27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** (2000) 1505 -1508

Analytic expression for neoclassical transport coefficients including finite banana-width effect near the magnetic axis

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It is well known that a particle orbit near the magnetic axis is quite different from the conventional banana orbit and a thin-banana approximation in the standard neoclassical transport theory is broken down. The revision of the standard theory around the magnetic axis has been done by many authors, and several expressions for the transport coefficients at the magnetic axis are presented [1-3]. In this paper, we present a simple interpolation formula between the neoclassical transport coefficients at the magnetic axis and the conventional ones for an axisymmetric toroidal plasma. Using this formula, we also discuss the possibility of a tokamak sustained only by the bootstrap current.

Let us consider a tokamak configuration $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$, where ψ is the poloidal flux and φ is the toroidal angle around the magnetic axis. The trajectory of a charged particle in this configuration is determined by using the constants of energy, magnetic moment and canonical angular momentum. The boundary in velocity space for a trapped particle that reverses the sign of $v_{\|} = \mathbf{v} \cdot \mathbf{B}/B$ was discussed in detail by Chu [4]. A particle with the parallel velocity $v_{\|}$ at a radial position r and a poloidal angle $\theta = \pi/2$ is approximately shown to reverse the sign of parallel velocity when the conditions

$$\left|\frac{v_{\|}}{v}\right| < \xi_+ \quad \text{for } e_a v_{\|} > 0 \qquad \text{and} \qquad \left|\frac{v_{\|}}{v}\right| < |\xi_-| \quad \text{for } e_a v_{\|} < 0$$

are satisfied, where e_a is a charge for species a, and the poloidal flux around the magnetic axis is approximated as $\psi = B_0 r^2/(2q_0)$ with the safety factor q_0 and the magnitude of magnetic field B_0 at the magnetic axis. The parameters $\xi_+(>0)$ and $\xi_-(<0)$ are two real roots of the quartic equation $\xi^4 + \delta_a\xi - \epsilon^2 = 0$, where the inverse aspect ratio $\epsilon = r/R_0$ with the major radius R_0, and $\delta_a = 2q_0\rho_a/R_0$ with the Larmor radius $\rho_a = v/|\Omega_a|$. We rewrite the trapping condition in terms of a variable $\lambda = (1 - v_{\|}^2/v^2)/B$ and propose the following approximate trapping condition

$$\frac{1}{B_{\min}} > \lambda > \frac{1-\xi_+^2}{B_0} \equiv \lambda_c^+ \quad \text{for} \quad e_a v_{\|} > 0,$$

$$\frac{1}{B_{\min}} > \lambda > \frac{1-\xi_-^2}{B_0} \equiv \lambda_c^- \quad \text{for} \quad e_a v_{\|} < 0.$$

The limiting forms for $\xi_\pm$ are given as follows: $\xi_\pm \to \pm\sqrt{\epsilon}$ when $\sqrt{\epsilon} >> \delta_a^{1/3}$, and $\xi_+ \to 0$ and $\xi_- \to -\delta_a^{1/3}$ when $\sqrt{\epsilon} << \delta_a^{1/3}$. Therefore, the trapping condition reduces to the conventional condition $1/B_{\min} \geq \lambda \geq 1/B_{\max}$ in the region where $\sqrt{\epsilon} >> \delta_a^{1/3}$

is satisfied. In the region $\sqrt{\epsilon} << \delta_a^{1/3}$, it reduces to the trapping condition at the magnetic axis [2].

Recently, Wang [2] proposed a simple theory for estimating the parallel current at the magnetic axis. In his theory, the parallel current is obtained from the conventional theory only by replacing the conventional trapping condition with the trapping condition at the magnetic axis. Following this idea, we take into account the finite banana-width effect near the magnetic axis by replacing the trapping boundary parameter λ_c with λ_c^+ for $e_a v_{||} > 0$ and with λ_c^- for $e_a v_{||} < 0$. Then, the bootstrap current $J_{\rm B}$ and Ohmic current $J_{\rm E}$ can be written as

$$J_{\rm B} = -\frac{IBn_eT_e}{< B^2 >}\left[L_1^{(\rm B)}\left(\frac{p_e'}{p_e} + \frac{1}{Z}\frac{T_i}{T_e}\frac{p_i'}{p_i}\right) + L_2^{(\rm B)}\frac{T_e'}{T_e} + L_3^{(\rm B)}\frac{T_i'}{ZT_e}\right], \quad (1)$$

$$J_{\rm E} = \frac{e^2 n_e \tau_{ee}}{m_e} L^{(\rm E)} \frac{< BE_{||} > B}{< B^2 >}, \quad (2)$$

where $\tau_{aa} = 3\sqrt{\pi}/(4\nu_{aa})$ with the collision frequency ν_{aa}, Z is the ion charge number, and $< \cdot >$ denotes a flux-surface average. The transport coefficients $L_k^{(\rm B)}$ and $L^{(\rm E)}$ are given by

$$\begin{bmatrix} L_1^{(\rm B)} \\ L_2^{(\rm B)} \\ L^{(\rm E)} \end{bmatrix} = \frac{1}{D_e}\begin{bmatrix} \mu_{e1}^\star(\mu_{e3}^\star + l_3) - \mu_{e2}^\star(\mu_{e2}^\star - l_2) \\ l_2\mu_{e3}^\star + l_3\mu_{e2}^\star \\ \mu_{e3}^\star + l_3 \end{bmatrix}$$

and $L_3^{(\rm B)} = (\sqrt{2}\mu_{i2}^\star/D_i)L_1^{\rm B}$, where $l_1 = Z$, $l_2 = 3Z/2$, $l_3 = \sqrt{2} + 13Z/4$, $D_e = (\mu_{e1}^\star + l_1)(\mu_{e3}^\star + l_3) - (\mu_{e2}^\star - l_2)^2$ and $D_i = \mu_{i1}^\star(\mu_{i3}^\star + \sqrt{2}) - \mu_{i2}^{\star 2}$. The viscosity coefficients $\mu_{ek}^\star$ are defined by

$$\begin{bmatrix} \mu_{a1}^\star \\ \mu_{a2}^\star \\ \mu_{a3}^\star \end{bmatrix} = \frac{\tau_{aa}}{n_a}\frac{8\pi}{3}\int_0^\infty dv \frac{v^4}{v_a^2}\nu_a^D(v)\frac{f_t^\star}{f_c^\star}f_{a0}\begin{bmatrix} 1 \\ v^2/v_a^2 - 5/2 \\ (v^2/v_a^2 - 5/2)^2 \end{bmatrix}.$$

Here $f_t^\star = 1 - f_c^\star$, $v_a = \sqrt{2T_a/m_a}$ is the thermal velocity, $\nu_a^D(v)$ is the deflection collision frequency, and

$$f_c^\star = \frac{3}{8} < B^2 > \left[\int_0^{\lambda_c^-}\frac{\lambda d\lambda}{< \sqrt{1-\lambda B} >} + \int_0^{\lambda_c^+}\frac{\lambda d\lambda}{< \sqrt{1-\lambda B} >}\right],$$

where we use the approximation $< \sqrt{1-\lambda B} > \sim \sqrt{1-\lambda B_0}$ for $\lambda > \lambda_c$. The viscosity coefficients $\mu_{ak}^\star$ reduce to the conventional ones μ_{ak} for $\sqrt{\epsilon} >> \delta_a^{1/3}$, and these coefficients at the magnetic axis become

$$\begin{bmatrix} \mu_{a1}^\star \\ \mu_{a2}^\star \\ \mu_{a3}^\star \end{bmatrix} = \delta_a^{1/3}\frac{2\pi\tau_{aa}}{n_a}\int_0^\infty dv \left(\frac{v}{v_a}\right)^{1/3}\frac{v^4}{v_a^2}\nu_a^D(v)f_{a0}\begin{bmatrix} 1 \\ v_a^2 - 5/2 \\ (v^2/v_a^2 - 5/2)^2 \end{bmatrix}$$

for $\delta_a^{1/3} << 1$, where we have redefined the parameter δ_a by using the normalized Larmor radius $\rho_a = v_a/|\Omega_a|$ instead of $v/|\Omega_a|$, and we will use this redefined δ_a from now. We note here that the effect of finite banana-width on the transport coefficients for the bootstrap current and the Ohmic current comes through the viscosity coefficients $\mu_{ak}^{\star}$. The transport coefficient $L_1^{(\mathrm{B})}$ for the bootstrap current is plotted as a function of ϵ for $\delta_e = 0.001$ in Fig.1. Comparing with the conventional transport coefficient plotted in dotted curve, we find that the transport coefficient $L_1^{(\mathrm{B})}$ is modified very near the magnetic axis and it remains finite on the axis. At the magnetic axis, our result agrees with the Wang's one when we approximate the collision operator by the pitch-angle scattering term.

Similar to the parallel current, we can estimate the effect of finite banana-width near the magnetic axis on the radial particle and heat fluxes by replacing the conventional viscosity coefficients μ_{ak} with newly defined ones $\mu_{ak}^{\star}$. For instance, the radial ion heat flux q_i is written as

$$\frac{q_i}{T_i} = \left(\frac{Ic}{e_i}\right)^2 \frac{n_i m_i}{\tau_{ii}} \frac{L_i}{< B^2 >} T_i', \qquad L_i = \frac{\sqrt{2}}{D_i}\left(\mu_{i2}^{\star 2} - \mu_{i1}^{\star}\mu_{i3}^{\star}\right),$$

where we have used the definition of heat flux averaged over not only a flux surface but also a typical orbit width [1]. The transport coefficient L_i is shown as a function of the inverse aspect ratio for $\delta_i = 0.05$ in Fig. 2. The dotted curve represents the conventional transport coefficient.

Finally, we discuss the impact of finite banana-width effect on the plasma equilibrium. The concept of a tokamak sustained only by the neoclassical bootstrap current is quite attractive. However, this completely bootstrapped tokamak is considered to be impossible since the bootstrap current vanishes at the magnetic axis and thus a seed current near the axis must be generated by an external source. The bootstrap current obtained by taking into account the finite banana-width effect remains finite at the magnetic axis. This result indicates that the completely bootstrapped tokamak may be possible.

In a straight cylindrical approximation, the poloidal magnetic field B_p is calculated by the Ampére's law

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rB_p\right) = \mu_0\left(J_B + J_E\right),$$

where μ_0 is the permeability of vacuum. Assuming the radial profiles for the pressure and temperature as

$$p_e = p_0\left(1 - \frac{r^2}{a^2}\right)^{\nu_{pe}} \qquad \text{and} \qquad T_e = T_i = T_0\left(1 - \frac{r^2}{a^2}\right)^{\nu_{Te}},$$

we solve the Ampére's law with the use of (1) and (2). The resulting ratio of the Ohmic current to the total current is plotted as a function of the poloidal beta $\beta_p \equiv \int(p_e+p_i)dV/(VB_{pa}^2/2\mu_0)$ in Fig. 3. This figure shows that the Ohmic current decreases with increasing β_p and finally it vanishes at the critical poloidal beta $\beta_{pc} = 1.86$ for $\nu_{pe} = 2$, $\nu_{Te} = 1$, and $\beta_{pc} = 1.75$ for $\nu_{pe} = 3$, $\nu_{Te} = 1$. The current profiles for the completely bootstrapped tokamak at the critical poloidal beta are depicted in Fig. 4.

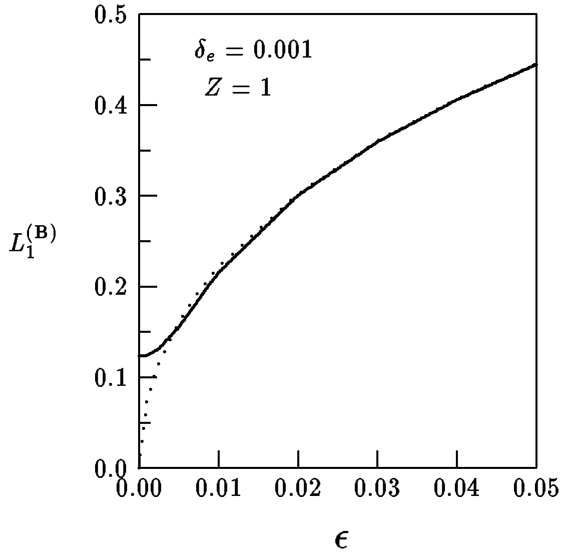

Fig. 1: *The transport coefficient* $L_1^{(\mathrm{B})}$ *for the bootstrap current versus the inverse aspect ratio* ϵ *for* $\delta_e = 0.001$ *and* $Z = 1$. *The dotted curve represents the conventional transport coefficient.*

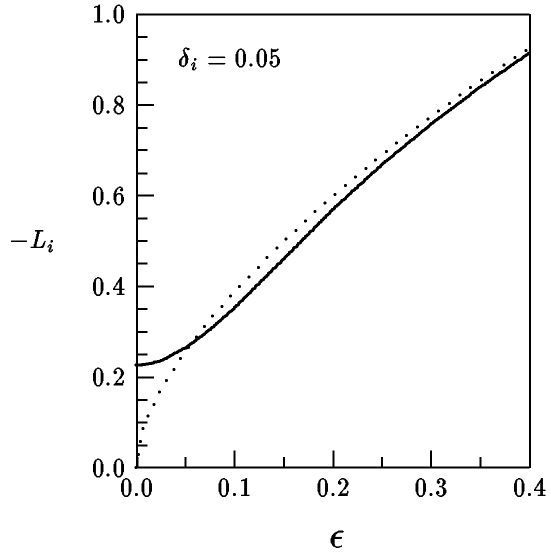

Fig. 2: *The transport coefficient* L_i *for the radial ion heat flux versus the inverse aspect ratio* ϵ *for* $\delta_i = 0.05$. *The dotted curve represents the conventional transport coefficient.*

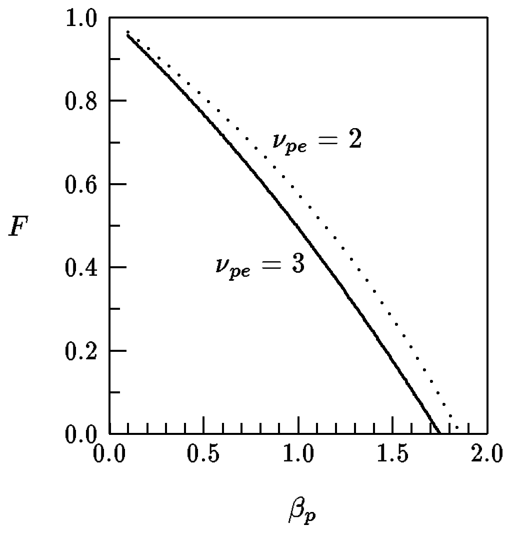

Fig. 3: *Ratio F of the Ohmic current to the bootstrap current as a function of poloidal beta* β_p *for* $\nu_{pe} = 3$ *(solid curve) and 2 (dotted curve).*

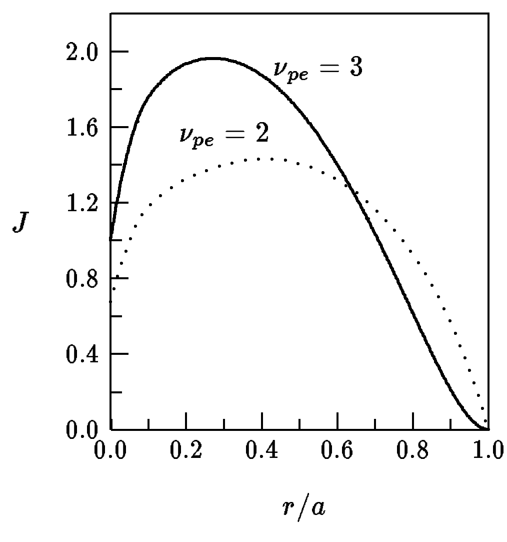

Fig. 4: *Current profiles in the completely bootstrapped tokamak for* $\nu_{pe} = 3$ *(solid curve) and 2 (dotted curve).*

This work was partially supported by a Grant-in-Aid for Scientific Research from Japan Society for the Promotion of Science.

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