Free boundary high-beta equilibria with high bootstrap current fraction in Spherical Tokamaks

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1 Introduction. ST power plant studies set a number of requirements on plasma equilibrium and stability properties [1]: high pressure driven equilibrium current fraction, normalized beta $\beta_N \sim 8$ stable against ballooning modes, ratio of plasma current to toroidal field current $I_p/I_{rod} \gtrsim 1$ and restrictions to the plasma triangularity to allow flaring of the centre column. There have been several studies devoted to the plasma equilibrium and stability optimization using fixed boundary equilibria [2, 3, 4] and first examples of the free boundary equilibrium computations were presented in [5].

In the present paper further spherical tokamak relevant equilibrium and stability studies are presented for nearly 100% bootstrap driven ballooning stable equilibria. The influence of the plasma shaping on external mode stability is investigated. The free boundary equilibrium code CAXE-F with optional bootstrap current prescription and the stability code KINX [6] are employed.

2 Free boundary bootstrap driven ST equilibria. The CAXE-F equilibrium code computes poloidal field coil (PFC) currents needed to maintain the equilibrium plasma boundary close to the prescribed set of control points. In the present version of the code the equilibrium profiles can be specified in several ways including using the bootstrap current for $< \mathbf{j} \cdot \mathbf{B} >$ and a prescribed p'. Exact limiter points and fixed PFC currents can also be specified. The bootstrap current is computed by the formulas given by Hirshman [7] for low collisionality plasmas (in the banana regime), specialized to the case of a two component plasma with equal temperature $T_e = T_i$.

The pressure profile was specified in the same form as in [2]: $p' = p_0(1 + 39\psi^3 - 40\psi^4)$, where ψ is normalized poloidal flux. The requirement of nearly 100% bootstrap current was imposed by $< \mathbf{j} \cdot \mathbf{B} > / < \mathbf{B} \cdot \nabla\phi > = < \mathbf{j} \cdot \mathbf{B} >_{bs} / < \mathbf{B} \cdot \nabla\phi >$. To make the $< \mathbf{j} \cdot \mathbf{B} > / < \mathbf{B} \cdot \nabla\phi >$ profile smooth near the magnetic axis the total parallel current profile was extrapolated by a parabola in $s = \sqrt{\psi}$ in the segment $s \in [0, 0.25]$ keeping the profile and its derivative in ψ continuous. The specification of the I_p/I_{rod} ratio is equivalent to the prescription of the normalized current $I_N = I[MA]/(a[m]B[T])$ with vacuum magnetic field at the plasma center: $I_N = 5AI_p/I_{rod}$, where A is plasma aspect ratio. The higher the value of I_N in a 100% bootstrap driven equilibrium the higher the resulting normalized beta β_N.

In [5] ballooning stable free boundary equilibria with $I_{bs}/I_p = 88\%$ were obtained. In these computations only three pairs of PFC were used and the lack of plasma shape control prevented the second stability access near the plasma edge if no driven current was assumed there.

According to [3] a moderate plasma cross section squareness at the outboard side is favorable for the second stability access. To control the plasma shape in the equilibrium computations control points at the analytically prescribed contour were chosen:

$$r = R + a\cos(\theta + \delta\sin\theta - \zeta\sin 2\theta),$$
$$z = Z + a\kappa\sin\theta,$$
$$\zeta = 0.5\left[\zeta_o + \zeta_i + \cos\theta(\zeta_o - \zeta_i)\right], 0 < \theta < 2\pi,$$

with $R = 3.416$, $a = 2.44$, $A = 1.4$, $\kappa = 3.0$, $\delta = 0.45$, $\zeta_o = 0.1$, $\zeta_i = -0.2$ being major radius, minor radius, aspect ratio, elongation, triangularity, outboard squareness and

inboard squareness respectively. Four pairs of the coils were used for plasma shape control (using control points and one limiter point at the inboard side in equatorial plane) and the divertor PFC pair was used for adjusting separatrix proximity to plasma boundary.

In such a way an equilibrium with $I_{bs}/I_p = 99.7\%$ of bootstrap and ballooning stable for $I_N = 7.5$ in the whole volume was computed. The equilibrium parameters are: $A = 1.4, \kappa = 3.19, \delta = 0.61, \psi_{edge}/\psi_{x-point} = 0.986, q_0 = 5.06, q_{95} = 11.62, \beta = 0.63, I_N = 7.5, \beta_N = 8.37$ (see Fig.1).

External kink stability margins of the equilibria are close to the considered before cases with high triangularity of the plasma boundary: marginal wall positions are $1.75, 1.5, 1.4$ for $n = 1, 2, 3$ modes respectively.
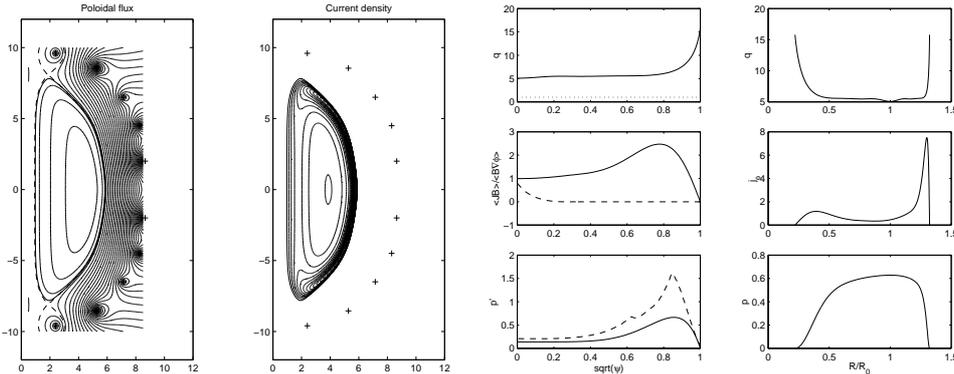

Fig.1 Moderate outboard squareness case.

When the outboard squareness value ζ_o is increased the resulting bootstrap driven equilibria have a non monotonic safety factor profile which can lead to internal global mode instability and internal/external mode coupling. An example of such equilibrium is shown in Fig.2. The equilibrium was obtain using the same procedure but with the value $\zeta_o = 0.3$ in the control contour prescription. The equilibrium parameters are: $A = 1.4, \kappa = 3.16, \delta = 0.59, \psi_{edge}/\psi_{x-point} = 0.978, q_0 = 6.05, q_{95} = 11.61, \beta = 0.62, I_N = 7.5, \beta_N = 8.28$. Due to interaction with internal modes the $n = 1$ mode is considerably more unstable for high outboard squareness equilibrium: $a_w/a = 1.25$. While the corresponding $n = 2$ value was not much changed $a_w/a = 1.42$ but an unstable $n = 3$ internal mode became localised between magnetic axis and local q profile maximum ($q = 6.48$) became unstable.
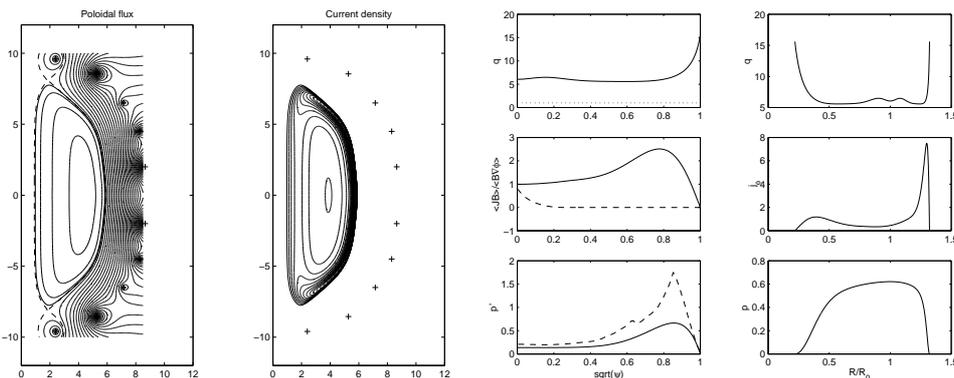

Fig.2 High outboard squareness case.

Lower plasma triangularity is beneficial for reducing the power dissipated in the flared centre column. On the other hand high triangularity is favorable for MHD stability. So an optimal configuration would be a balance between these two factors. One more PFC was added to control the inboard squareness and triangularity. Due to the negative inboard squareness of the resulting free boundary equilibrium there was no problem to maintain ballooning stability (ballooning stable $\beta_N = 8.96, I_N = 7.9$ can be reached with such plasma shape) in accordance to [3]. The equilibrium parameters are: $A =$

$1.4, \kappa = 3.12, \delta = 0.54, \psi_{edge}/\psi_{x-point} = 0.989, q_0 = 5.06, q_{95} = 9.94, \beta = 0.64, I_N = 7.5, \beta_N = 8.52,$. Global kink stability marginal a_w/a are lower : 1.25, 1.25 and 1.35 for $n = 1$, 2 and 3. However the marginal wall positions are very sensitive to the β_N value: $a_w/a(n = 1) = 1.45$ for $\beta_N = 8.3$.
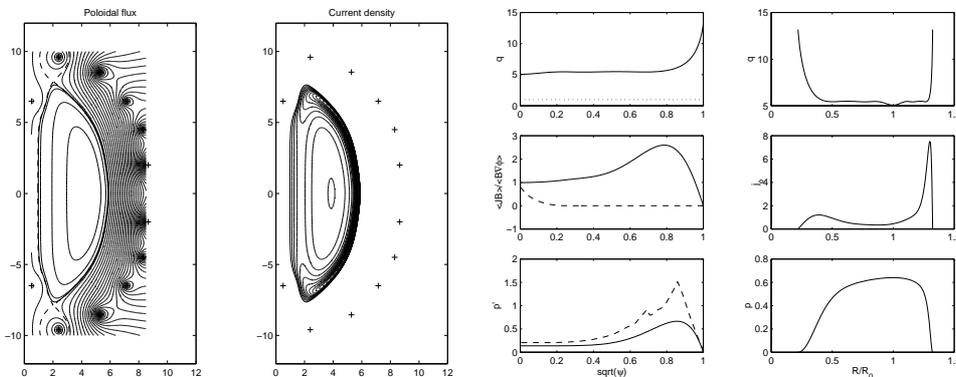

Fig.3 Lower triangularity case with inboard coils.

3 Influence of separatrix and triangularity on global mode stability.

To check the influence of the separatrix on global kink mode stability a new series of equilibria was recomputed. It is not sufficient to "scrape-off" outer magnetic surfaces and use the resulting equilibrium with new plasma boundary and non zero current density there. Its stability properties would be rather defined by the current density pedestal than the separatrix proximity.

The equilibrium series was generated as follows: the initial equilibrium (the equilibrium from [5] Fig.4 with $\psi_{edge} = 0.993\psi_{x-point}$) was "scraped-off" to define a new boundary using magnetic surface with $\psi = \psi_b$. The equilibrium was then recomputed with the fixed in normalized ψ profiles of p' and ff' (both vanishing at the boundary). Marginal conformal wall positions a_w/a for the resulting equilibrium series are given in the table below:

ψ_b/ψ_{edge}	$n = 1$	$n = 2$
1	1.79	1.61
0.995	1.74	1.60
0.99	1.58	1.58
0.98	1.59	1.56

In [5] the initial equilibrium stability margins were compared to another free boundary equilibrium with $\psi_{edge} = 0.9775\psi_{x-point}$: marginal $a_w/a = 1.34$ and 1.45 for $n = 1$ and $n = 2$ respectively. However the equilibrium from the recomputed series with $\psi_b/\psi_{edge} = 0.98$ is considerably more stable. The reason for this is higher triangularity of the equilibrium with the "scraped" boundary: $\delta = 0.602$, $A = 1.41$ ($\delta = 0.566$, $A = 1.385$ in the initial equilibrium) versus $\delta = 0.556$, $A = 1.387$. The same effect leads to a little bit more stable $n = 1$ mode for $\psi_b/\psi_{edge} = 0.98$ than for $\psi_b/\psi_{edge} = 0.99$. Another conclusion from the comparison is that the stabilizing effect of the separatrix is much less pronounced for the $n = 2$ mode than the triangularity influence.

It is also interesting to compare stability of higher n modes for the same pair of equilibria with nearly the same proximity to the separatrix but different triangularity. The respective values of the conformal wall marginal positions are given in the table below.

n	$\delta = 0.602$	$\delta = 0.556$
1	1.59	1.34
2	1.56	1.45
3	1.41	1.36
5	1.28	1.26
9	1.19	1.18

The new version of the KINX code with ballooning factor extraction [8] was used to compute external modes for the equilibria from the Section 2. High-n mode computations were performed. The convergence of the eigenvalues with the mesh size is greatly enhanced with ballooning factor extraction also for low-n modes.
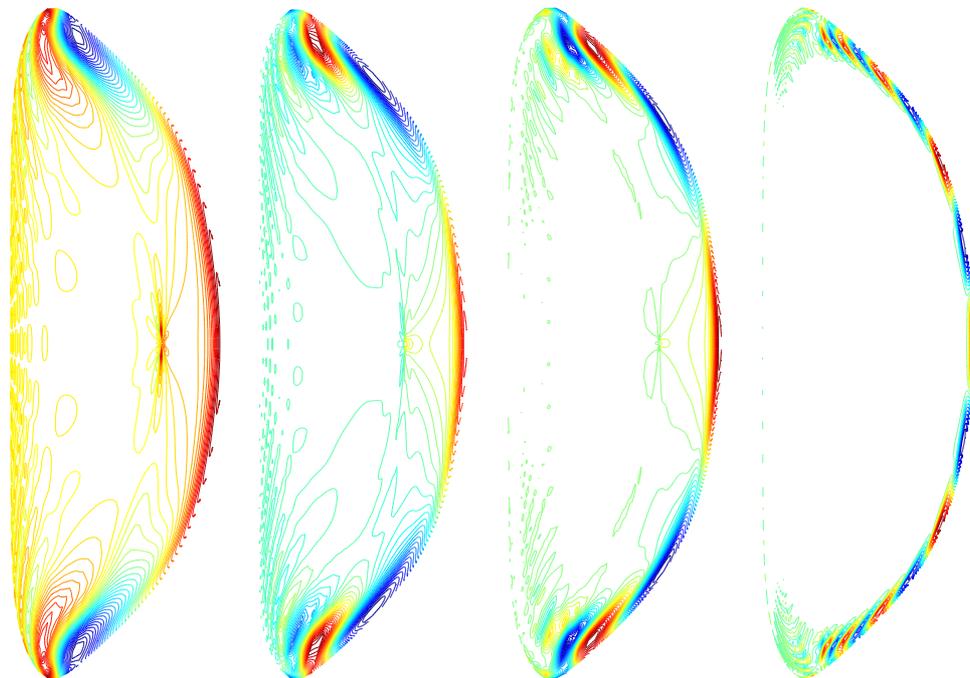

Fig.4 Normal displacement level lines for $n = 1$, $n = 2$, $n = 3$ and $n = 10$ modes (left to right), $a_w/a = 2.0$. Moderate outboard squareness case ($q_{edge} = 15.7$).

4 Conclusions.

Plasma boundary shape control via extra PFC can be used to maintain ballooning stability in bootstrap driven equilibria. Moderate outboard squareness (~ 0.1) is beneficial for ballooning stability near the plasma edge. High outboard squareness results in non monotonic safety profile in bootstrap driven equilibria and global mode stability deterioration for the pressure profiles considered.

Inboard PFC coils can control plasma triangularity/inboard squareness. Negative inboard squareness is beneficial for ballooning mode stability. External kink mode stability deteriorates for lower triangularity equilibria. However there is also a strong dependence of marginal wall positions on the value of normalized β_N.

Proximity of the separatrix to the plasma boundary and increased triangularity are stabilizing for external kink modes, though both these effects are decreasingly pronounced for higher-n modes.

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