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ELECTRON BEAM ACCELERATION AND POTENTIAL FORMATION INDUCED BY COMPTON SCATTERING OF EXTRAORDINARY WAVES

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Abstract

It has been proved theoretically that high-energy or relativistic electron beam acceleration along and across a magnetic field and an electric field transverse to the magnetic field are induced by Compton scattering of almost perpendicularly propagating extraordinary waves.

1. Introduction

High-energy or relativistic electron beam acceleration along and across a magnetic field and the generation of an electric field transverse to the magnetic field, both induced by Compton scattering of almost perpendicularly propagating extraordinary waves, are investigated theoretically on the basis of the single-particle theory[2,3,5,6]. Compton scattering occurs via nonlinear Landau damping of two extraordinary waves interacting nonlinearly with the electron beam, satisfying the resonance condition of $\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}-(k_{\perp}-k_{\perp}')v_{\mathrm{d}}-(k_{||}-k_{||}')v_{\mathrm{b}}=m\omega_{\mathrm{ce}}$ (m=0, ±1), where v_{b} and v_{d} are the parallel and perpendicular velocities of the electron beam, respectively[1,3]. The transport equation derived from the single-particle theory shows that two extraordinary waves accelerate or decelerate the electron beam in the $\boldsymbol{k}''$ direction ($\boldsymbol{k}''=\boldsymbol{k}-\boldsymbol{k}'$). Simultaneously, an intense cross-field electric field $\boldsymbol{E}_0=\boldsymbol{B}_0\times\boldsymbol{v}_{\mathrm{d}}/c$ is generated via dynamo effect owing to perpendicular drift of the electron beam to satisfy the generalized Ohm's law, which means that this cross-field electric drift is identical to the $\boldsymbol{E}\times\boldsymbol{B}$ drift[2,4-6]. The single-particle theory is greatly useful for easy and straightforward understanding of physical mechanism of the electron beam acceleration and the generation of the cross-field electric field, although the rigorously exact transport equations are derived from Vlasov-Maxwell equations.

2. Kinetic Equation for an Electron Beam

In order to derive the transport equation, the single-particle theory for Compton scattering of extraordinary waves, which is convenient for understanding of detailed physical mechanisms, was developed. The kinetic equation for an electron beam with the velocity $\boldsymbol{v}$, which includes the cross-field electron drift velocity $\boldsymbol{v}_{\mathrm{d}}$ and the cross-field electric field $\boldsymbol{E}_0$, is given by[3]

$$m_e \mathrm{d}\boldsymbol{v}/\mathrm{d}t = -e(\boldsymbol{E}_0 + \boldsymbol{E}_1) - (e/c)\boldsymbol{v}\times(\boldsymbol{B}_0 + \boldsymbol{B}_1) \quad . \tag{1}$$

In addition it is assumed that the following generalized Ohm's law for a uniform collisionless plasma is satisfied simultaneously:

$$\boldsymbol{E}_0 + \boldsymbol{v}_d\times\boldsymbol{B}_0 /c = 0 \quad . \tag{2}$$

Here,

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}_d t + \boldsymbol{v}_{||} t + \boldsymbol{r}_k + \boldsymbol{r}_k{}^* + \boldsymbol{r}_{k'} + \boldsymbol{r}_{k'}{}^* + \boldsymbol{r}_{k''}{}^{(2)} + \boldsymbol{r}_{k''}{}^{(2)*} \quad ,$$

$$\boldsymbol{v} = \boldsymbol{v}_d + \boldsymbol{v}_{||} + \boldsymbol{v}_k + \boldsymbol{v}_k{}^* + \boldsymbol{v}_{k'} + \boldsymbol{v}_{k'}{}^* + \boldsymbol{v}_{k''}{}^{(2)} + \boldsymbol{v}_{k''}{}^{(2)*} \quad ,$$

$$\boldsymbol{E}_1 = \boldsymbol{E}_k + \boldsymbol{E}_k{}^* + \boldsymbol{E}_{k'} + \boldsymbol{E}_{k'}{}^* \quad ,$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_k + \boldsymbol{B}_k{}^* + \boldsymbol{B}_{k'} + \boldsymbol{B}_{k'}{}^* \quad ,$$

$$\boldsymbol{k}\times\boldsymbol{E}_k = (\omega_k/c)\boldsymbol{B}_k \quad ,$$

where $\boldsymbol{v}_d=c\boldsymbol{E}_0\times\boldsymbol{B}_0/B_0{}^2=(v_d , 0, 0)$, $\boldsymbol{v}_{||}=(0, 0, v_{||})$, and $E_0=(v_d/c)B_0$. The Larmor radius of an electron beam is assumed to be zero. The background uniform stationary electric and magnetic fields are given by $\boldsymbol{E}_0=(0, E_0 , 0)$ and $\boldsymbol{B}_0=(0, 0, B_0)$. $\boldsymbol{E}_k=\boldsymbol{E}_k{}^{(1)}$ and $\boldsymbol{B}_k=\boldsymbol{B}_k{}^{(1)}$ are the first-order electric and magnetic fields of extraordinary waves, respectively, and $\boldsymbol{r}_k=\boldsymbol{r}_k{}^{(1)}+\boldsymbol{r}_k{}^{(3)}$, and $\boldsymbol{v}_k=\boldsymbol{v}_k{}^{(1)}+\boldsymbol{v}_k{}^{(3)}$ are the first- and third-order oscillating terms of an electron beam. The beat-wave is assumed to be absent, that is, $\boldsymbol{E}_{k''}{}^{(2)}=\boldsymbol{B}_{k''}{}^{(2)}=0$. It is noted that $\boldsymbol{v}_d$ is identical to the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity.

Equation (2) means that $\boldsymbol{v}_d$ is determined by $\boldsymbol{E}_0$ and $\boldsymbol{B}_0$. Simultaneously it can be also stated that the cross-field electric field $\boldsymbol{E}_0$ is enhanced or suppressed when the mechanism of acceleration and deceleration of the electron beam exists. Namely Eq. (2) shows $\boldsymbol{E}_0$ caused by the dynamo effect of $\boldsymbol{v}_d$. Accordingly we find that the cross-field electric field $\boldsymbol{E}_0$ can be generated by the cross-field electron beam acceleration due to Compton scattering of extraordinary waves[2,4-6]. The dynamo effect results from the electrical charge separation caused by the Lorentz force $-e\boldsymbol{v}_d\times\boldsymbol{B}_0/c$ so that the created electric field $\boldsymbol{E}_0=\boldsymbol{B}_0\times\boldsymbol{v}_d/c$ is balanced with the Lorentz force[4,6].

From Eqs. (1) and (2) we can get the following fourth-order kinetic equations:

$$m_e \mathrm{d}\boldsymbol{v}^{(4)}/\mathrm{d}t = e\boldsymbol{D} \quad , \tag{3}$$

$$\boldsymbol{E}_0{}^{(4)} + \boldsymbol{v}_d{}^{(4)}\times\boldsymbol{B}_0 /c = 0 \quad , \tag{4}$$

$$\boldsymbol{D} = -(\boldsymbol{k}\cdot\boldsymbol{v}_k{}^{(3)})\boldsymbol{E}_k{}^*/\Omega_k - (\boldsymbol{k}'\cdot\boldsymbol{v}_{k'}{}^{(3)})\boldsymbol{E}_{k'}{}^*/\Omega_{k'} + (\boldsymbol{k}'\cdot\boldsymbol{v}_k{}^{(1)})(\boldsymbol{k}'\cdot\boldsymbol{v}_{k''}{}^{(2)*})\boldsymbol{E}_{k'}{}^*/\Omega_k\Omega_{k''}{}^*$$
$$- (\boldsymbol{k}\cdot\boldsymbol{v}_{k'}{}^{(1)})(\boldsymbol{k}\cdot\boldsymbol{v}_{k''}{}^{(2)})\boldsymbol{E}_k{}^*/\Omega_{k'}\Omega_{k''} - \boldsymbol{v}_k{}^{(3)}\times\boldsymbol{B}_k{}^*/c - \boldsymbol{v}_{k'}{}^{(3)}\times\boldsymbol{B}_{k'}{}^*/c$$

$$- \boldsymbol{v}_{\mathbf{k}''}{}^{(2)}\times[(\boldsymbol{k}\cdot\boldsymbol{v}_{\mathbf{k}'}{}^{(1)})\boldsymbol{B}_{\mathbf{k}}{}^{*}/\Omega_{\mathbf{k}'} \;+\; (\boldsymbol{k}'\cdot\boldsymbol{v}_{\mathbf{k}}{}^{(1)*})\boldsymbol{B}_{\mathbf{k}'}/\Omega_{\mathbf{k}}]\,/c$$
$$+\boldsymbol{v}_{\mathbf{k}}{}^{(1)}\times(\boldsymbol{k}'\cdot\boldsymbol{v}_{\mathbf{k}''}{}^{(2)*})\boldsymbol{B}_{\mathbf{k}'}{}^{*}/c\Omega_{\mathbf{k}''}{}^{*} \;-\; \boldsymbol{v}_{\mathbf{k}'}{}^{(1)}\times(\boldsymbol{k}\cdot\boldsymbol{v}_{\mathbf{k}''}{}^{(2)})\boldsymbol{B}_{\mathbf{k}}{}^{*}/c\Omega_{\mathbf{k}''} \quad , \tag{5}$$

where $\Omega_{\mathbf{k}}=\omega_{\mathbf{k}}-k_\perp v_d-k_{||}v_{||}$, $\Omega_{\mathbf{k}'}=\omega_{\mathbf{k}'}-k_\perp' v_d-k_{||}' v_{||}$, $\Omega_{\mathbf{k}''}=\omega_{\mathbf{k}''}-k_\perp'' v_d-k_{||}'' v_{||}$, and $\boldsymbol{v}^{(1)}$, $\boldsymbol{v}^{(2)}$ and $\boldsymbol{v}^{(3)}$ are given by the first-, second- and third-order kinetic equations of Eq. (1). It can be proved easily that $D_z/D_x=k_{||}''/k_\perp''$ and $D_y=0$ ($k_y=k_y'=k_y''=0$), thereby the kinetic equation (3) demonstrates that the acceleration or deceleration of the electron beam occurs only in the $\boldsymbol{k}''$ direction. Equation (4) shows the fourth-order cross-field electric field caused by the change of the cross-field drift velocity ($E_0{}^{(4)}=(v_d{}^{(4)}/c)B_0$), that is, $\boldsymbol{E}_0{}^{(4)}$ is generated by the dynamo effect of $\boldsymbol{v}^{(4)}$, as has been clarified previously[2,4-6]. The first four terms on the right-hand side of Eq. (5) express the pondermotive force due to the wave electric field (E^x, E^z), and the remaining five terms result from the nonlinear $\boldsymbol{v}\times\boldsymbol{B}$ Lorentz force due to the wave magnetic field (B^x, B^z) or the wave electric field (E^y).

3. Electron Beam Acceleration

First, we consider the Compton scattering of $m=0$. In Eqs. (3) and (5) only the nonlinear singularity $\omega_{\mathbf{k}''}-k_\perp''v_d-k_{||}''v_{||}=0$ should contribute to the acceleration and deceleration of the electron beam. Assuming the drifted Maxwellian velocity distribution function of the electron beam with the drift velocities of $\boldsymbol{v}_b$ and $\boldsymbol{v}_d$, the transport equation indicating the temporal evolution of the momentum density of the electron beam can be derived from integrating Eq. (3) multiplied by $n_b g_b$ in the velocity-space[3]. It shows that the acceleration rate of the electron beam is proportional to $\boldsymbol{k}''h_{0\mathbf{k}''}\exp(-h_{0\mathbf{k}''}{}^2)$, where $h_{0\mathbf{k}''}=(\omega_{\mathbf{k}''}-k_\perp''v_d- k_{||}''v_b)/k_{||}''v_{tb}$. Namely, the acceleration or deceleration of the electron beam occurs in the $\boldsymbol{k}''$ direction. Thus, we see that when $(\omega_{\mathbf{k}''}-k_\perp''v_d)/k_{||}''>v_b$ ($h_{0\mathbf{k}''}>0$), the electron beam can be accelerated efficiently. On the contrary, when $(\omega_{\mathbf{k}''}-k_\perp''v_d)/k_{||}''<v_b$ ($h_{0\mathbf{k}''}<0$), the electron beam can be decelerated efficiently. For $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}<\omega_h$ (ω_h being the upper-hybrid frequency), the term due to the pondermotive force becomes dominant. For $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}>\omega_R$ (ω_R being the right-hand cutoff frequency), the term due to the nonlinear $\boldsymbol{v}\times\boldsymbol{B}$ becomes dominant. Accordingly, it is found that the efficient acceleration and deceleration of the electron beam occur when $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}<\omega_h$ and $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}>\omega_R$, as has been shown previously[3].

Second, we consider the Compton scattering of $m=\pm1$. In Eqs. (3) and (5) the nonlinear singularity of $\omega_{\mathbf{k}''}-k_\perp''v_d-k_{||}''v_{||}-m\omega_{ce}=0$ should provide another contribution to the acceleration and deceleration of the electron beam. The obtained transport equation shows that the acceleration rate of the electron beam is proportional to $\boldsymbol{k}''\exp(-h_{m\mathbf{k}''}{}^2)$, where $h_{m\mathbf{k}''}=(\omega_{\mathbf{k}''}-k_\perp''v_d-k_{||}''v_b-m\omega_{ce})/k_{||}''v_{th}$. This states that the acceleration of the electron beam occurs always in the $\boldsymbol{k}''$ direction in the vicinity of $h_{m\mathbf{k}''}\cong0$. Moreover, it is found in the same way as the case of m=0 that the efficient acceleration of the electron beam takes place when $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}<\omega_h$ and $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}>\omega_R$.

4. Conclusion

It was verified theoretically on the basis of the single-particle theory that the high-energy or relativistic electron beam acceleration along and across the magnetic field and an cross-field electric field can be induced by Compton scattering of almost perpendicularly propagating extraordinary waves for $|h_{0\mathbf{k}''}|\cong 1$ and $h_{m\mathbf{k}''}\cong 0$ (m=±1). The cross-field electric field $\boldsymbol{E}_0=\boldsymbol{B}_0\times\boldsymbol{v}_d/c$ is generated via the dynamo effect due to the perpendicular acceleration of the electron beam to satisfy the generalized Ohm's law. The efficient acceleration and deceleration of the electron beam in the $\boldsymbol{k}''$ direction result from the pondermotive force when $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}<\omega_h$, and from the $\boldsymbol{v}\times\boldsymbol{B}$ nonlinear force when $\omega_{\mathbf{k}},\omega_{\mathbf{k}'}>\omega_R$.

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