

Wakefield Generation by Incoherent Laser Beams

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Abstract

The generation of large electric fields by intense incoherent laser beams has been considered. For this purpose, we employ a wave kinetic equation for incoherent photons, taking into account the electron number density and electron mass variations in the fields of broad band intense laser beams. The electron density perturbation associated with the wakefield, in turn, is significantly affected by the relativistic ponderomotive force of incoherent laser beams. The governing nonlinear equations are numerically solved to obtain the wake electric field profiles.

Recently, significant progress [1] has been made in ultra-intense laser beam-plasma interactions for generating strong wakefields in order to accelerate the electrons to extremely high energies. The ponderomotive force of the intense laser pulse excites an electron plasma wave (EPW) with a phase speed close to the speed of light [2]. Since the laser excited high phase speed plasma waves have very strong electric fields, the plasma based electron accelerators (the laser beat wave, the laser wake field, and self-modulated laser wake field) are superior than the conventional accelerators. The idea of the laser wakefield acceleration is also being exploited for understanding the origin of high energy gamma ray bursts in astrophysical environments. Previous studies of the plasma based accelerators have dealt with coherent laser beams. However, in practice, laser beams always have some finite bandwidth [3] and the laser intensity distribution may have a broad spectrum. Accordingly, in this paper we investigate the wakefield generation by incoherent laser beams in plasmas.

We consider the propagation of an incoherent circularly polarized laser radiation pulse

in the z direction, so that the the perpendicular component (with respect to the z direction) of wave vector potential is represented as

$$\mathbf{A}_\perp = \mathbf{A}_{\perp k}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) \exp(-i\omega t + ikz) + \text{complex conjugate}, \quad (1)$$

where ω and k are the frequency and the wavenumber of the laser pulse, respectively. In a nonlinear plasma, they are related by

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\gamma \omega^2}, \quad (2)$$

where c is the speed of light in vacuum, ω_{pe} is the electron plasma frequency, and γ is the Lorentz factor in the high intensity wave, namely,

$$\gamma \approx \left(1 + \frac{e^2}{m_e^2 c^4} \sum_k |\mathbf{A}_{\perp k}|^2 \right)^{1/2} \simeq (1 + I)^{1/2} \quad (3)$$

where e is the magnitude of the electron charge and m_e is the electron rest mass. It turns out that the relativistic electron mass variation in the wave field causes a reduction of the effective electron number density

$$n_{eff} = \frac{n_e}{\gamma}. \quad (4)$$

As a result, the plasma is transparent in the relativistic induced transparency regime, where $n_c < n_e < \gamma n_c$ with $n_c = m_e \omega^2 / 4\pi e^2$ as a critical density for light at frequency ω .

The dynamics of incoherent laser beams is governed a wave kinetic equation

$$\frac{\partial I}{\partial t} + c \frac{\partial I}{\partial x} = \frac{\omega_p^2}{2kc} \frac{\partial}{\partial x} \left(\frac{n_e/n_0}{\sqrt{1+I}} \right) \frac{\partial I}{\partial k}, \quad (5)$$

where $\omega_p = (4\pi n_0 e^2 / m_e)^{1/2}$, n_0 is the unperturbed electron number density, and

$$\omega^2 = \frac{4\pi e^2 n_e(x, t)}{m_e \sqrt{1+I}} + k^2 c^2 \sim k^2 c^2 \quad (6)$$

for very high-frequency laser beams.

The electron number density variation ($n_e - n_0$) associated with the longitudinal plasma waves in the presence of incoherent laser beams is determined from the continuity equation

$$\frac{\partial n_e}{\partial t} + \frac{\partial J_e}{\partial x} = 0, \quad (7)$$

the relativistic momentum equation

$$\frac{\partial p_e}{\partial t} + v_e \frac{\partial p_e}{\partial x} = -eE - m_e c^2 \frac{\partial}{\partial x} \sqrt{1 + I}, \quad (8)$$

and Poisson's equation

$$\frac{\partial E}{\partial x} = 4\pi e(n_0 - n_e), \quad (8)$$

where $J_e = n_e v_e$, $p_e = m_e v_e \gamma_e$ is the relativistic momentum, $\gamma_e = (1 - v_e^2/c^2)^{-1/2}$ is the gamma factor involving the electron fluid velocity v_e , and E is the wake electric field. The second term in the right-hand side of eq. (8) is the relativistic ponderomotive force of incoherent photons.

Introducing the transformation $\chi = (\omega_p/c)(x - v_p t)$, where v_p is the plasma wave phase speed, we obtain from eq. (5)

$$\frac{dI}{d\chi} \left[(1 - \beta_p) + \frac{1}{2} \Omega_p^2 \left(\frac{\partial I}{\partial \nu} \right) \frac{\eta}{(1 + I)^{3/2}} \right] = \Omega_p^2 \frac{\partial I}{\partial \nu} (\sqrt{1 + I}) \frac{d\eta}{d\chi}, \quad (9)$$

where

$$I(\chi, k) = I_0(\chi) \exp \left[-\frac{(k - k_0)^2}{2k_w^2} \right] = I_0(\chi) \exp \left(-\frac{\nu^2}{2} \right),$$

$$\beta_p = \frac{v_p}{c}, \quad \Omega_p^2 = \frac{\omega_p^2}{k_w k c^2},$$

$$\eta(\chi) = \frac{n_e}{n_0} = \frac{\Gamma_0 \sqrt{1 + P_e^2}}{P_e - \beta_p \sqrt{1 + P_e^2}},$$

and

$$\Gamma_0 = \frac{P_0 - \beta_p \sqrt{1 + P_0^2}}{\sqrt{1 + P_0^2}},$$

with $P_0 = P_e(\chi = 0)$

We also obtain from eqs. (6)-(8)

$$\frac{dP_e}{d\chi} = \eta \frac{d}{d\chi} (\Psi - \sqrt{1+I}) \quad (10)$$

and

$$\frac{d^2\Psi}{d\chi^2} = \eta - 1, \quad (11)$$

where $\Psi = e\phi/m_e c^2$ and ϕ is the wake potential.

Equations (9)-(11), which are the main results of our paper, have been numerically solved in order to obtain the wakefield profiles for $I_0(0) = 10$, $\nu = 0.7$, and for two values of Ω_p^2 ($= 0.07, 0.028$). It turns out that the magnitude of the wake electric field is reduced due to broadband laser intensity distributions. Physically, this happens because energy of broad band laser beams is spread over the entire k spectrum, so that the work done by the laser beam on the electrons is impeded.

To summarize, we have considered the excitation of longitudinal plasma waves by incoherent laser beams in plasmas. The wave kinetic equation for intense laser beams and the driven (by the relativistic ponderomotive force of laser beams) plasma wave equations have been cast in the form of coupled nonlinear differential equations in a frame moving with the phase speed of the plasma waves. The governing nonlinear equations are then numerically solved to obtain the wake field profiles that are created by arbitrary large amplitude incoherent laser beams. The results of our investigation should be applicable to the electron acceleration by wakefields in laboratory and astrophysical environments.

References

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