Resonant Heating with Large Amplitude Sub-Cyclotron Waves

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Resonant heating of particles in a confining magnetic field has been examined by many authors and is of importance in the heating of magnetically confined laboratory as well as extraterrestrial plasmas[1–7]. Recently it has been shown [8] that, at sufficiently large wave amplitude, wave heating well below the cyclotron frequency is possible. The simplest problem possible is that of a particle gyrating in a constant magnetic field $\vec{B}_0 = B_0\hat{z}$ acted upon by an electrostatic plane wave propagating perpendicularly to $\vec{B}_0$.

The Hamiltonian for this system is $H = (\vec{p} - \vec{A})^2/2 + \Phi(x,t)$ with the vector potential $\vec{A} = -B_0 y\hat{x}$. Take the units of time to be given by Ω_c, the cyclotron frequency, let the electrostatic wave be given by $\Phi = \Phi_0 cos(kx - \omega t)$, and set the velocity parallel to $\vec{B}_0$, $v_z = 0$. Dimensionless parameters characterizing the heating problem are $k\rho$, with $\rho = v/\Omega_c$ the cyclotron radius, $k^2\Phi_0 = k\Delta x$, the nonlinearity parameter, giving the ratio of particle displacement caused by the wave to wave length, and ω/Ω_c. The equations of motion become $\dot{v}_x = v_y + k\Phi_0 sin(kx - \omega t)$, $\quad v_y = -x + x_0$, giving $d^2x/dt^2 + x = x_0 + k\Phi_0 sin(kx - \omega t)$. First consider this equation for $s \equiv k(x - x_0) \ll 1$. Letting $2T = kx_0 - \omega t$ and keeping only lowest order in s we have

$$\frac{d^2 s}{dT^2} + \left[\frac{4}{\omega^2} - \frac{4k^2\Phi_0}{\omega^2} cos(2T)\right] s = \frac{4k^2\Phi_0}{\omega^2} sin(2T) \tag{1}$$

i.e., a Mathieu equation with unstable solutions for $\omega \simeq 2/q$ with q integer, indicating the existence of large amplitude solutions for these values of ω. This response is due to resonance consisting of an integer number of cyclotron oscillations per wave oscillation.

Consider a Poincaré section of $k\rho$, $\psi = kx - \omega t$, by taking points when $v_y = 0, \dot{v}_y > 0$. Resonances exist for $\omega = 2/q$ for all integer q, associated with the unstable domains of the associated Mathieu equation. Secularities are not found at fractional frequencies in the standard Hamiltonian analysis[1] because the wave field is considered only to first order, a higher order analysis demonstrates their existence. Figure 1a shows an example of the resonances and the extent of the stochastic domain for $\omega = 1/4$, with $k^2\Phi_0 = 0.77$. Heating of an initially cold distribution proceeds to the maximum limit given by good KAM surfaces in a few hundred cyclotron periods. Even at a wave frequency of 1/10 of the cyclotron frequency a Poincaré plot is quite stochastic for $k^2\Phi_0 = 1$. The onset of chaos at large wave amplitude as a function of ω is shown in Fig. 1b.

Alfvèn waves have been observed or predicted to be present in plasmas with parameters ranging from those of laboratory to space and astrophysical environments. Previous theoretical investigations of heating mechanisms have nearly always been based on the existence of the primary cyclotron resonance, ie $\omega - k_z u_z \pm n\Omega_c \simeq 0$ where ω and $\vec{k}$ are respectively the angular frequency and wave vector of the Alfvèn wave, $\vec{B}_0 = B_0\hat{z}$

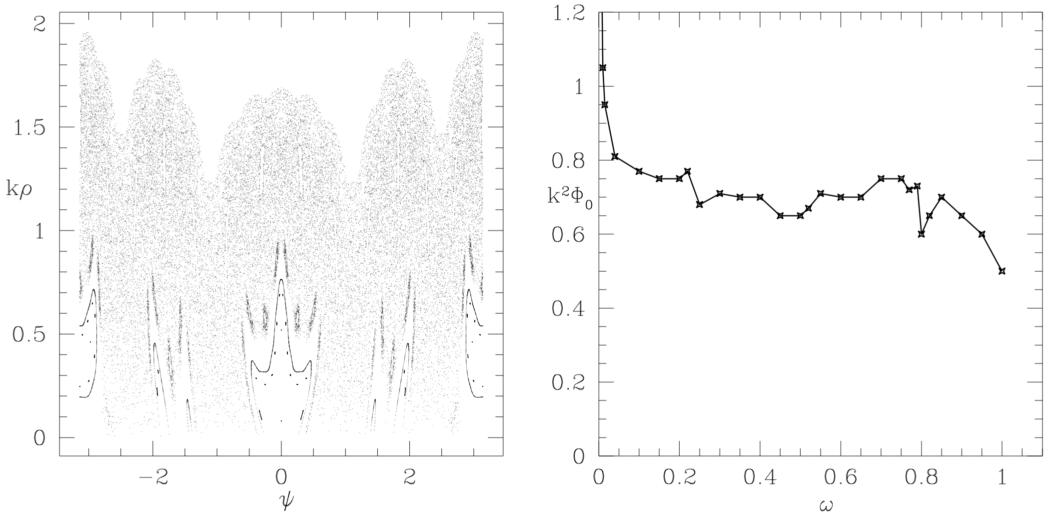

Figure 1: a. Poincaré plot for Electrostatic wave, $k^2\Phi_0 = 0.77$, $\omega = 1/4$. b. Stochastic threshold vs ω.

is the confining magnetic field, $\vec{u}$ is the laboratory frame particle velocity, $n \geq 1$ and $\pm$ corresponds to right $(+)$ and left $(-)$ circular polarization. Such resonances will change the magnetic moment $\mu = u_\perp^2/2B_0$ leading to pitch angle scattering and heating. Since for the Alfvèn waves $\omega \simeq k_z v_A$ with $v_A = B/(4\pi n_0 m_i)^{1/2}$ the Alfvèn velocity, the cyclotron resonance condition becomes $k_z v_z = k_z(u_z - v_A) \simeq \pm n\Omega_c$, where $\vec{v}$ is the particle velocity in the wave frame. Noting that typically $|k_z v_A| < |n\Omega_c|$ the resonance condition generally requires that u_z be super Alfvènic, a condition often not satisfied. Wu and coworkers[9] have examined nonlinear interactions between ions and Alfvèn waves under nonresonant conditions using a one dimensional $(\vec{k} = k_z\hat{z})$ model, finding that while the Alfvèn waves can lead to large amplitude oscillations in the ion motion, there is no stochastic heating. That nonzero $\vec{k}_\perp$ is necessary for stochastic heating has been noted earlier[10], but only for cases in which the cyclotron resonance condition was satisfied.

For a sufficiently large-amplitude, obliquely propagating $(\vec{k} = k_z\hat{z} + \vec{k}_\perp)$ wave, there indeed exists efficient stochastic ion pitch angle scattering and heating by the Alfvèn wave even when $k_z v_z$ is only a small fraction of Ω_c. Note for cold ions in the laboratory frame $v_z = -v_A$ so $k_z v_z = -\omega$ and this condition becomes $\omega \ll \Omega_c$. The physics of this stochastic heating is qualitatively similar to the electrostatic case, discussed above. To demonstrate this similarity, consider a linearly polarized Alfvèn wave in the laboratory frame X, Y, Z, given by $\vec{B}_w = B_w \hat{y} cos(\psi)$ with $\psi = \vec{k} \cdot \vec{X} - \omega t$. Let the ions be initially cold in the laboratory frame, so that $\omega = -k_z v_z = k_z v_A$. Again take the units of time to be given by Ω_c, and normalize the field to B_0. In the wave frame $\vec{x} = \vec{X} - v_A t\hat{z}$ we have $\psi = k_x x + k_z z$ and the velocity $v = v_A$ is constant in time. Dimensionless numbers characterizing the problem are then $k_x v$, $k_z v = \omega/\Omega_c$, and the wave magnitude B_w/B_0.

The equations of motion become $\dot{v}_x = v_y - v_z B_w cos\psi$, $\quad v_y = x_0 - x$, $\quad \dot{v}_z = v_x B_w cos\psi$, giving $d^2x/dt^2 + x = x_0 - v_z B_w cos\psi$. To first order in B_w we have $d^2x/dt^2 + x = x_0 - v_z(0) B_w cos\psi$, equivalent to the electrostatic case, with $\omega/\Omega_c = k_z v_z(0)/\Omega_c$ playing the role of the frequency of the electrostatic wave, and $k_x v_z(0) B_w/(B_0\Omega_c)$ the nonlinearity parameter. Thus there are resonances at many values of particle pitch in the wave frame. However, note that $k_x = 0$ implies no nonlinear interaction.

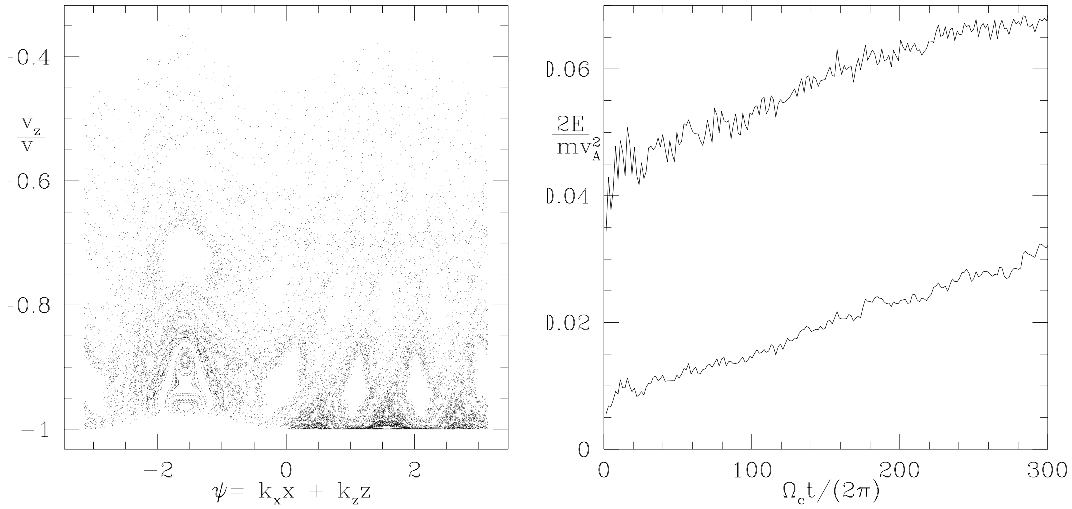

Figure 2: a. Stochasticity produced by a single circularly polarized Alfven wave with $\delta B/B = 0.25$ and $\omega = \omega_c/4$. b. Ion heating due to this Alfven wave showing $E_\perp$ (larger) and $E_\parallel$ vs time.

We shall consider in the following only a left hand circularly polarized Alfven wave. Thus we have, again in the wave frame $\vec{B}_w = -B_w \hat{x} cos(\alpha) sin(\psi) + B_w \hat{y} cos\psi + B_w \hat{z} sin(\alpha) sin(\psi)$ with $\psi = k_x x + k_z z$ and $tan(\alpha) = k_x/k_z$. In the laboratory frame the wave propagates in the $+z$ direction, and in the wave frame $v_z/v = -1$ for an initially cold ion distribution. Figure 2a shows a Poincaré plot for a left hand circularly polarized wave with $B_w = 0.25$, $k_x v = 0.27$, $\omega = 0.25$, formed by taking points when $v_y = 0$ and $\dot{v}_y > 0$. All particles were initiated with $v_z/v < -0.99$, ie the initial ion distribution in the lab frame was cold. Ions can readily diffuse from $v_z/v = -1$ to values near -0.4. In Figure 2b is shown the heating of an initially cold distribution. Since the distribution begins with $\lambda = -1$ the ions mainly gain perpendicular energy and thus we see $E_\perp > E_\parallel$ with $E_\perp$ and $E_\parallel$ respectively the energies perpendicular and parallel to $\vec{B}_0$. Two hundred cyclotron periods is sufficient to heat to $v \simeq 0.25 v_A$ for these parameters. These results may provide an interesting new mechanism of solar corona heating by Alfven waves[11]. The ion distribution produced in this figure has a perpendicular thermal velocity of $250 km/sec$ taking $v_A \sim 10^3 km/sec$ in the lower solar corona. Since energization increases with ω/Ω_c, this heating mechanism will preferentially energize partially ionized heavier mass (lower Ω_c) ions. These features are consistent with observations[12].

The existence of resonances at fractions of the cyclotron frequency is a generic phenomenon and may be expected to occur for other types of waves. This process also leads to a mechanism that directly transfers energy from Super-Alfvenic ions to thermal ions in high β plasmas. The mechanism involves the excitation of compressional Alfvén eigenmodes (CAEs) in the frequency range with $\omega \leq \omega_{ci}$. The broadband turbulence resulting from the large number of excited modes causes stochastic diffusion in velocity space, which transfers wave energy to thermal ions. This effect may be important on NSTX [13], and may scale up to reactor scenarios. This has implications for low aspect ratio reactor concepts, since it potentially allows for the modification of the ignition criterion.

Spherical torus experiments typically operate at very low values of the axial toroidal magnetic field (0.3T to 0.6T). On NSTX, which operates with 80keV deuterium neutral

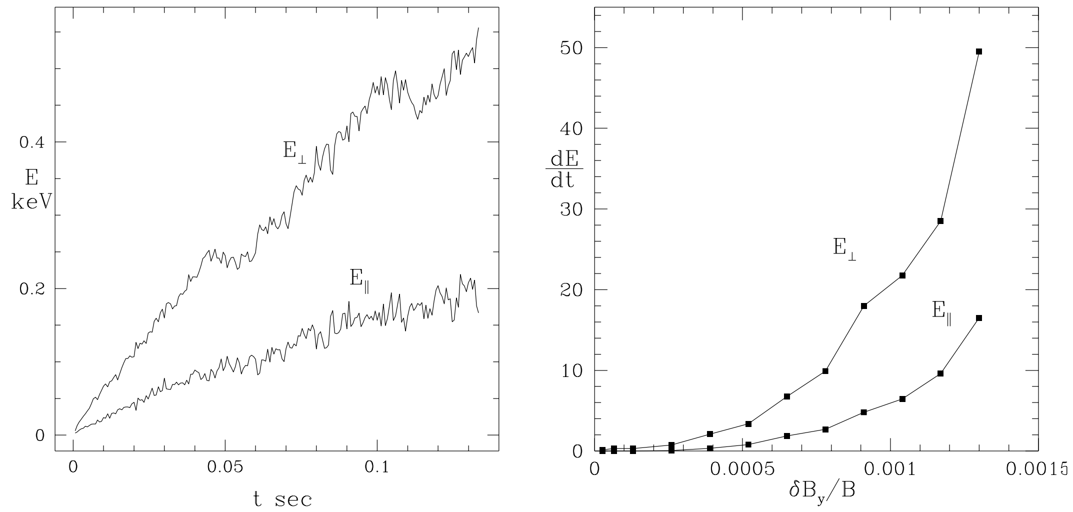

Figure 3: Heating simulation using wave spectrum with $0.2\omega_{ci} < \omega < 0.6\omega_{ci}$ similar to compressional Alfven waves predicted and observed in NSTX.

beams and 0.3T toroidal field, the neutral beam velocity is typically given by $v_{beam} \sim 4v_A$. The existence of a large class of super-Alfvénic particles can change the regime from one of weakly interacting waves to one where the wave amplitudes become large enough to modify the thermal particle energy. The energy flow is from the fast particles to the waves by resonance, and from the waves to the thermal particles by stochastic heating. In tokamaks generally the heating particles are at best weakly super-Alfvénic and only a small fraction of their total energy is available. In Fig. 3 is shown the results of a simple slab geometry heating simulation, using a spectrum of modes estimated to exist in NSTX, which demonstrates the feasability of this heating scenario.

In conclusion, large amplitude waves are capable of producing extensive stochastic heating of a magnetized plasma at frequencies well below the cyclotron frequency.

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