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Drift Wave Eigenmodes in Toroidally Rotating Tokamak Plasma

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Introduction

The drift waves in rotating toroidal plasma are studied for an axisymmetric, large-aspect-ratio tokamak with concentric and circular magnetic surfaces. Plasma rotation has significant toroidal component attributed e.g. to Neutral Beam Injection and relatively minor poloidal component due to inherent diamagnetic drift of hot ions. Low-frequency drift type electrostatic oscillations of low-β plasma are considered in assumptions of almost adiabatic electrons and plasma quasineutrallity. For the case of moderate velocity shear, a strong coupling approximation is used which accounts for the toroidal coupling of a large number of normal modes centered on the neighboring rational surfaces. The derived eigenmode equation gives either marginally stable global drift modes or propagating drift waves. We show that toroidal rotation plays key role in the formation of global modes and in stabilization of the universal drift instability.

Basic Equations

We consider low-β toroidal plasma configuration which rotates both toroidally and poloidally. The plasma is confined by inhomogeneous magnetic field $\mathbf{B}$ with vanishing component along the equilibrium density $n_0(r)$ gradient. For low frequency electrostatic oscillations, electrons behave almost adiabatically and their perturbed density follows the Boltzmann relation

$$\frac{n_e}{n_0} = \frac{e\phi}{T_e}(1 - i\delta) \quad \text{with} \quad 0 \leq \delta << 1 \tag{1}$$

where ϕ is the perturbed electrostatic potential and $-e$ and T_e are the electron charge and temperature, respectively. The phase shift δ between the perturbed density and potential describes the dissipative part of electron density distribution.
In equilibria, the rotation velocity for the ion fluid is

$$\boldsymbol{v}_0(r) = \boldsymbol{v}_{0p}(r) + \boldsymbol{v}_{0t}(r)$$

where the toroidal component $\boldsymbol{v}_{0t}(r)$ is attributed to median plane NBI and the poloidal one $\boldsymbol{v}_{0p}(r)$ to the inherent diamagnetic drift of hot ions.

Considering electrostatic quasineutral perturbations, where the fluctuating parts are of the order of $\epsilon = r/R \sim \omega/\omega_{ci}$ with respect to the equilibrium quantities, we linearise the ion fluid equations. In the first order, the continuity equation obtains the form

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v_0} \cdot \nabla\right)\left[\frac{e\phi}{T_e}(1 - i\delta)\right] + \nabla \cdot \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \ln n_0 = 0 \tag{2}$$

Separating the equation of motion into parallel and perpendicular to the magnetic field components, we get

$$\frac{\partial v_{\|}}{\partial t} + \hat{\boldsymbol{b}} \cdot (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v} + \hat{\boldsymbol{b}} \cdot (\boldsymbol{v} \cdot \nabla)\boldsymbol{v_0} = -\frac{e}{m_i}\left[1 + \frac{T_i}{T_e}(1 - i\delta)\right] \hat{\boldsymbol{b}} \cdot \nabla\phi \tag{3}$$

for the parallel components and

$$\boldsymbol{v}_\perp = \boldsymbol{v_E} + \frac{\hat{\boldsymbol{b}}}{\omega_{ci}} \times \left[\frac{\partial \boldsymbol{v_E}}{\partial t} + (\boldsymbol{v_0} \cdot \nabla)\boldsymbol{v_E} + (\boldsymbol{v_E} \cdot \nabla)\boldsymbol{v_0}\right] \tag{4}$$

for the perpendicular ones where the usual drift wave ordering was considered. The $E \times B$ drift is given by

$$\boldsymbol{v_E} = \frac{c}{B}\left[1 + \frac{T_i}{T_e}(1 - i\delta)\right] \hat{\boldsymbol{b}} \times \nabla\phi$$

Choosing the simple toroidal coordinate system (r, φ, θ), we express the magnetic field as $\mathbf{B} = B_p(r)\hat{\mathbf{e}}_\theta + B_t(1 + \epsilon\cos\theta)^{-1}\hat{\mathbf{e}}_\varphi$, according to the tokamak approximation. Drift modes are coupled in the poloidal direction due to toroidicity effects. The mode localised on the rational surface $r = r_0$, defined by $m_0 - nq(r_0) = 0$ (m, n the poloidal and toroidal mode number and $q(r) = rB_t/RB_p(r)$ is the safety factor) is coupled poloidally with the modes localised on the neighbouring surfaces $r_0 \pm \Delta r$ such that $m_0 \pm 1 = nq(r_0 \pm \Delta r)$. So, we seek for solutions which in terms of the azimuthal mode spectrum are centred around the poloidal mode m_0. Thus, we use an appropriate Fourier expansion

$$f(r, \theta, \varphi, t) = \exp(im_0\theta - in\varphi - i\omega t)\sum_l f_l(r)\exp(il\theta)$$

where f stands for ϕ and $v_{\|}$. For further progress, we expand the coefficients that are functions of r into Taylor series in the vicinity of the reference rational surface $r = r_0$ and we reduce the equations by eliminating $v_{\|}$, into the following one

$$\begin{aligned}\frac{\partial^2\phi_l}{\partial x^2} + [A_1 + A_2(l - k_\theta\rho s x)]\frac{\partial\phi_l}{\partial x} + \left[A_3 + A_4(l - k_\theta\rho s x) + A_5(l - k_\theta\rho s x)^2\right]\phi_l + \\ + [A_6 + A_7(l - k_\theta\rho s x)](\phi_{l+1} + \phi_{l-1}) + [A_8 + A_9(l - k_\theta\rho s x)]\frac{\partial}{\partial x}(\phi_{l+1} - \phi_{l-1}) = 0\end{aligned} \tag{5}$$

This is the model equation which describes the mode structure of a drift wave in a rotating tokamak plasma around surface r_0. It identifies an infinite system of coupled equations for the poloidal harmonics ϕ_l with $l = 0, \pm1, \pm2, \ldots$. The radial flux surface coordinate r has been replaced by the dimensionless local coordinate $x = (r - r_0)/\rho$, where ρ is the ion Larmour radius at temperature $(T_e + T_i)$. The coefficients $A_{1\ldots9}$ depend on plasma parameters and rotation velocity characteristics.

Strong Coupling Approximation

In the strong coupling approximation [Horton et al., Phys. Fluids **21**, 1366 (1978)], a large number Δl ($m_0 \gg \Delta l \gg 1$) of poloidal harmonics is assumed to be coupled due to toroidicity effects. In this limit, the discrete set of functions $\phi_l(x)$ can be replaced by a continuous function $\phi(x, l)$ of two variables,

$$\phi_{l\pm1}(x) \to \phi(x, l) \pm \frac{\partial\phi(x, l)}{\partial l} + \frac{1}{2}\frac{\partial^2\phi(x, l)}{\partial l^2}$$

The mode equation obtains the 2D form but can be reduced to an ordinary second order differential equation by introducing the single variable $y = x - l/k_\theta \rho s$. The resulted equation can be written in Weber form by introducing an appropriate potential function $n(y)$ and shifting linearly the variable y to ψ.

$$\frac{\partial^2 n}{\partial \psi^2} + \left(\Lambda - \zeta\psi^2\right) n = 0 \tag{6}$$

This equation can be seen as Scrödinger equation with effective "energy"

$$\Lambda = \frac{k_\theta \rho}{a_0}\frac{\omega_{ci}}{\omega'}\frac{\rho}{r_n}\left[1 - \frac{\omega'}{\omega_*} + \frac{r_0}{R} + \frac{r_0 r_n}{\rho R}\mathcal{V}(1 - i\tau) - \frac{\rho}{r_0}\frac{V_{0p}}{c_s}(1+\xi_p) + \frac{\rho}{q_0 R}\frac{V_{ot}}{c_s}\xi_t + i\frac{\tau}{k_\theta \rho}\frac{r_n}{\rho}\frac{\omega'}{\omega_{ci}}\right]$$

and effective "potential"

$$\zeta = -{a_0}^{-1}\left(\frac{\rho}{R}\frac{\omega_{ci}}{\omega'}\frac{k_\theta \rho s}{q_0}\right)^2$$

where

$$a_0 = 1 + \frac{\omega_{ci}}{\omega'}\frac{1}{k_\theta R s^2}\left[2s - 1 + \frac{r_0}{2r_n} + \frac{r_0}{2\rho}\mathcal{V}(1 - i\tau)\right]$$

A Doppler-shifted frequency has been introduced through $\omega' = \omega + k_\varphi V_{0t} - k_\theta V_{0p}$, due to plasma rotation, where $k_\theta = m_0/r_0$ and $k_\varphi = n/R$. High order corrections have been neglected with respect to ϵ, supposing that ω' is of the order of dimagnetic drift frequency ω_*. The scale of inhomogeneity is $r_n = -(d\ln n_0/dr)^{-1}_{r=r_0}$, the magnetic shear parameter $s(r) = rq'(r)/q(r)$, the local poloidal/toroidal velocity $V_{0p,t} = v_{0p,t}(r_0)$, their shear $\xi_{t,p} = r_0 V'_{0p,t}/V_{0p,t}$ and $\tau = \delta T_e/(T_i + T_e)$. Furthermore, the dimensionless "velocity factor" $\mathcal{V}$ has been defined through $\mathcal{V} = \frac{V_{0p}}{c_s} + \frac{r_0}{q_0 R}\frac{V_{0t}}{c_s}$.
The solutions of (6) are of two different types depending on the effective potential. The plasma rotation has crucial role while it determines the well or anti-well character of the potential and thus, the type of the drift eigenmodes.

Global Drift Modes

When $\zeta > 0$, the eigenfunctions of Eq.(6) are of the form of $\exp(-y^2)H_N(y)$ (H_N is the Hermite polynomial of N^{th} order) and the corresponding eigenvalues are defined by the dispersion equation

$$\Lambda/\sqrt{\zeta} = 2N + 1 \qquad\qquad N = 0, 1, 2.. \tag{7}$$

The corresponding solutions of Eq.(6) describe a non-propagating drift eigenmode localised inside a "potential well", i.e. the **global mode**. This eigenmode is broad in its radial dependence being composed of a superposition of localised -coupled- quasimodes. The necessary conditions ($\zeta > 0$, $\Lambda > 0$) for the formation of global drift modes are satisfied for positive shifted Doppler eigenfrequencies of the interval

$$1 + \frac{r_n r_o}{\rho R}\mathcal{V} < \frac{\omega'_0}{\omega_*} < -\frac{r_o r_n}{2R\rho}\frac{1 + k_\theta^2\rho^2}{k_\theta^2\rho^2 s^2}\left[\mathcal{V} + \frac{\rho}{r_n} + (2s-1)\frac{2\rho}{r_0}\right]$$

and for negative shifted Doppler eigenfrequencies

$$-1 - \frac{r_n r_o}{\rho R}\mathcal{V} < \frac{|\omega'_0|}{\omega_*} < \frac{r_o r_n}{2R\rho}\frac{1 + k_\theta^2\rho^2}{k_\theta^2\rho^2 s^2}\left[\mathcal{V} + \frac{\rho}{r_n} + (2s-1)\frac{2\rho}{r_0}\right]$$

It is worthwhile to note here that for rotating plasma, necessary condition for global mode formation turns to be

$$\omega_0' \left(\mathcal{V} + \frac{\rho}{r_n} \right) < 0$$

supposing that $4s << (r_0 + 2r_n) r_n^{-1}$. The dispersion equation (7) can be reduced to simple dispersion relations for some limited cases. So, for frequency $\omega' < 0$ that;

$$\frac{|\omega_0'|}{\omega_*} << \frac{r_o r_n}{2R\rho} \frac{1 + k_\theta^2 \rho^2}{k_\theta^2 \rho^2 s^2} \left(\mathcal{V} + \frac{\rho}{r_n} \right)$$

with

$$\frac{27(N+1/2)^2}{2k_\theta^2 \rho^2 q_0^2} \frac{r_n^3}{R^3} \frac{r_0}{\rho} \left(\mathcal{V} + \frac{\rho}{r_n} \right) \gg \left[1 + \frac{r_0 r_n}{\rho R} \left(\mathcal{V} + \frac{\rho}{r_n} \right) \right]^3$$

we obtain the following dispersion relation

$$\left(\frac{|\omega_0'|}{\omega_*} \right)^3 \simeq \frac{(2N+1)^2}{k_\theta^2 \rho^2 q_0^2} \frac{r_n^3}{R^3} \frac{r_0}{2\rho} \left(\mathcal{V} + \frac{\rho}{r_n} \right)$$

Propagating Modes

When $\zeta < 0$, the solutions of Eq.(6) are given by the Hermite functions H_ν and are bounded in the whole interval of y, only when $\nu = N = 0, 1, 2, ...$ The dispersion equation in this case is given by

$$\Lambda / i \sqrt{|\zeta|} = 2N + 1 \tag{8}$$

and the corresponding solutions of (6) describe **radial propagating drift waves** with characteristic frequency

$$\frac{\omega_0'}{\omega_*} \simeq 1 + \frac{r_0}{R} + \frac{r_0 r_n}{\rho R} \mathcal{V} - \frac{\rho}{r_0} \frac{V_{0p}}{c_s} (1 + \xi_p) + \frac{\rho}{q_0 R} \frac{V_{0t}}{c_s} \xi_t$$

and amplitude growth/decrement rate given by

$$\frac{\gamma}{\omega_*} \simeq \frac{\omega_0'}{\omega_{ci}} \frac{r_n}{\rho} \frac{\tau}{k_\theta \rho} - \tau \frac{r_0 r_n}{R\rho} \mathcal{V} - (2N+1) \frac{\omega_0'}{|\omega_0'|} \frac{s r_n}{q_0 R} \sqrt{1 + \frac{\omega_{ci}}{\omega_0'} \frac{r_0}{2R k_\theta \rho s^2} \left(\mathcal{V} + \frac{\rho}{r_n} \right)}$$

The "universal" drift instability term and the stabilisation character of magnetic shear (for $\omega' > 0$) appear clearly in the latest equation. Nevertheless, plasma rotation has crucial role being control mean in drift stability.

Conclusions

The eigenmode equation describing coupled drift waves in rotating toroidal plasma has two classes of solutions depending on plasma rotation velocity. The global drift mode has a structure of a quasimode, localised in radial direction with a small wavenumber along the confining magnetic field. It includes a large number of rational magnetic surfaces due to toroidal coupling of the modes localised on the neighbouring magnetic surfaces. This mode corresponds to the bounded state in a potential well, which is marginally stable. The propagating drift waves correspond to unbounded states which leave the magnetic surface on which they are excited. The "universal" instability can be compensated by the magnetic field shear but crucial role and control feature on drift stabilisation has the rotation velocity.