

Ambipolar Effects in ECR Heating of a Plasma Column

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Electron cyclotron resonance heating (ECRH) of plasma is a problem which is of substantial interest to researches working in the field of control fusion. Most of theoretical studies of the processes taking place during the ECRH has not been taken into account effects connected with the presence of charge separation electric field. However these effects might be significant in the case when the plasma inhomogeneous in a plane transversal to external magnetic field. Specifically the quasi-stationary ambipolar electric field can influence the evolution of electron temperature. As it is shown below the effect of the charge separation results in limitation of the electron energy increase even within nonrelativistic consideration.

The exactly solvable physical model implies that there is an axially symmetric column of collisionless plasma with two sorts of particles (electrons and ions). The column is located in a uniform magnetic field directed along the axis of symmetry of the column, z , and in a uniform transversal electric field rotating with electron cyclotron frequency, ω_0 :

$$\mathbf{B} = B_0 \mathbf{e}_z, \quad \mathbf{E}_{ext} = E_0 (\cos(\omega_0 t) \mathbf{e}_x + \sin(\omega_0 t) \mathbf{e}_y), \quad B_0, E_0 = const, \quad \omega_0 \equiv \frac{eB_0}{m_e c} \quad (1)$$

The plasma column is considered to be homogeneous along the z -axis. Therefore, the potential of charge-separation electric field, φ , depends only on one spatial variable r which is the distance from the axis of symmetry of the column: $r \equiv \sqrt{x^2 + y^2}$. Particles of each sort are described by distribution functions f_α which do not depend on z . Therefore, they satisfy the following Vlasov kinetic equations:

$$\frac{\partial f_\alpha}{\partial t} + (\mathbf{v}, \nabla_{\mathbf{r}}) f_\alpha - \frac{Z_\alpha e}{m_\alpha} (\nabla_{\mathbf{r}} \varphi, \nabla_{\mathbf{v}}) f_\alpha + \frac{Z_\alpha e}{m_\alpha} (\mathbf{E}_{ext} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}], \nabla_{\mathbf{v}}) f_\alpha = 0, \quad (2)$$

$$\mathbf{r} \equiv (x, y), \quad \mathbf{v} \equiv (v_x, v_y), \quad \nabla_{\mathbf{r}} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right), \quad \nabla_{\mathbf{v}} \equiv \left(\frac{\partial}{\partial v_x}, \frac{\partial}{\partial v_y} \right), \quad (\alpha = i, e)$$

where $Z_\alpha e$ and m_α are the charge and mass of particles of sort α , respectively, ($Z_e = -1$, $Z_i = Z$). The commonly accepted approach to analyzing dynamics of a sufficiently dense plasma is quasi-neutral approximation. It implies that the charge separation field provides the following relationship to be fulfilled:

$$\sum_{\alpha} Z_\alpha n_\alpha(\mathbf{r}, t) \approx 0, \quad n_\alpha(\mathbf{r}, t) \equiv \int f_\alpha(\mathbf{v}, \mathbf{r}, t) d\mathbf{v} \quad (3)$$

But for the case of the presence of the external field \mathbf{E}_{ext} rotating with the high frequency ω_0 the relationship (3) is valid only for the densities n_α averaged over period of electron's cyclotron oscillations $T \sim 1/\omega_0$, i.e.

$$\frac{1}{T} \int_t^{t+T} [Zn_i(\mathbf{r}, \tau) - n_e(\mathbf{r}, \tau)] d\tau \approx 0$$

A fruitful approach to analysis of Eqs. (2) – (3) is based on the method of moments developed in [1, 2]. Specifically, following this method, the dynamics of a plasma column can be described by the following moments of the distribution functions:

$$\langle r_k \rangle_\alpha \equiv \frac{1}{N_\alpha} \int \int r_k f_\alpha(\mathbf{v}, \mathbf{r}, t) d\mathbf{v} d\mathbf{r}, \quad N_\alpha \equiv \int \int f_\alpha(\mathbf{v}, \mathbf{r}, t) d\mathbf{v} d\mathbf{r}, \quad \langle v_k \rangle_\alpha, \quad (4)$$

$$\langle r_k r_j \rangle_\alpha \equiv \frac{1}{N_\alpha} \int \int r_k r_j f_\alpha(\mathbf{v}, \mathbf{r}, t) d\mathbf{v} d\mathbf{r}, \quad \langle r_k v_j \rangle_\alpha, \quad \langle v_k v_j \rangle_\alpha, \quad (5)$$

where N_α is the total number of particles of sort α per unit length of the plasma column; r_k , r_j , v_k , v_j denote the x - and y -components of the transverse radius-vector \mathbf{r} and velocity \mathbf{v} , respectively; and moments $\langle v_k \rangle_\alpha$, $\langle r_k v_j \rangle_\alpha$, $\langle v_k v_j \rangle_\alpha$ are defined in analogy with (4), (5).

The set of equations for the moments of the first order is equivalent to the equations of motion of one particle in the given electric and magnetic fields:

$$\frac{d}{dt} \langle x \rangle_\alpha = \langle v_x \rangle_\alpha, \quad \frac{d}{dt} \langle y \rangle_\alpha = \langle v_y \rangle_\alpha, \quad \omega_\alpha \equiv \frac{Z_\alpha e B_0}{m_\alpha c}, \quad \mu \equiv \frac{Z m_e}{m_i}, \quad (6)$$

$$\frac{d}{dt} \langle v_x \rangle_\alpha - \omega_\alpha \langle v_y \rangle_\alpha = -\frac{Z_\alpha e}{m_\alpha} \left\langle \frac{\partial \varphi}{\partial x} \right\rangle_\alpha + \frac{Z_\alpha e}{m_\alpha} E_0 \cos(\omega_0 t), \quad \omega_e = -\omega_0, \quad (7)$$

$$\frac{d}{dt} \langle v_y \rangle_\alpha + \omega_\alpha \langle v_x \rangle_\alpha = -\frac{Z_\alpha e}{m_\alpha} \left\langle \frac{\partial \varphi}{\partial y} \right\rangle_\alpha + \frac{Z_\alpha e}{m_\alpha} E_0 \sin(\omega_0 t), \quad \omega_i = \mu \omega_0. \quad (8)$$

Based on the results obtained in [2] for the case $E_0 = 0$ we shall search for solution of basic equations (2) – (3) with:

$$e\varphi(\mathbf{r}, t) = -\frac{m_e}{2} \nu^2(t) r^2, \quad \nu(t) \ll \omega_0, \quad |d\nu/dt| \ll \omega_0^2. \quad (9)$$

Here ν is the frequency of electron's oscillations in the charge separation electric field which is considered to be much smaller than the electron cyclotron frequency, ω_0 . In the frames of the model (9) the general solution of the Eqs. (6) – (8) can be presented via linear combination of free oscillations of the moments $\langle r_k \rangle_\alpha$, $\langle v_k \rangle_\alpha$ and constrained oscillations on the frequency ω_0 . For the case of interest ($\nu \ll \omega_0$, $\nu^2 < \mu \omega_0^2$, $\mu \ll 1$) assuming that the field \mathbf{E}_{ext} has been switched on adiabatically, one can keep only forced oscillation in the solution and supposes for the ions $\langle r_k \rangle_i \approx 0$, $\langle v_k \rangle_i \approx 0$. For the electrons the approximate solution is presented in a form:

$$\langle x \rangle_e = A_1 \cos(\omega_0 t) - A_2 \sin(\omega_0 t), \quad \langle y \rangle_e = A_1 \sin(\omega_0 t) + A_2 \cos(\omega_0 t), \quad (10)$$

$$\omega_0 \frac{d}{dt} A_1 + \nu^2 A_2 = 0, \quad \omega_0 \frac{d}{dt} A_2 - \nu^2 A_1 = \omega_0^2 a, \quad a \equiv \frac{cE_0}{\omega_0 B_0} \quad (11)$$

where A_1, A_2 change insignificantly on the time interval $T \sim 1/\omega_0$. In frames of the described approach the equations for the moments of the second order $\langle r^2 \rangle, \langle xv_x + yv_y \rangle_\alpha, \langle xv_y - yv_x \rangle_\alpha$ and $\langle v_x^2 + v_y^2 \rangle_\alpha \equiv \langle v^2 \rangle_\alpha$ include only slow functions of time. Therefore, in the case of interest (two-component symmetrical plasma column) the moments $\langle r^2 \rangle_\alpha$ are equal for different sorts of particles due to the quasi-neutral approximation (3). The spatial structure of a plasma column, respectively, can be described by one scalar function of time $l(t) \equiv \sqrt{\langle r^2 \rangle}$ which satisfy the following equations:

$$\frac{1}{2} \frac{d^2}{dt^2} l^2 = \sum_\alpha \mu_\alpha \left[\langle v^2 \rangle_\alpha + \omega_\alpha \langle xv_y - yv_x \rangle_\alpha \right] - \frac{\mu}{1 + \mu} \omega_0^2 a A_1(t) = \quad (12)$$

$$\langle v^2 \rangle_e - \omega_0 \langle xv_y - yv_x \rangle_e - \nu^2(t) l^2 - \omega_0^2 a A_1(t), \quad \mu_\alpha \equiv \frac{m_\alpha N_\alpha}{M},$$

$$\frac{d}{dt} j_e = \omega_0^2 a A_2(t), \quad \frac{d}{dt} j_i = 0, \quad j_\alpha \equiv \langle xv_y - yv_x \rangle_\alpha + \frac{\omega_\alpha}{2} l^2, \quad (13)$$

$$\frac{d}{dt} \langle v^2 \rangle_\alpha = -\nu^2(t) \frac{d}{dt} l^2 + 2\omega_0 a (\omega_0^2 + \nu^2) A_2(t), \quad \frac{d}{dt} \langle v^2 \rangle_i = \mu \nu^2 \frac{d}{dt} l^2 \quad (14)$$

where $M \equiv \sum_\alpha m_\alpha N_\alpha$ is the total mass per unit length of the plasma column, and the function $A_{1,2}(t)$ satisfies (11).

Without the external electric field (i.e. $E_0 = 0$) the Eqs.(12) – (14) describe the harmonic oscillations of the plasma column cross-section at the low-hybrid frequency, $\Omega \equiv \sqrt{\mu} \omega_0$ ($\mu \equiv Zm_e/m_i$) [2]:

$$\frac{d^2}{dt^2} l^2 + \Omega^2 l^2 = 2 \sum_\alpha \mu_\alpha \left[\langle v^2 \rangle_\alpha + \omega_\alpha j_\alpha \right] = const, \quad l^4 \left[\nu^2(t) + \frac{1 - \mu}{4} \omega_0^2 \right] = const. \quad (15)$$

However there is a particular stationary solution: $l = l_{st}$ with $\nu = 0$.

In the presence of ECRH ($\mathbf{E}_{ext} \neq 0$) starting with the conditions $l^2(0) \equiv l_0^2 = l_{st}^2$ and $\nu(0) = 0$ the parameters of the plasma column – $l^2, \langle v^2 \rangle_\alpha, j_e, \nu$ – became slow functions of time. In this case the quasi-stationary dynamics of the column is described by the relationships which follow from the set (12) – (14):

$$\frac{d^2}{dt^2} l^2 + \left(\frac{\mu\omega_0}{4}\right)^2 \left(1 - \frac{l_0^2}{l^2}\right)^3 [l^2 + l_0^2] = \frac{2}{1+\mu} \omega_0^2 a^2, \quad (16)$$

$$\left[\frac{\mu\omega_0^2}{4} - \nu^2\right] l^4 = const, \quad \langle v^2 \rangle_e - (1+\mu)\omega_0^2 l^2 = const, \quad (17)$$

Note, that the Eq. (16) corresponds to nonlinear oscillations of the plasma cross-section $l^2(t)$ between the initial value l_0^2 and the maximum l_{max}^2 . Respectively the Eq. (17) results in limitation of the electron energy increase even within nonrelativistic consideration. Specifically, in the case of not very strong heating, i.e. $E_0/B_0 \ll \mu\omega_0 l_0/c$ one can obtain:

$$\left(\frac{l_{max}}{l_0}\right)^2 - 1 \simeq \frac{1}{(1+\mu)^{1/3}} \left(\frac{8a}{\mu l_0}\right)^{2/3} \ll 1, \quad (18)$$

$$(\Delta\langle v^2 \rangle_e)_{max} \equiv \langle v^2 \rangle_e|_{l=l_{max}} - \langle v^2 \rangle_e|_{l=l_0} \approx \left(\frac{8(1+\mu)}{\mu} \frac{E_0}{B_0}\right)^{2/3} \left(\frac{\omega_0 l_0}{c}\right)^{4/3} c^2 \quad (19)$$

For $E_0/B_0 \gg \mu\omega_0 l_0/c$:

$$l_{max}^2 \approx \frac{1}{1+\mu} \left(\frac{8c}{\mu\omega_0} \frac{E_0}{B_0}\right)^2; \quad (\Delta\langle v^2 \rangle_e)_{max} \approx \left(\frac{8}{\mu} \frac{E_0}{B_0}\right)^2 c^2 \quad (20)$$

The relativistic limitation for the electron energy in the case $E_0 \ll B_0$ can be written as

$$(\Delta\langle v^2 \rangle_e)_{max} \approx \left(\frac{4E_0}{B_0}\right)^{2/3} c^2 \quad (21)$$

Therefore we can state that the ambipolar effect in ECR heating can be considerable if initial cross section of the plasma column is small enough, i.e. $(\omega_0 l_0/c)^2 \ll \mu$.

In the later case the limitation of electron energy is given by

- 1) Eq. (19) when $\frac{E_0}{B_0} \ll \frac{\mu\omega_0 l_0}{c} \ll \mu^{3/2}$;
- 2) Eq. (20) when $\frac{\mu\omega_0 l_0}{c} \ll \frac{E_0}{B_0} \ll \mu^{3/2}$;
- 3) Eq. (21) when $\frac{E_0}{B_0} \gg \mu^{3/2}$.

References

- [1] D.S.Dorozhkina and V.E.Semenov, JETP **89** (3), 468 (1999).
- [2] D.S.Dorozhkina and V.E.Semenov, Phys.Rev.E **61** (3), 3058 (2000).