

Simulation of Electromagnetically Induced Transparency in Plasma

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1. Introduction

Electromagnetically induced transparency (EIT) means that an electromagnetic wave is enabled to propagate through an otherwise opaque medium by the interaction with a second wave. In plasma, the coupling mechanism is a modulation of the plasma frequency, which determines the refractive properties of the plasma. This can be due to a relativistic increase of the electron mass, or due to a variation in electron density caused by longitudinal plasma oscillations driven by the ponderomotive force associated with the beat of the waves.

The original study of EIT by Harris [1] employs a three-wave model, incorporating two transverse electromagnetic waves – with frequencies $\omega_0 > \omega_p$, $\omega_s < \omega_p$, where ω_p is the plasma frequency – and a longitudinal plasma wave at the difference frequency $\omega_- = \omega_0 - \omega_s$. It predicts transparency if the latter is slightly lower than ω_p . More recent investigations of EIT have addressed three issues:

- 1) Matsko *et al.* [2], and Gordon *et al.* [3] found that the conditions for EIT are affected by plasma oscillations at the anti-Stokes frequency $\omega_a = \omega_0 + \omega_-$ as well, whereas in the original model scattering of the wave at ω_0 occurs only into the Stokes wave at ω_s .
- 2) Gordon *et al.* [3] also studied the possibility to achieve transparency in finite plasma; it proved difficult to establish the crucial phase relations between the waves starting from a surface.
- 3) While Refs. [1-3] assume that the amplitude of the wave at the lower frequency is small, we have started to explore the conditions of EIT for **two** waves of **comparable**, weakly relativistic amplitudes [4].

2. Coupled Propagation in Plasma

2.1. Relativistic Fluid Equations

We start from a description of the plasma as cold electron fluid. We take the relativistic corrections to the electron mass into account since they are of the same order of magnitude as the ponderomotive coupling. In one dimension the relevant equations read (see *e.g.* [5]):

$\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} - \frac{n}{\gamma} \right) \bar{a} = \bar{0}$	$\frac{\partial}{\partial t} E = \frac{n}{\gamma} p$	$\frac{\partial}{\partial z} E = 1 - n$	$\frac{\partial}{\partial t} p = -E - \frac{\partial}{\partial z} \gamma$
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where \bar{a} is the transverse vector potential, scaled by mc^2/e , E is the longitudinal electric field, scaled by $\omega_p mc^2/e$, p is the longitudinal momentum, scaled by mc , n is the electron density, scaled by the unperturbed density n_0 , and $\gamma = \sqrt{1 + \bar{a}^2 + p^2}$ is the Lorentz factor.

Time is scaled by $1/\omega_p$, and length by c/ω_p , where the plasma frequency is given by

$\omega_p = \sqrt{4\pi n_0 e^2 / m}$, m and e are the electron mass and charge, and c is the vacuum speed of light.

We assume a vector potential consisting of three waves, the pump \vec{a}_0 , and Stokes / Anti-Stokes waves $\vec{a}_{s,a}$, respectively:

$$\vec{a} = \vec{a}_0 + \vec{a}_s + \vec{a}_a \quad \vec{a}_j = a_j [\cos(\theta_j)\vec{e}_x + \sin(\theta_j)\vec{e}_y] \quad \theta_j = k_j z - \omega_j t + \varphi_j$$

We choose circularly polarized waves in order to avoid harmonics of their frequencies, and to restrict the beat wave to the difference frequency and wave number (for co-rotating waves):

$$\omega_- \equiv \omega_a - \omega_0 = \omega_0 - \omega_s \quad k_- \equiv k_a - k_0 = k_0 - k_s$$

Frequencies ω_j are scaled by ω_p , and wave numbers k_j by ω_p / c .

2.2. Expansion in Powers of the Vector Potential

We expand the quantities E , p , and n corresponding to longitudinal plasma waves up to second order in the vector potential a . To this order, we find that the density modulations are driven by the relativistic ponderomotive force:

$$\left(\frac{\partial^2}{\partial t^2} + 1 \right) (n - 1) \approx \frac{\partial^2}{\partial z^2} \gamma \quad \gamma \approx \sqrt{1 + \vec{a}^2} \approx 1 + \vec{a}^2 / 2$$

For the above vector potential, we find, neglecting terms of second order in $a_{s,a}$, which are assumed small compared to the pump:

$$\vec{a}^2 \approx a_0^2 + 2a_0(a_s + a_a)\cos(\theta_-) \quad n = 1 + a_0(a_s + a_a)\frac{k_-^2}{\omega_-^2 - 1}\cos(\theta_-)$$

2.3. Dispersion Relations

Substituting the expressions for n and γ into the equation for \vec{a} and retaining only terms up to first order in $a_{s,a}$, we arrive at dispersion relations for the three transverse waves:

$$D_0 = 0 \quad D_s a_s = Q(a_s + a_a) \quad D_a a_a = Q(a_s + a_a)$$

with

$$D_j = k_j^2 - \omega_j^2 + 1 - \alpha \quad Q = \alpha \frac{k_-^2 - \omega_-^2 + 1}{1 - \omega_-^2} \quad \text{and } \alpha = a_0^2 / 2.$$

The factors D_j contain the reduction of the plasma frequency due to the relativistic mass increase, while Q accounts for the modifications of the refractive properties due to the beat wave. The solvability of the equation system for $a_{s,a}$ requires

$$\begin{aligned} (D_s - Q)(D_a - Q) - Q^2 &= D_s D_a - Q(D_s + D_a) = 0 \\ (1 - \omega_-^2) \left[(k_-^2 - \omega_-^2)^2 - 4(k_0 k_- - \omega_0 \omega_-)^2 \right] - 2\alpha (k_-^2 - \omega_-^2)(k_-^2 - \omega_-^2 + 1) &= 0 \end{aligned}$$

For given ω_0 , α ($\Rightarrow k_0$), and ω_- , this is a fourth-order equation for k_- . For transparency, at least one of the roots must be real (for real frequencies).

For comparison, if the Anti-Stokes wave can be neglected, the dispersion relation becomes

$$D_s - Q = 0 \quad \left| \quad (1 - \omega_-^2) \left[(k_-^2 - \omega_-^2) - 2(k_0 k_- - \omega_0 \omega_-) \right] - \alpha (k_-^2 - \omega_-^2 + 1) = 0 \right.$$

which is a quadratic equation for k_- and can be solved analytically. Close to the plasma resonance, $\omega_- = 1 + \delta$, $|\delta| \ll 1$, this equation predicts a transparency band for $\delta < 0$. However, Gordon *et al.* [3] pointed out that in the vicinity of the resonance the Stokes and Anti-Stokes amplitudes are of comparable magnitude, so that the simplified dispersion relation cannot be used there.

The fourth-order equation has approximate solutions $k_- = \pm 1 \mp \delta \left[2(\omega_0 \mp k_0)^2 / \alpha + 1 \right]$, $k_- = \pm \sqrt{-\delta \left[(4\omega_0^2 - 1) / \alpha - 2 \right]}$ in this regime. Note that the second pair is real for negative δ .

3. Numerical Simulation

3.1. Hydrodynamic Code

In order to check the assumptions and approximations made in the analytical theory, we performed numerical simulations, using a code implementing the fully relativistic hydrodynamic equations for the plasma motion, and solving the electric field propagation in a leapfrog scheme. Compared to Particle-in-Cell simulations, results from a hydrodynamic simulation should contain less noise, while a one-dimensional system does not yet require excessive computation times.

3.2. Boundary Conditions

In the simulations, the plasma was set up using two different boundary conditions:

- Either the plasma region was bounded by short regions of (almost) zero density, so that electromagnetic waves incident from vacuum could be simulated.
- Or the plasma region extended over the entire simulation range; at both ends the physical quantities outside this range were assumed to be mirror images of those inside. This allowed to treat the hypothetical case where the electromagnetic waves are generated inside the plasma, and effects of the vacuum-plasma interface can be neglected.

In either case, the initial electron density was homogeneous (inside the plasma for a)), and equal to the (constant) ion density. The electromagnetic fields were applied as boundary values, typically as superposition of oscillations at two (or three) distinct frequencies.

4. Results and Conclusions

In general, our simulations confirmed the findings of Gordon *et al.* [3]: although theoretically possible in infinite plasma, EIT in a bounded system appears to be very hard or impossible to achieve. Typically, close to the surface or source of the waves, plasma oscillations upshift part of the Stokes wave to the pump or Anti-Stokes frequencies, which can freely propagate, so that the corresponding energy is carried away. This can be seen in Fig.2.

In Ref. [3], the argument is given that the Stokes wave cannot effectively generate the necessary density modulations. We tried to use a short intense pulse to generate a wake and thus pre-form the plasma; this should provide the approximate modulations for the case $k_- \approx 1 \approx \omega_-$. The results were, however, not decisively different from the case without wake. It may be possible to improve the matching of the beat wave and the wake.

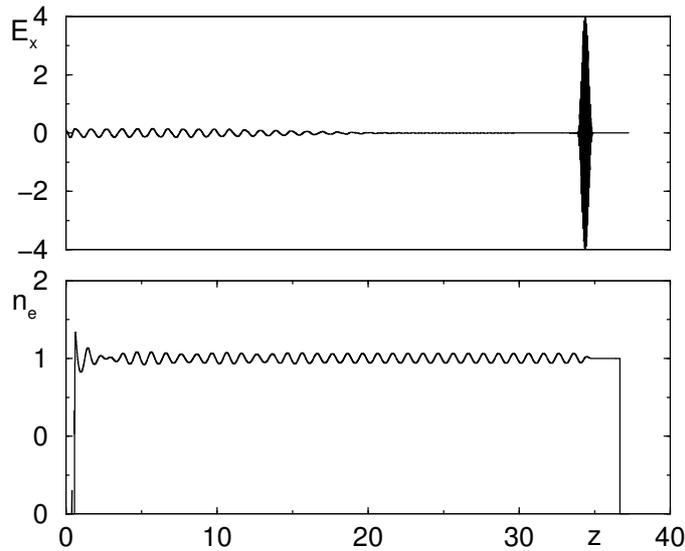


Figure 1: Snapshots of transverse electric field, propagating from left to right (top), and electron density (bottom). Note the short pulse generating the wake.

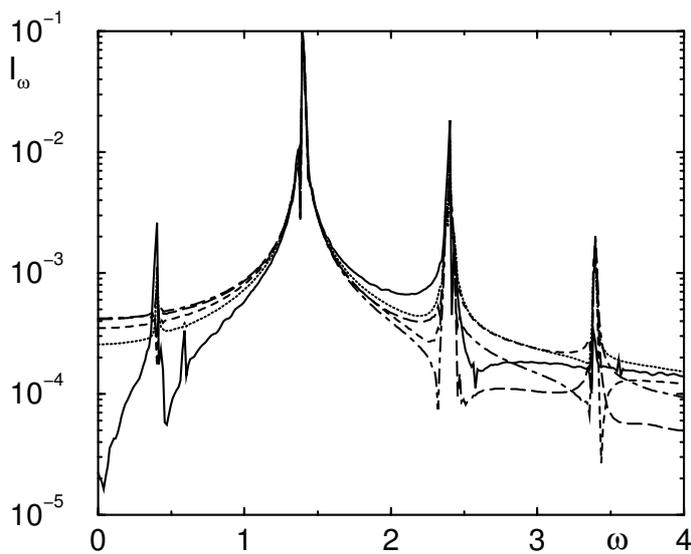


Figure 2: Spectral intensity of the transverse electric field at equidistant positions from the surface; the order is solid=0, dotted= d , dashed= $2d$, long-dashed= $3d$, dot-dashed= $4d$; $d=3.66$

References

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