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Inhomogeneous Plasma Parametric Decay Instability Driven By Frequency Modulated Pump

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The pump phase modulation was discussed as a possible way of parametric decay instability (PDI) suppression starting from 60th. According to results of homogeneous plasma theory [1], it can serves as an effective method for parametric instability control. However the analysis carried out in inhomogeneous plasma model for very fast pump phase modulation have revealed high stability of convective amplification coefficient, which appeared to be non-sensitive to modulation [2]. Such stability was recently confirmed in experiment [3] and shown in numerical computations also for absolute PDI [4], where the threshold conditions for instability suppression in inhomogeneous plasma were determined.

In the present paper it is shown both experimentally and in theory that such non-sensitivity holds only for the pump frequency modulation faster than the decay wave transient time. In the case of slower modulation, both harmonic or stochastic, the PDI enhancement may take place instead.

The experiment was carried out in the linear plasma device "Granit" [3]. The Trivelpiece-Gould (TG) pump wave (f_0 =2480 MHz) was excited in the inhomogeneous plasma with a waveguide. The backscattering parametric instability $l_0 \rightarrow l_0' + s$ was observed under these conditions in previous experiments [3], utilizing a monochromatic pump. The reflected fundamental TG mode - l_0' and ion acoustic wave, propagating along the magnetic field in the direction of decreasing plasma density, are produced by this decay. The satellite, red-shifted by 3 MHz, appears in the spectrum of the signal reflected by the plasma. The effect of the harmonic pump frequency modulation on this PDI was investigated in wide modulation frequency region for 0.1MHz < f_m < 10MHz [3]. The suppression was observed only for f_m >0.8MHz and for large frequency deviations Δf > 50MHz. The effect was strong close to the instability threshold, which was increased by a factor of 3 for Δf = 150MHz. Far from instability threshold the influence of modulation on the instability was weaker. At smaller modulation frequencies f_m <0.5MHz there was no suppression of PDI, on contrary, a pronounced growth of the ion acoustic wave was observed. A more detailed investigation of this wave in a specially performed scattering experiment revealed its strong amplitude modulation at frequency f_m. The results of time resolved stroboscopic measurements of the amplitude of the scattering signal are shown in **Fig. 1** a,b for modulation frequencies f_m = 0.2MHz and f_m = 0.35MHz.

As it is seen, the amplitude modulation is especially pronounced for specific frequency deviations (Δf = 60MHz for f_m = 0.2MHz and Δf = 40MHz for f_m = 0.35MHz), for which the non-harmonic distortions are maximal. The observed phenomena looks like a resonance effect for which the condition $\Delta f\, f_m$=const holds.

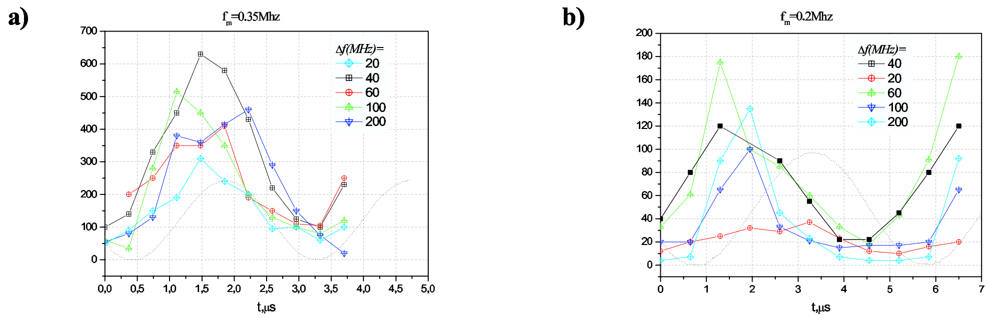

Fig. 1 *Experimentally observed PDI wave amplitude behavior for different frequency deviation Δf.*

The theoretical investigation of this resonance phenomenon is carried out in the framework of coupled equations for slow varying wave amplitudes a_1 and a_2 of oppositely propagating plasma waves:

$$\begin{aligned} \frac{\partial a_1(x,t)}{\partial t} + v_1\frac{\partial a_1(x,t)}{\partial x} + \nu_1 a_1(x,t) = \gamma_0 a_2(x,t)e^{i\Phi(x,t)} \\ \frac{\partial a_2(x,t)}{\partial t} + v_2\frac{\partial a_2(x,t)}{\partial x} + \nu_2 a_2(x,t) = \gamma_0^* a_1(x,t)e^{-i\Phi(x,t)} + S(x,t) \end{aligned} \qquad (1)$$

where $\Phi(x,t) = \int^{x} \Delta\kappa(x')dx' + \delta\Phi(x - v_0 t)$ is a phase mismatch caused by plasma inhomogeneity and pump wave frequency modulation, v_j - group velocities, v_0 - pump wave group velocity, ν_j - damping coefficients, γ_0 - is maximal PDI growth rate and $S(x,t)$ - acoustic wave source. Here and below dimensionless parameters are used $t = tv_1/\ell$, $v_i = v_i/|v_0|$, $x = x/\ell$, $\nu_i = \nu_i\,\ell/|v_0|$, $i = 0,1,2$. It is supposed that $v_1 > 0$, $v_2 < 0$, $v_0 < 0$ and $|v_0| = v_1$

The results of numerical modeling in the decay point $x_d = 0$ vicinity, where $\Delta\kappa(x) = x/\ell^2$ for harmonic frequency modulation resulting in

$$\Phi(x,t) = x^2/2\ell^2 + A/\Omega\cos\left[\Omega\left(t - x/v_0\right)\right] \qquad (2)$$

are shown in Fig. 2 for $v_2 = -0.2$, $\gamma_0 = 0.65$ and $A = 1$. At high modulation frequency $\Omega > 0.5$ decay wave amplification is not affected by modulation. On contrary, at lower modulation frequency $\Omega < 0.4$ the amplification is much higher, moreover it demonstrates well pronounced modulation at frequency Ω, which has much in common to that observed in Fig. 1. The physical reason for this effect is convective losses suppression occurring when the decay point velocity, moving under pump frequency modulation, coincides with decay wave group velocity [5]. The amplification coefficient in this case, according to [5] takes form

$$S = \exp\left\{\frac{\pi\gamma_0^2\ell^2}{|v_1 - v_d||v_2 - v_d|}\right\} \qquad (3)$$

where v_d is the decay point velocity. In the case the frequency of wave a_2 is prescribed by thermal fluctuations, modeled by the source term in (1), the decay point velocity is given by simple condition $v_d = -\frac{2\ell^2}{v_0}\frac{d\omega_0}{dt}$. For harmonic frequency modulation (2) the resonant increasing of amplification is expected at $\frac{2\ell^2}{v_0} A\Omega \cos\left[\Omega\left(t - x/v_0\right)\right] = v_2$. At $v_2 v_0 < 2\ell^2 A\Omega$ such increases should occur twice each period of modulation, but the strongest amplification is foreseen for $A\Omega = \frac{v_2 v_0}{2\ell^2}$ when the two resonances are overlapped. The last conclusion is in a nice agreement with above experimental observation $\Delta f\, f_m$=const (see **Fig. 1** a,b). Strictly speaking expression (3) is valid for linear frequency modulation resulting in constant decay point speed. In case of arbitrary modulation the substitution $a_j = b_j \exp\left\{ i \frac{v_k v_0}{(v_j - v_0)(v_k - v_j)} \frac{(x - v_j t)^2}{2\ell^2} \right\}$, $k \neq j$ and transition to the moving coordinate system $y = x - v_0 t$ converts system (1) into the form suitable for analytical treatment:

$$\frac{\partial b_1(y,t)}{\partial t} + (v_1 - v_0)\frac{\partial b_1(y,t)}{\partial y} = \gamma_0 b_2(y,t) e^{i\phi(y)}$$

$$\frac{\partial b_2(y,t)}{\partial t} + (v_2 - v_0)\frac{\partial b_2(y,t)}{\partial y} = \gamma_0^* b_1(y,t) e^{-i\phi(y)}$$

where $\phi(y) = \frac{y^2}{2\ell_*^2} + \delta\phi(y)$, $\ell_*^2 = \ell^2 \frac{(v_1 - v_0)(v_2 - v_0)}{v_1 v_2}$ and damping, source terms are omitted. The amplification coefficient is estimated from this system in WKB approximation as:

$$S = \exp\left(\int_{y_1}^{y_1} \sqrt{\frac{1}{\ell_d^2} - \frac{1}{4}\left(\phi'(y)\right)^2}\, dy \right) \qquad (4)$$

where $\ell_d^2 = \gamma_0^{-2}(v_1 - v_0)(v_2 - v_0)$ and y_i are zeros of the square root. In case $\delta\phi = \frac{y^4}{12L^4}$, modeling situation of the most effective interaction for harmonic modulation, we compare (see **Fig. 3**) expression (4) predictions and computational results based on (1).

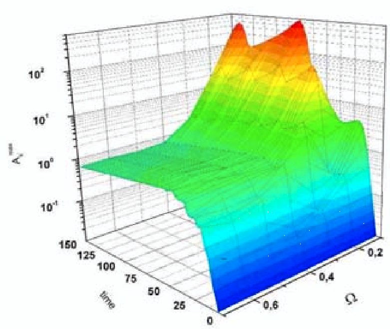

Fig. 2 *Decay wave amplitude dependence on time and modulation frequency Ω for V_1=1.0, V_2=-0.2, γ_0=0.65*

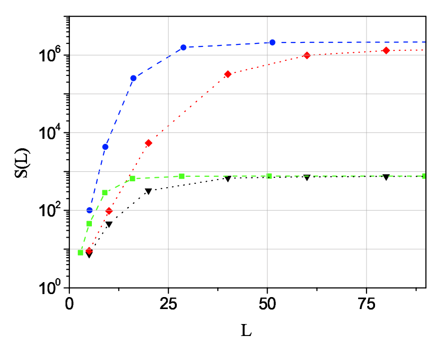

Fig. 3 *Amplification coefficient dependence on the nonlinear phase mismatch parameter L for code calculations: Vd=0.0, Vd=-0.12, analytical results: Vd=0.0, Vd=-0.12*

It should be stressed that resonant enhancement of convective amplification coefficient is possible not only for linear or harmonic pump frequency modulation, but also for stochastic modulation as it was shown in numerical modeling. The pump phase modulation was taken in the form:

$$\delta\Phi_\Omega(x,t) = -(32\pi)^{1/4}\left(\tau_c/T\right)^{1/2} \Delta \sum_{j=1}^{\infty} \exp\left\{-\left(\omega_j \tau_c\right)^2/4\right\} \cos\left(\omega_j\left(t - x/v_0\right) + \vartheta_j\right)/\omega_j$$

where $\omega_j = 2\pi j/T$, T - time longer then any time scale of the problem, ϑ_j - random phase from $[0,2\pi]$ interval. This representation provides statistically uniform, gaussian frequency modulation possessing correlation function $\langle \delta\Omega(t)\delta\Omega(t')\rangle = \Delta^2 \exp\left(-(t-t')^2/2\tau_c^2\right)$, $\langle \delta\Omega \rangle = 0$, $\left\langle \delta\Omega^2 \right\rangle^{1/2} \equiv \Delta$. As it is seen in Fig. 4, where variation of decay wave amplitude is shown for different random phase sets, in the case of fast phase modulation ($\tau_c = 1$) amplitude growth is much slower than that associated with growth rates γ_0 or $\gamma_0\sqrt{v_2/v_1}$ and saturates at level prescribed by amplification coefficient $\exp\left[\dfrac{\pi\gamma_0^2\ell^2}{v_1 v_2}\right]$ in agreement with [2]. On contrary, for slow modulation ($\tau_c = 16$) shown in Fig. 5, fast bursts of growth are observable and average level of amplification is much higher.

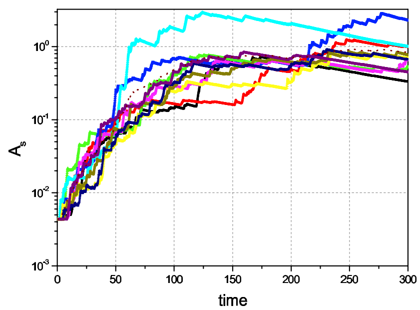

Fig. 4 *Slow wave amplitude behavior for different stochastic phase realization, τ_c=1.0, Δ=9.6*

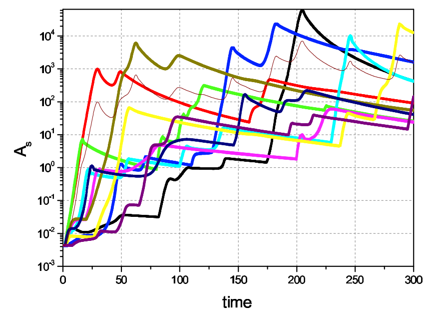

Fig. 5 *Slow wave amplitude behavior for different stochastic phase realization, τ_c=16.0, Δ=9.6*

Conclusion

It should be underlined that the conclusion of inhomogeneous plasma theory about weak influence of the pump frequency modulation on the PDI level is only applicable to the case of modulation faster than transient time of daughter waves in the decay region. If this condition is violated, the PDI enchantment become possible due to the effect of resonant suppression of convective losses from the moving decay region. This effect manifests itself in the form of short giant bursts of decay wave amplitudes. It can be used for selective excitation of PDI in experiment with chirped frequency laser pulses.

Acknowledgements

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