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Suppression of runaway electron avalanches by radial diffusion

P. Helander[1], L.-G. Eriksson[2], and F. Andersson[3]

[1] *EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, U.K.*
[2] *Association EURATOM-CEA sur la Fusion, CEA Cadarache, France*
[3] *Dept of Electromagnetics, Chalmers University of Technology, Göteborg, Sweden*

Abstract
The kinetic theory of runaway electron avalanches caused by close Coulomb collisions is extended to account for radial diffusion. This is found to slow down the growth of avalanches. An approximate analytical formula for the growth rate is derived and is verified by a Monte Carlo code. As the poloidal magnetic flux that is available to induce an electric field in a tokamak is limited, avalanches can be prevented by sufficiently strong radial diffusion. The requisite magnetic fluctuation level is sensitive to the mode structure. It is estimated to be $\delta B/B \sim 10^{-3}$ for parameters typical of large tokamaks.

Introduction

The generation of runaway electrons in disruptions can be a severe problem since their loss to the first wall may cause localized surface damage. Magnetic flucutations may be helpful to avoid runaway production since they increase the loss rate of fast electrons [1]. The present paper investigates to what extent the presence of radial diffusion caused by magnetic fluctuations can prevent the production of runaways in large tokamaks where so-called secondary runaway generation occurs. In plasmas with large current, close collisions between thermal electrons and existing runaways can lead to exponential multiplication of the latter – a runaway avalanche [2-4]. The point is that a single such collision can kick the thermal electron above the critical energy for runaway acceleration by the electric field. The total number of *e*-foldings caused by this mechanism in a disruption can be estimated as [4]

$$\int \gamma_r dt \sim I/I_A \ln\Lambda, \tag{1}$$

where I is the plasma current and $I_A = 4\pi m_e c/\mu_0 e = 0.017$ MA is the Alfvén current. Consequently, in small tokamaks this mechanism is not effective, but becomes the dominant mechanism in a next-step device. Mathematically, secondary runaway production is governed by the kinetic equation

$$\frac{\partial f}{\partial t} - \frac{eE_{\|}\xi}{m_e c}\left(\frac{\partial f}{\partial p} - \frac{2\lambda}{p}\frac{\partial f}{\partial \lambda}\right) = C(f) + S, \tag{2}$$

for the electron distribution f [4]. The velocity-space coordinates are the relativistic momentum p, which has been normalized to $m_e c$ (with m_e the rest mass), and $\lambda = p_\perp^2/p^2 B = (1-\xi^2)/B$, where $\xi = p_{\|}/p$ and B is the magnetic field. Coulomb collisions are described by the right-hand side of the equation, where

$$C(f) = \frac{1}{\tau p^2}\left[\frac{\partial}{\partial p}\left(1+p^2\right) f + \sqrt{1+p^{-2}}\,\frac{1+Z_{\rm eff}}{2}\frac{\partial}{\partial \xi}\left(1-\xi^2\right)\frac{\partial f}{\partial \xi}\right]$$

is the Fokker-Planck collision operator and $\tau = 4\pi\varepsilon_0^2 m_e^2 c^3/n_e e^4 \ln\Lambda$ the collision time for relativistic electrons. $Z_{\rm eff}$ is the effective ion charge. Relativistic effects associated with the thermal population have been neglected but are retained for the fast electrons. S is a source term of fast electrons produced by close collisions between existing runaways and slow electrons. Its form is given by Rosenbluth and Putvinskii [4], who solved Eq (2) analytically in several limits and constructed an interpolation formula for the runaway production rate

$$\gamma_r = \frac{d\ln n_r}{dt} \simeq \frac{E-1}{\tau\ln\Lambda}\sqrt{\frac{\pi\varphi}{3(Z_{\rm eff}+5)}}\left(1-\frac{1}{E}+\frac{4\pi(Z_{\rm eff}+1)^2}{3\varphi(Z_{\rm eff}+5)(E^2+4/\varphi^2-1)}\right)^{-1/2}. \tag{3}$$

Here $E = \left|E_{\|}\right|/E_c$, $\varphi = 1-1.46\varepsilon^{1/2}+1.72\varepsilon$ describes the effects of finite toroidicity, $\varepsilon = r/R$ is the inverse aspect ratio, and $E_c = m_e c/e\tau$ is the critical electric field below which no runaway is possible [5]. Thus, we have approximately $\gamma_r\tau \sim (E-1)/2\ln\Lambda$. The estimate (1) is obtained by integrating this growth rate over time, assuming $E_{\|} \gg E_c$, using $E_{\|} \simeq L(dI/dt)/(2\pi R)$, and approximating the plasma inductance by $L \simeq \mu_0 R$.

Radial diffusion

The analysis reviewed above assumes that there is no loss of runaway electrons. In practice, however, these particles undergo radial transport due to magnetic fluctuations. Here, we calculate the reduction of the avalanche growth rate caused by radial diffusion described by the addition of a term

$$\frac{1}{r}\frac{\partial}{\partial r} rD\frac{\partial f}{\partial r} \tag{4}$$

on the right-hand side of (2). The diffusion coefficient D of fast electrons is thought to be governed by magnetic turbulence and to depend sensitively on the turbulence mode structure and the electron energy. The confinement of runaways improves significantly as they are accelerated by the electric field, so that $D(p)$ decreases with increasing p.

An approximate analytical solution to (2) with the addition of (4) can be found by noting that there is a separation of time scales in the problem. The acceleration to relativistic speed of an electron above the runaway threshold occurs on a time scale $\tau_{acc} = \tau/E$, but the time scale for avalanche growth is much longer, $\gamma_r^{-1} \sim (2\ln\Lambda)\tau_{acc}$. The momentum space can thus be divided into a low-energy region ($p < p_*$) where the avalanche mechanism operates undisturbed, and a high-energy region ($p > p_*$) where the accelerating runaways undergo radial diffusion. The flux of runaways from the former to the latter region is given by (3). Integrating (2) in the latter region over $p_\perp$ gives for a beam-like distribution ($p \simeq p_{\|}$)

$$\tau\frac{\partial F}{\partial t} + (E-1)\frac{\partial F}{\partial p} = \frac{\tau}{r}\frac{\partial}{\partial r} rD(p)\frac{\partial F}{\partial r}, \tag{5}$$

where $F = \int f d^2 p_\perp$. The high-energy population governed by (5) is fed from below (in p) by the avalanche mechanism, which operates in the region $p < p_*$ according to (3) and thus provides a boundary condition on $F(p_*)$,

$$(E-1)F(p_*,r,t) = \gamma_r\tau n_r = \gamma_r\tau\int_{p_*}^{\infty} F(p)dp. \tag{6}$$

The fastest growing solution to Eqs (5) and (6) is

$$F(p,r,t) = J_0(kr)\exp\left[\gamma t - \frac{\tau}{E-1}\int_{p_*}^{p}\left(\gamma + k^2 D(p')\right)dp'\right], \tag{7}$$

where $k = 2.4/a$ and the growth rate γ is determined by the equation

$$E-1 = \gamma_r\tau\int_{p_*}^{\infty} dp\, \exp\left[-\frac{\tau}{E-1}\int_{p_*}^{p}\left(\gamma + k^2 D(p')\right)dp'\right]. \tag{8}$$

Normally in a disruption $E \gg 1$, so that the runaway threshold is very low, $p_* \ll 1$. We can then take $p_* = 0$ in (8), which determines the growth rate γ if the runaway diffusion coefficient $D(p)$ is known. If the latter is independent of p, we simply obtain

$$\gamma = \gamma_r - k^2 D. \tag{9}$$

As already remarked, in practice $D(p)$ decreases with increasing p; in fact, the integral

$$\int_0^\infty D(p)\, dp \tag{10}$$

usually converges fairly rapidly. In this case, (8) can be simplified by noting that $\gamma\tau/(E-1) \lesssim 1/2 \ln\Lambda \ll 1$. The integration of the term involving $D(p')$ can then be extended to infinity, which gives the growth rate

$$\gamma = \gamma_r \exp\left(-\frac{k^2\tau}{E-1} \int_0^\infty D(p)\, dp \right). \tag{11}$$

Monte Carlo simulation

In order to verify these ideas and to simulate the avalanche process, we have constructed a Monte Carlo code, ARENA (Avalanche of Runaway Electrons, Numerical Analysis code). This code solves the orbit-average of (2) along the lines described in Ref [4], but with the additional diffusion term (4). The Monte Carlo solution of this equation involves following a large number of Monte Carlo particles in phase space (r, p, λ); the phase space positions of these particles are changed periodically, at time intervals Δt, by applying Monte Carlo operators representing collisions, the electric field, and the diffusion term (4). The ARENA code has been benchmarked (without diffusion) against the Rosenbluth-Putvinski formula (3), which has been found to be accurate in all physically relevant cases. Adding radial diffusion, we have used the code to verify the approximate analysis given above. If D is taken to be independent of p, the growth rate is found to drop linearly with increasing D, in practically exact agreement with (9), see Fig 1. In the more realistic case where D does depend on momentum, we have verified (11) by running the code with a radial diffusion coefficient given by $D(p) = D_0 e^{-(p/\Delta p)^2}$, for various values of the width Δp. The runaways were initialized with the density profile $n_r(r) = n_{r0}[1 - 0.9(r/a)^2]^{0.1}$, and $E = 100$. Fig 2 shows the growth rate obtained from the simulations and the one from (11) as functions of Δp. Their agreement indicates that (11) is an accurate estimate of the runaway growth rate caused by the diffusion. If Δp becomes very large the agreement is worse since the integral (10) then converges slowly. In such cases the more accurate expression (8) should be used to calculate the growth rate.

Conclusions

Our results demonstrate that radial diffusion slows down runaway avalanches, so that the growth rate γ_r calculated in Ref 4 is reduced according to (8). If the integral (10) converges quickly enough, this equation is reduced to the more explicit Eq (11). At first sight, this expression suggests that radial diffusion cannot prevent avalanches, since the growth rate (11) cannot become negative. (The reason for this is that the confinement time of the fast electrons approaches infinity as they are accelerated.) However, in practice it is sufficient to reduce the growth rate to a level where there are not enough volt-seconds of induced electric field to achieve significant avalanche growth. To estimate the magnetic fluctuation level δB required

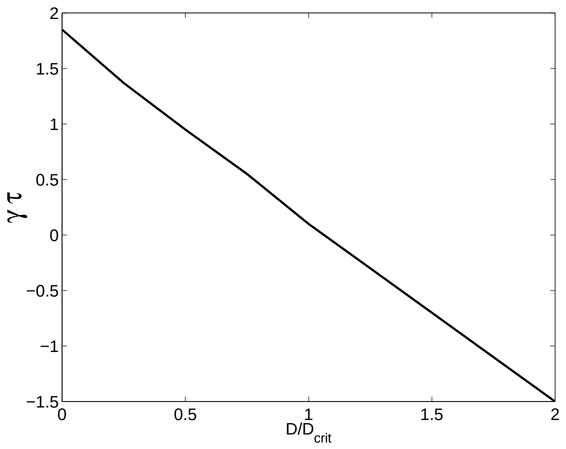

Figure 1: *The normalized avalanche growth rate* $\gamma\tau$ *vs the diffusion coefficient in the case when the latter is assumed to be independent of momentum. As predicted by Eq (9),* γ *decreases linearly with* D *and vanishes when* $D = D_{\rm crit} = \gamma_r (a/2.4)^2$.

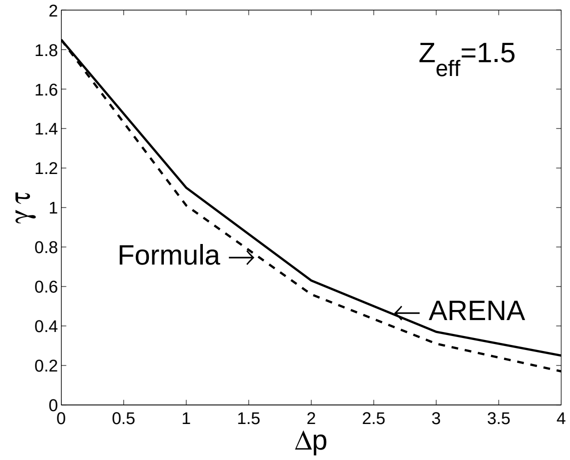

Figure 2: *The normalized growth rate* $\gamma\tau$ *vs* Δp *for the diffusion coefficient* $D(p) = D_0 e^{-(p/\Delta p)^2}$, *with* $D_0 = R^2/\tau$, $a/R = 0.31$, *and* $Z_{\rm eff} = 1.5$. *The solid line shows the result from the numerical simulation, and the dotted line the analytical approximation (11).*

for this, we use the approximate expression (1) modified by the reduction factor from Eq (11),

$$\int \gamma dt \sim \frac{I}{I_A \ln \Lambda} \exp\left(-\frac{k^2\tau}{E-1}\int_0^\infty D(p)\, dp\right). \tag{12}$$

We assume that $D(p)$ is equal to the Rechester-Rosenbluth value $D_{RR} = \pi q v_{\|} R (\delta B/B)^2$ for small p and falls off at higher energies when the orbit width becomes comparable to the mode width of the magnetic turbulence. This determines the value of the integral (10), which we write as $\int D(p)\, dp = p_{\rm crit} \pi q c R (\delta B/B)^2$, where $p_{\rm crit}$ characterizes the speed of convergence. Requiring that the number of e-foldings (12) should not be larger than some number N defining the maximum tolerable avalanche size implies the following lower limit on the fluctuation level,

$$\frac{\delta B}{B} \gtrsim \frac{a}{2.4}\sqrt{\frac{E-1}{\pi q c R \tau p_{\rm crit}} \ln\left(\frac{I}{N I_A \ln \Lambda}\right)}.$$

This limit is practically independent of the argument within the logarithm, and becomes for typical large-tokamak parameters about 10^{-3}. This greatly exceeds the fluctuation level in quiescent plasmas but is not unrealistic in a disruption. Thus, it appears possible that the naturally occurring, or any externally induced, magnetic fluctuations could significantly reduce the size of secondary runaway avalanches.

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