Flute instability of the neutral current sheet
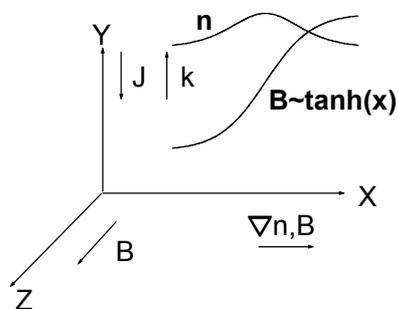
Shaikhislamov I.F.

Institute of Laser Physics, pr. Lavrentyeva 13/3, Novosibirsk, 630090, Russia

Introduction

Lower-hybrid drift or flute instability of plasma has a long-standing history of laboratory and theoretical research. It was observed in theta-pinch and laser-produced plasma experiments as well as in magnetosphere artificial releases. It is now well established that it may be understood as a coupling between current drift and magnetosonic or lower-hybrid wave and described by Hall-MHD equations [1]. Hall dynamics influences plasma behavior at a spatial scale of the ion inertia length. Because of it, Hall effects are also viewed as one of the probable mechanisms of structuring of the neutral current sheet in the problem of magnetic field reconnection [2]. However, the flute instability of the neutral current sheet has not been extensively analyzed in the reconnection problem so far. In the work [3] the early phase of instability development was numerically investigated, while in [4] dispersion equation was obtained and analyzed. The purpose of this work is to show by analytical analyses that, in general, the neutral current sheet is unstable if its width is of order of ion inertia length and to investigate by numerical simulation the nonlinear evolution on the Alfven time scale.

Formulation of the problem and linear analyses

We consider long stationary neutral current sheet. Two x-y dimensions restrict the problem. Magnetic field has only one z-component as shown in the sketch. Harris profile of magnetic field is used in the slab geometry, assuming that initial gradients exist only along x coordinate. Plasma is in MHD-equilibrium state. For analyses collisionless EMHD equations are applied in the quasi-neutrality approximation. Electromagnetic effects are ignored and useful but not essential approximation of adiabatic initial temperature profile: $T \sim n^{\gamma-1}$ is used. The spatial scales involved are from electron to ion inertia lengths. Resulting equations are:

$$\frac{\partial n}{\partial t} + \nabla \cdot (\vec{V}n) = 0, \qquad \frac{dp}{dt} + \gamma p \nabla \cdot \vec{V} = 0, \qquad nM\frac{d\vec{V}}{dt} = -\nabla\left(\frac{B^2}{8\pi} + p\right)$$

$$\frac{\partial B}{\partial t} + \nabla \cdot (\vec{V}B) = \lambda_i^2 \omega_{ci} \frac{\nabla B \times \nabla n}{n} + \nabla \cdot \left(\lambda_e^2 \frac{d_e}{d_e t} \nabla B\right)$$

$$\lambda_e^2 = \frac{m_e c^2}{4\pi ne^2}, \qquad \lambda_i^2 \omega_{ci} = \frac{cB}{4\pi ne}, \qquad p = nT$$

For small perturbations ~exp(ω·t-k$_y$·y) with wavenumber k the dispersion equation in a local approximation follows as:

$$\left(1 + \tilde{k}^2\right)\cdot\tilde{\omega}^3 + \varepsilon_b\left(\tilde{c}_s^2 - \tilde{k}^2\right)\cdot\tilde{\omega}^2 - \left(1 + \left(1 + \tilde{k}^2\right)\cdot\tilde{c}_s^2\right)\cdot\tilde{\omega} + \varepsilon_b \tilde{c}_s^2 \tilde{k}^2 = 0$$

$$\tilde{\omega} = \frac{\omega}{\tilde{k}\omega_H}, \quad \tilde{k} = k\lambda_e, \quad \varepsilon_b = \lambda_i \frac{dB}{Bdx}, \quad \tilde{c}_s^2 = \gamma\frac{4\pi nT}{B^2} = \frac{\gamma}{2}\beta$$

This cubic dispersion equation may be found in many other works on the Lower-Hybrid Drift Instability (LHDI) treated in the frame of Hall-MHD analyses. As it is well known from the previous studies, the plasma becomes unstable at sufficiently large values of $\varepsilon_b \geq 1$ and kλ_e~1. Maximum increment corresponds to the wave number $k\lambda_e \approx \tilde{c}_s$. The instability may be interpreted as the coupling between the drift and magneto-sonic or lower-hybrid waves. In the figure 1 the threshold of instability is shown as a function of outside beta β_o=8πp$_o$/B^2 in terms of the full current width 2L/λ_i for the Harris profile.
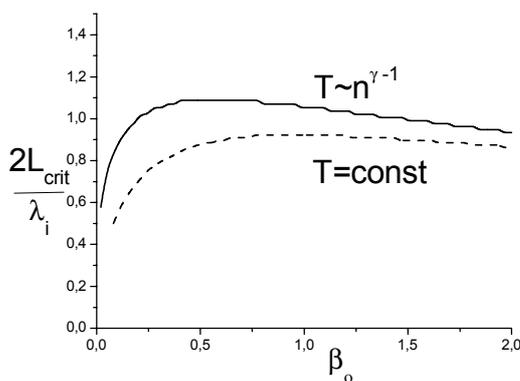

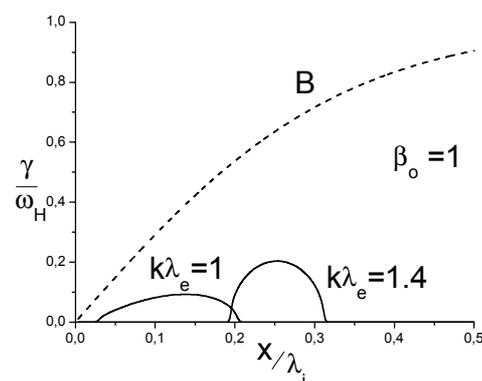*Figure 1. Critical width of the current sheet as a function of outside plasma beta.*

Figure 2. Increment distribution along the Harris sheet.

For comparison the case of constant temperature profile is presented by dashed line as well. The sheet becomes unstable at full width approximately equal to the ion-inertia length. The distribution of the increment along the sheet with width 2L/λ_I=0.7 is demonstrated in the figure 2 for two wavelengths. Toward the neutral line it decreases roughly as ~B.

Numerical simulation

For simulation presented here the sheet parameters were taken as $\beta_o=1$, $2L/\lambda_i=0.7$, the width being slightly over-critical. There is no forced inflow of plasma and physical dissipation except the numerical one. The boundary conditions are taken periodic in y-direction and free in x-direction. The number of grid points is 500 in both directions. Electron mass is $m=M/2500$. The evolution isn't sensitive to the initially imposed perturbations after the instability catches up. The measured small perturbation growth rate $\sim 0.125 \cdot \omega_H$ and flutes wavelength $k\lambda_e \approx 1$ are close to the linear analyses prediction. The non-linear evolution starts around $t \cdot \omega_{ci} \approx 1.5$. Its most pronounced feature is flutes of density that, once formed, gradually expand toward neutral line and move with magneto-sonic speed along the current. The profiles of averaged magnetic field and that of fluctuations at various times are shown in the figure 3.
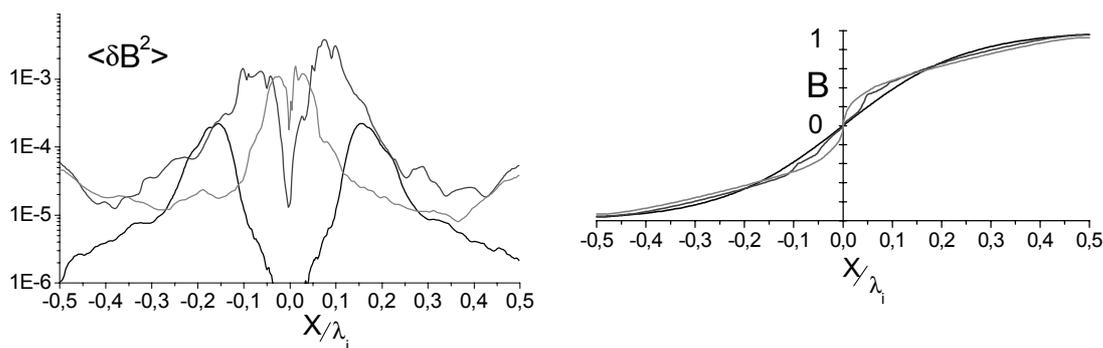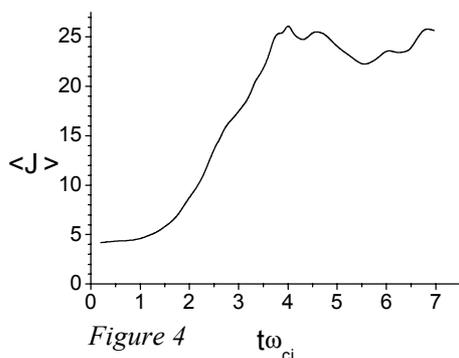
Figure 3. Profiles of averaged magnetic field fluctuations (left) and of averaged magnetic field (right) at moments $t \cdot \omega_{ci} =0.5$ (black), 1.5 (dark gray) and 3 (gray).

It demonstrates how the unstable region gradually shifts to the neutral line while mean magnetic field moves toward it and forms a steep drop with over-fall approximately ± 0.3. This drop gradually compresses up to a few grid points, regardless of the grid size. Without collisions the maximum current density becomes extremely large and isn't numerically resolved. In the figure 4 the evolution of current density is shown which is averaged over the distance of electron inertia length λ_e that contains 10 grid points. Even the average current density exceeds initial one by more than 6 times. Current rise takes approximately 2 ion cyclotron times.

Figure 4

Discussion

The Hall term in the chosen geometry can be written as the convection-like, divergence free non-linear motion of magnetic filed. This motion conserves magnetic flux and energy as well. It allows the magnetic field to evolve apart from the density. If there is instability and fluctuations ∂B grow, then the energy of averaged magnetic profile should be decreasing. As the part of magnetic energy contained in fluctuations is redistributed to the plasma energy, so the whole magnetic energy is decreasing a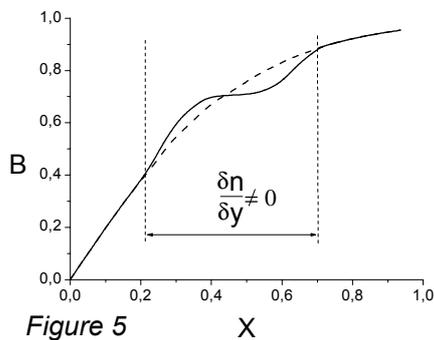s well. In time, at the point of maximum growth the current decreases and instability stops, while it becomes more prominent where the current increases, as demonstrated in figure 5. Due to this current redistribution and slow acoustic expansion the whole current sheet becomes involved in the Hall-induced evolution. In the end current diminishes to the degree at which it becomes stable. However, due to flux conservation, in the area near the neutral line a sharp field discontinuity appears.

Figure 5

The study confirmed the results of previous works that current sheet is subjected to Lower-Hybrid Drift Flute-like instability if its width becomes smaller than ion inertia length. It was shown for the first time that in the course of non-linear evolution magnetic field is convected toward the neutral line. Magnetic field restructuring releases magnetic energy that feeds instability. Near the neutral line current density increases by ~10 times. Sharp growth phase takes place in a span of a few ion cyclotron times. Current rise is such that relative electron-ion velocity may exceed electron thermal velocity, thus triggering Buneman instability, strong dissipation and subsequent disruption phase.

1. J.D. Huba, A.B. Hassam and D. Winske 1990 *Phys. Fluids* **B2** (7), 1676

2. J.C. Dorelli and J. Birn 2001 *Physics of Plasmas* **8** (9), 4010

3. D. Winske 1981 *Phys. Fluids* **24** (6), 1069

4. P.H. Yoon, Anthony T.Y. Lui and Chia-Lie Chang 1994 *Phys. Plasmas* **1** (9), 3033