LOCALIZED BALLOONING MODES IN COMPACT QUASIAXIALLY SYMMETRIC STELLARATORS*

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Introduction

Understanding of ballooning mode stability boundaries may lead to performance improvement of toroidal devices through control of disruptions. Toroidally localized ballooning modes have been found as precursors to high beta disruptions on TFTR[1] arising in conditions of *n*=1 kink mode asymmetry. Recent optimization has shown that magnetohydrodynamic stability as well as good particle confinement are likely to be achievable in the National Compact Stellarator Experiment (NCSX), a compact, quasiaxially symmetric stellarator (QAS) for values of the plasma near β = 4%[2]. The configuration, with a major radius of 1.42 m, an aspect ratio of 4.4, a toroidal magnetic field 1.2-1.7 T and 6MW of neutral beam heating, is stable to MHD instabilities, and is expected to be limited by high-*n* kink and ballooning modes. This paper describes the ballooning eigenvalue isosurfaces for NCSX, the first step in an examination of the kinetic stabilization of the ballooning beta limit using a hybrid WKB approach[3].

Eigenvalue Isosurfaces of the Quasiaxially Symmetric Stellarator

The VVBAL module of the TERPSICHORE code suite[4] has been used to calculate the ballooning instability for several NCSX equilibria (Fig. 1) above the design point (β = 4.1%) with the VMEC code. The displacement of the flux surface grows with growth rate γ; $\xi \propto \exp(i\omega t) \propto \exp(\gamma t)$. We define the eigenvalue $\lambda = -\omega^2$; positive values of λ denote instability, while negative values denote stability. For β = 4.3% and β = 6.8% we have assembled a datacube of ballooning eigenvalues $\lambda(s,\alpha,\theta_k)$, of size (126,101,21). s is the toroidal flux, α is the field line variable. Roughly, within $\pm\pi$, the ballooning parameter θ_k determines where the eigenfunction is a maximum. Figure 2 shows the plasma iota of the two equilibria. These are weak shear plasmas, with vanishing shear near the edge.

The isosurfaces of λ describe the possible trajectories of rays of the ballooning equation. Consequently they characterize the quantization conditions that are used to find the maximum wave vector, and thereby kinetic stabilization of the ballooning beta limit. The QAS isosurfaces are found to exhibit unusual and unique topologies for the two configurations. Distinct structures occur in different ranges of λ in the stable spectrum: a) at $\lambda = -0.15$, a helical structure is found near the plasma edge, rotating about an axis nearly parallel to?the θ_k axis, with fins stretching toward the plasma center; b) at $\lambda = -0.45$, cylinders are found nearly constant in θ_k, localized in s and α. At $\beta = 6.8\%$, similar structures in the stable spectrum occur, although more global in extent.

The unstable spectra are less complex, consisting primarily of planes and topologically cylindrical and spherical isosurfaces near the outer edge of the plasma, where shear goes to zero and the instability is more easily driven. In general, there is a weak dependence on the ballooning angle θ_k, stronger dependence on the field line α and quite strong dependence on the radial parameter s. At $\beta = 4.3\%$ topologically spherical isosurfaces are found for the maximum eigenvalues, indicative of strong quantum chaos[3]. This description "quantum chaos" for the paths of rays of the ballooning equation does not mean that the plasma behavior is chaotic, but that the mathematics of a quantum chaotic scattering problem can be used for instabilities far above the marginal point of the equilibrium. At lower values of the eigenvalue, isolated unstable cylindrical and planar isosurfaces conjoin, as the eigenvalue λ drops to zero, and the isosurface is no longer simply connected. At $\beta = 6.8\%$ the bands break up at maximum eigenvalues. The configuration is Mercier stable at both values of β.

Comparison with a related axisymmetric tokamak shows that the structures of the stable and unstable spectra of the QAS arise from the complexity of the magnetic configuration.

Finite Larmor Radius Stabilization of the Beta Limit

In practice, only finite-*n* modes can be unstable due to finite ion Larmor radius (FLR) stabilization. Finite-*n* ballooning mode stability calculations with a three-dimensional, linear MHD code for a two-field period QAS configuration showed that the finite *n* ballooning modes ($n \sim 20$) are significantly more stable than the infinite-*n* results. For H1 and LHD finite *n* ballooning modes have been examined by applying the WKB ballooning formalism and semi-classical quantization or quantum chaos theory, depending on the topology of the isosurfaces[3]. The validity of the hydrodynamic, fluid model for MHD breaks down and kinetic corrections are required if the condition $(k_\perp \rho_i)^2 << 1$ is not satisfied. Here $k_\perp$ is the wave vector perpendicular to the field line, and ρ_i is the ion Larmor radius, which for the QAS is ~1cm. Near the beta limit the ballooning rays at the marginal point ($\lambda=0$) will propagate on an isosurface having topology of conjoined cylinders, with axes parallel to θ_k. It does not appear that either the Einstein-Keller-Brillouin semiclassical quantization method or the quantum chaos approach applies.

Anderson localization

Symmetry breaking localization of the ballooning mode in stellarator plasmas has been identified for LHD, H1 and HSX[5] as analogous to Anderson localization[6] of electron eigenfunctions in condensed matter. Our calculations show that localization of the QAS eigenfunction increases for reduced plasma shear (Fig. 3). Each flux surface has a different shape, changing the poloidal angle at which the eigenfunction is maximized. Weak shear increases localization of the eigenfunction, in contrast to the tokamak case[7]. The most localized modes in this geometry occur in the region where global magnetic shear is weakest, including at the shear reversal surface itself, demonstrating the existence of Anderson localization in the QAS.

Conclusion

We find Anderson localization of the ballooning mode in the QAS and have obtained eigenvalue isosurfaces with which to examine kinetic stabilization of β. A new method of regularizing the eigenfunction to estimate $k_\perp$ will be needed for the QAS at the beta limit, since the topology of the marginal point isosurfaces is not simply connected.

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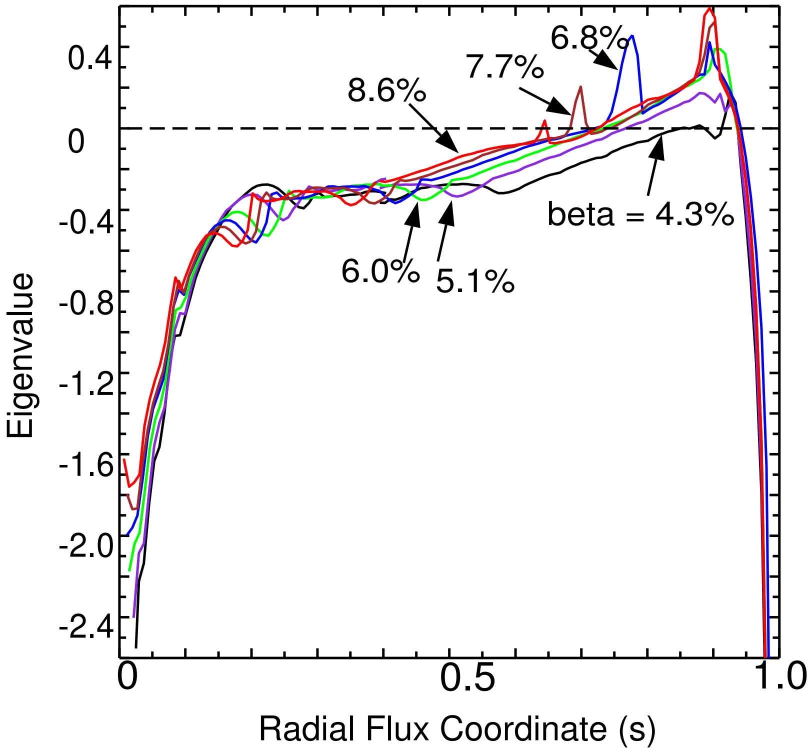

Figure 1. Ballooning eigenvalues for NCSX above the design point. α=0, θk=0.

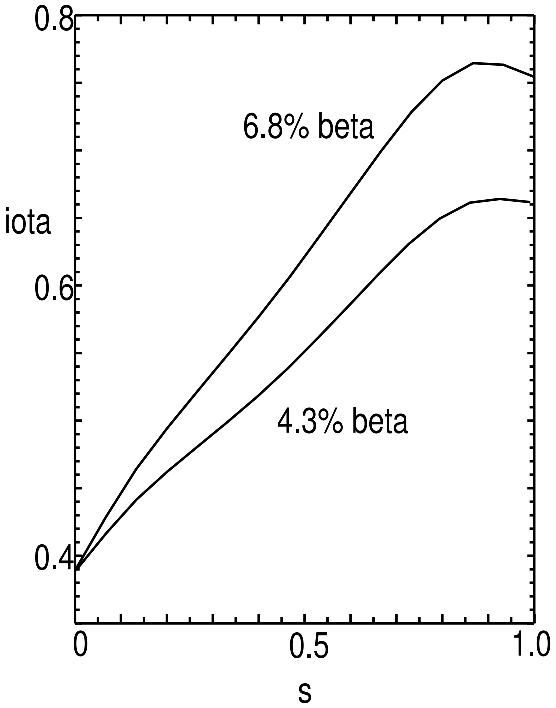

Figure 2. Iota profiles for equilibria above the design point beta.

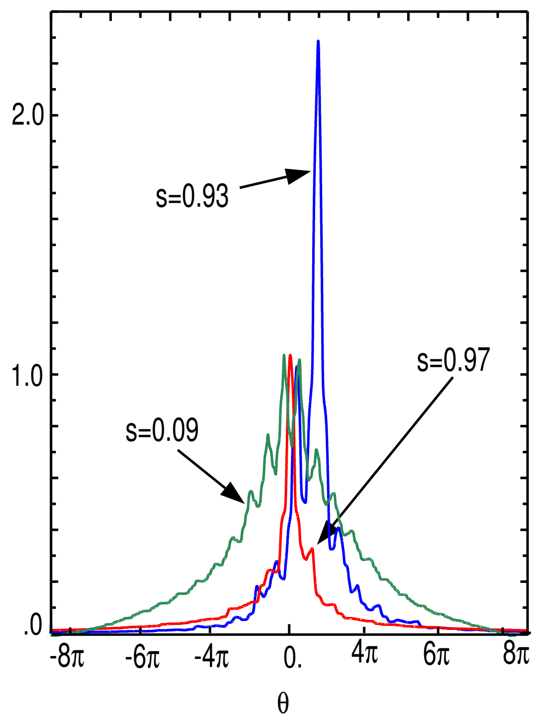

Figure 3. Eigenfunction localization in poloidal angle near the plasma edge, labeled by s, the edge normalized toroidal flux.