

Behaviour of Rotation and Radial Electric Field in Collision Dominated Plasma Edge with Steep Gradients

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Tokamak experimental results show evidence of a close connection between the sudden onset of a poloidal rotation in the edge regions of plasma and the transition to the “H-mode” confinement regime. As expected, reversion of the rotation velocity is also tied to a transition from the “H-mode” to the “L-mode” with a drastic change in the confinement characteristics. Various examinations have revealed that the spin-up tendencies can be driven by any mechanism [1-3], that would tend to favor particle accumulation on the outer portions of a tokamak magnetic surface, relative to the inner portion via particle, momentum or energy sources. Plasma rotation is always accompanied by a radial electric field, whose origin seems to be due to various competing effects. A further consequence of an unstable rotation is that, a poloidally asymmetric particle transport may also render the radial electric field unstable [3].

The more recent observations indicate an important change in the rotation velocity during the rise of a transport barrier in the main body of the plasma. To accommodate the steep gradients of density and temperature near the transport barriers and the plasma edge within the theory, the revisited neoclassical theory [4-6] has introduced shorter gradient lengths, i.e., $L_{\perp} \ll r \ll qR$. Furthermore, it was necessary to use the Mikhailowskii-Tsypin corrections [7] to the Braginskii’s stress tensors. To analyse such poloidal or toroidal rotation related phenomena in a collision dominated toroidal plasma near steep gradients within the framework of revisited neoclassical theory, we use the fluid equations including mass and momentum sources. When the parameter $\Lambda_1 \equiv (v_i / \Omega_i)(q^2 R^2 / rL_{\psi})$ is larger than 1/3, then the revisited theory introduces additional terms into the parallel momentum equation [4-6]. As a result, one obtains the ambipolarity equation,

$$m_i N_i \frac{\partial U_{\phi,i}^{(1)}}{\partial t} = \frac{\partial}{\partial r} \left[\eta_{2,i} \left(\frac{\partial U_{\phi,i}^{(1)}}{\partial r} - \frac{0.107 q^2}{1+Q^2/S^2} \frac{\partial \ln T_i}{\partial r} \frac{B_\phi}{B_\theta} U_{\theta,i}^{(2)} \right) \right] \quad (1)$$

$$+ J_r B_\theta - m_i \oint \frac{d\vartheta}{2\pi} h^2 S_i^N U_{i\varphi} + \oint \frac{d\vartheta}{2\pi} h^2 \vec{S}_i^M \cdot \vec{e}_\varphi$$

where, $h = 1 + (r/R_0) \cos \vartheta$, $Q = [4B_\phi U_{\theta,i}^{(2)} - 2.5(T_i/e_i) \partial \ln N_i^2 T_i / \partial r] B^{-1}$,

$S = (2r \chi_{\parallel,i} N_i^{-1}) / q^2 R^2$; J_r is radial polarization current, and parallel heat diffusion coefficient is $\chi_{\parallel,i} = 3.9 P_i / m_i v_i$. Similarly, one obtains an equation given in (2) for the poloidal velocity for circular cross sections, including also the time derivative of the poloidal velocity:

$$m_i N_i (1 + 2q^2) \frac{B_\theta}{B_\phi} \frac{\partial U_{\theta,i}^{(2)}}{\partial t}$$

$$= \frac{3\eta_{0,i}}{2R^2} \left[U_{\theta,i}^{(2)} + 1.833 (e_i B_\phi)^{-1} \frac{\partial T_i}{\partial r} \right] - 0.54 \frac{\eta_{2,i}}{1+Q^2/S^2} q^2 \frac{e_i B_\phi}{T_i} \frac{\partial \ln T_i}{\partial r} \left[\frac{T_i}{e_i B_\theta} \frac{\partial U_{\phi,i}^{(1)}}{\partial r} \right] \quad (2)$$

$$+ \frac{1}{2} U_{\phi,i}^{(1)2} - U_{\phi,i}^{(1)} \frac{B_\phi}{B_\theta} \left(U_{\theta,i}^{(2)} - \frac{T_i}{e_i B_\phi} \frac{\partial \ln N_i^2 T_i}{\partial r} \right) + 1.90 \frac{B_\phi^2}{B_\theta^2} \left(U_{\theta,i}^{(2)} - 0.8 \frac{T_i}{e_i B_\phi} \frac{\partial \ln N_i^{1.6} T_i}{\partial r} \right)^2$$

$$+ J_r B_\phi$$

The term with the time derivative on l.h.s. was omitted in the revisited theory [6] as it is of a smaller order. However, this term gains importance on a faster time scale, for example, during the poloidal spin-up this will be the governing term. Indeed, the time scale for poloidal spin-up is much shorter than the toroidal one. Thus, we have a coupled system of partial differential equations which governs the both rotation velocities near edge and a steep gradient. Hence, the coupled poloidal and toroidal rotations here represent a multiple time scale process. The Pfirsch-Schlüter factor for inertia enhancement on the left depends on the collisional regime and can be modified for the plateau regime [8]. Transferring to a faster time scale defined by $\tau = t/\epsilon$ these two equations can be rewritten shortly as,

$$m_i N_i \frac{\partial U_\phi}{\partial t} = \frac{\partial}{\partial \xi} \left(\eta_{2,i} \frac{\partial U_\phi}{\partial \xi} \right) - \frac{\partial}{\partial \xi} (\eta_{2,i} F(\xi, U_\theta)) \quad (3)$$

$$\varepsilon(1+2q^2)m_iN_i \frac{B_\theta}{B_\phi} \frac{\partial U_\theta}{\partial t} = G_1(\xi, U_\theta) - G_2(\xi, U_\theta)U_\phi + G_3(\xi, U_\theta)U_\phi^2 + G_4(\xi, U_\theta) \frac{\partial U_\phi}{\partial \xi} \quad (4)$$

where stretched variable ξ is defined as $\xi = (r - r_{sep}) / L_\psi$ and $0 < \varepsilon \ll 1$. On the faster time scale $\tau = t / \varepsilon$, the upper equation yields in the lowest order: $\partial U_\phi / \partial \tau = 0$. The second equation becomes an ordinary diff. equation in τ , which can be integrated as

$$\tau = \int dU_\theta \left[\frac{1 + K(U_\theta + L)^2}{aU_\theta^3 + bU_\theta^2 + cU_\theta + d} \right] + \text{Const.} \quad (5)$$

where K, L, a, b, c, d are functions of position that are assumed to be slowly varying.

Hence, the τ is found as

$$\tau = A \ln(U_\theta - a) + B \ln(U_\theta - b) + C \ln(U_\theta - c) + \text{Constant} \quad (6)$$

provided that the denominator of the integral has three real roots. If there is only one real root then the solution can be written as $\tau = A \ln(U_\theta - a) + B \ln(U_\theta^2 + b^2) + \text{Constant}$. The stability requirement for U_θ on the fast time scale ($\tau > 0$) can now be stated as follows:

i) In the three real roots case, U_θ may approach only to the largest of a, b, c . If its particular coefficient among $A, B,$ and C is negative, then there may be a stable solution for U_θ approaching to this largest root for growing time, provided it is not too far away from this limit initially. A more precise description would depend on the numerical values of all coefficients.

ii) In the single real root case, the stability of U_θ depends only on the sign of A . Namely, for $A < 0$, U_θ approaches to the limit a for growing τ if its initial value is not too far away from this limit.

The problem must also be analysed in the slower time scale t . In this limit, we can drop the time derivative on the left hand side of Eqn. 4. The resulting two equations can be treated as a

quasilinear partial differential equation as it was done by Daybelge, et.al. [9]. The stability of U_θ and U_φ , then, was shown to depend on various coefficients in the resulting equation.

The steady state solutions which are of great interest can be also calculated numerically as in [10-11]. A representative case is shown below in Fig. 1.

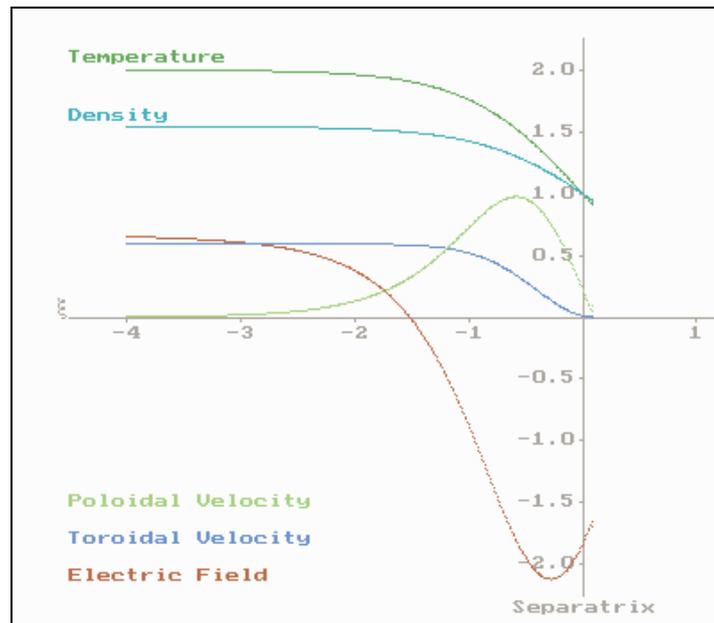


Figure 1. Steady state solutions of the toroidal and parallel momentum equations yielding normalized toroidal and poloidal velocities, U_φ , U_θ (For details, see [10]).

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