

Theory of Voids in Dusty Plasmas

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Abstract

It is proposed that the plasma voids, which are characterized as regions of significant ion and electron density depletions in association with negative plasma potentials, are self-consistent stationary solutions of Poisson's equation in which the electron density response is Boltzmannian and the ion density response is non-Maxwellian due to the ion trapping in large amplitude plasma potentials. The voids shrink when the dust grains are added into the plasma, and they tend to disappear if the dust number density is sufficiently high.

Dust free regions (known as voids) inside a dust cloud have been observed in silane [1] and other dust laden-plasmas [2-5]. Samsonov and Goree [2] observed that as dust particles in a sputtering plasma grew in diameter, the void was developed by a sudden onset of a filamentary mode in which the ionization rate and dust number density were both modulated. On the other hand, in their microgravity experiment (where dust particles are already sufficiently large) Morfill *et al.* [4] observed centimeter-size stable dust voids occurred without any initial turbulent phase. Experimental observations [1-5] thus reveal that a dusty plasma is not always composed of a homogeneous distribution of dust particles, but under some conditions, it is accompanied by dust free regions (voids). Although there have been some analytical and numerical efforts [6,7] to model voids in dusty plasmas, the underlying physics of the void formation is not fully understood. In this paper, we present a theoretical model to explain the formation of plasma voids (holes) in the absence of dust particles, as well as to explain the role of dust particles on the

size of the voids. Specifically, we propose here that the plasma voids are self-consistent solutions of Poisson's equation in which electrons follow the Boltzmann distribution and ions obey a vortex-like distribution [8,9] because of their trapping in large amplitude plasma potentials.

We consider a one-dimensional, collisionless, unmagnetized dusty plasma consisting of electrons following a Boltzmann distribution, ions obeying a trapped/vortex-like distribution, and negatively charged dust particles providing the stationary background. The electrostatic potential associated with the charge density perturbation is described by Poisson's equation

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_e - \alpha n_i + \alpha - 1, \quad (1)$$

where n_e (n_i) is the electron (ion) number density normalized by its equilibrium value n_{e0} (n_{i0}), ϕ is the electrostatic plasma potential normalized by $k_B T_i / e$ (T_i is the ion temperature, k_B is the Boltzmann constant, and e is the magnitude of the electron charge), ξ is the space variable normalized by $\lambda_{Dm} = (k_B T_i / 4\pi n_{e0} e^2)^{1/2}$, and $\alpha = n_{i0} / n_{e0}$. We note that $\alpha = 1$ represents no dust particle present in the plasma, but $\alpha > 1$ represents the presence of negatively charged dust particles in the background. Thus, Eq. (1) is valid in a plasma with a uniform distribution of dust particles.

The number density of electrons, which follow the Boltzmann distribution, is given by

$$n_e = \exp(\sigma \phi), \quad (2)$$

where $\sigma = T_i / T_e$. To model the ion distribution in the presence of trapped particles, we employ a vortex-like ion distribution of Schamel [8] which solves the ion Vlasov equation [8,9]. Thus, we have for the ion number density n_i for $\beta < 0$ [8,9]

$$n_i = \exp\left(-\frac{u_0^2}{2}\right) \left[I(-\phi) + K\left(\frac{u_0^2}{2}, -\phi\right) + \frac{2}{\sqrt{\pi|\beta|}} W_D(\sqrt{\beta}\phi) \right], \quad (3)$$

where u_0 is the frame speed normalized by the ion thermal speed $v_{Ti} = (k_B T_i / m_i)^{1/2}$, and $|\beta| (= T_h / T_{it})$, which is the ratio of the free ion temperature T_i to the trapped ion

temperature T_{it} , is a parameter determining the number of trapped ions. Furthermore, we have denoted $I(x) = [1 - \text{erf}(\sqrt{x})] \exp(x)$, $W_D(x) = \exp(-x^2) \int_0^x \exp(y^2) dy$, and $K(x, y) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{\pi}{2}} d\theta \sqrt{x} \cos \theta \exp(-y \tan^2 \theta + x \cos^2 \theta) \text{erf}(\sqrt{x} \cos \theta)$.

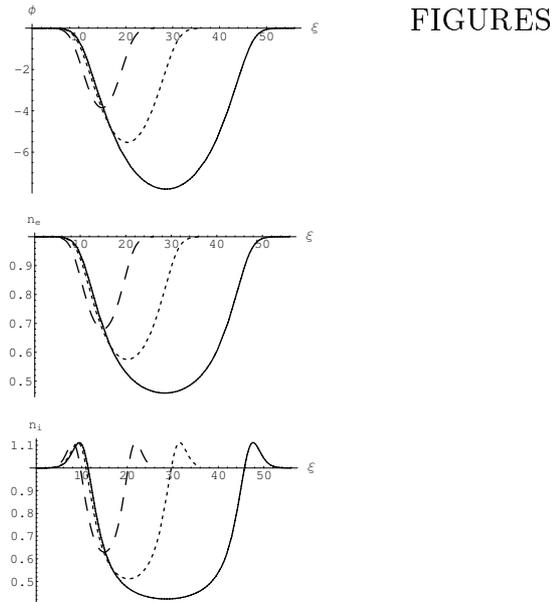
We are interested here to investigate arbitrary amplitude standing ($u_0 = 0$) plasma voids/holes. Substituting eqs. (2) and (3) into eq. (1), multiply it by $d\phi/d\xi$, and integrating the resultant equation we obtain the energy law for a pseudo-particle of unit mass in the pseudo-potential $V(\phi)$ [9]. Letting $V(\phi_m) = 0$, the amplitude (depth) ϕ_m of the voids/holes is determined from the nonlinear relation

$$(1 - \alpha)\phi_m + \frac{1}{\sigma}[1 - \exp(\sigma\phi_m)] + \alpha \exp\left(-\frac{u_0^2}{2}\right) [1 - P(-\phi_m, \beta) - H\left(\frac{u_0^2}{2}, 0, -\phi_m\right)] = 0. \quad (4)$$

The width Δ of these voids/holes can be related with the depth of the classical potential $|V_{min}|$ and the amplitude ϕ_m by the formula $\Delta = \frac{\phi_m}{\sqrt{|V_{min}|}}$, It is clear from eqs. (4) that for an arbitrary value of the amplitude it is not possible to find a simple analytical expressions for the amplitude (depth) ϕ_m and the width Δ . Therefore, to study the properties (amplitude and width) of arbitrary amplitude voids (holes), we have numerically solved eqs (1), (2) and (3) and displayed the plasma potential profiles and associated electron and ion density depletions in figure 1. The latter depicts that as we increase the dust particle number density, the depth as well as the width of the voids/holes (localized potential structure, electron and ion density depletions) decrease. Furthermore, as the dust particle number density is increased, the depth as well as the width of the voids (holes) decrease, and after a certain value of the dust particle number density, the voids (holes) disappear.

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FIGURES

FIG. 1. The upper, middle and lower plots show the variation of $|\phi_m|$, n_e , and n_i with ξ , respectively, for $\beta = -0.9$, $u_0 = 0$, $\sigma = 0.1$, $\alpha = 1$ (solid curve), $\alpha = 1.1$ (dotted curve), and $\alpha = 1.5$ (dashed curve).

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