two-fluid system of equations and exploring the effect of the toroidicity, made explicit by the specialised Fourier transformation (4) the infinite set of differential-difference equations (5) will be achieved. In order to reduce the Eq. (5) into analytically more manageable equations we use the strong and the weak coupling approximations. In both these approximations the differential-difference equations (5) is reduced to the form of the stationary Shrödinger equation. Using methods developed in quantum mechanics we are now able to get the final dispersion relations of the ship waves. It has been shown that we get two kind of waves. Global modes coupling magnetic surfaces at the edge to those in the core and propagating modes. Remembering that the ship waves under consideration are stationary structures ($\omega = 0$), the dispersion relation will give us the spatial extent of those structures. In the case of the strong coupling approximation we will get both global and propagating modes and also that the propagating modes experience shear convectiv damping which can be reversed into growth by the rotation velocity. For the weak coupling approximation only global modes are possible. Moreover, all global modes have an accumulation point.