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Phase Space Reconstruction of Current-Driven Instabilities in a Magnetised Plasma Column

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1. Introduction

When an electron current is drawn along the field lines of a magnetised plasma column, a number of high amplitude, strongly nonlinear ion instabilities can be excited. We have intensively investigated the two most important types of these instabilities in the Innsbruck Q-machine, i.e., the **p**otential **r**elaxation **i**nstability (PRI) and the **e**lectrostatic **i**on-**c**yclotron **i**nstability (EICI) [1]. Both instabilities have been excited simultaneously by drawing an electron current to a heated tantalum collector. There is a strong interaction between the two instabilities, which leads to an amplitude and frequency modulation of the EICI by the PRI.

Here we would like to present the results of a nonlinear dynamical analysis of the observed phenomena. For this purpose, we have recorded the time series of the collector current with a sampling rate of 500 kHz. For analysing the dynamics in the state space of our plasma system, we do not need the derivatives to form a coordinate system which describes the structure of orbits in the phase space. Instead, we can use directly the so-called time-advanced variables. The space constructed in this way is called the reconstructed space. After a sensible choice of the time delay and the embedding dimension for each of our signals, we have plotted the reconstructed space, thereby obtaining 3D Poincaré maps. Expectedly, we obtain torus-shaped maps for the cases where both instabilities appear simultaneously with similar amplitudes.

2. Experimental results

The experiments have been performed in the Innsbruck single-ended Q machine in a potassium plasma, produced on a 6 cm diameter tungsten hot plate, heated to about 2200 K. A circular tantalum limiter, inserted 3.6 cm in front of the hot plate, reduces the diameter of the plasma column to 3.5 cm, thus providing a flatter radial density profile. The distance between the plasma source (the hot plate) and the collector is 27.5 cm. The plasma density was in the range $10^8 < n_{pl} < 10^9$ cm^{-3}, and the ion and electron temperatures where $T_i \cong T_e \cong$ 0.2 eV. The confining magnetic field was $0.05 < B < 0.2$ T.

Both instabilities were excited simultaneously by drawing an electron current to a 10 mm diameter heated tantalum collector [2]. By slowly increasing the voltage on the collector, for $B = 0.13$ T, first the EICI appears with 67 kHz approximately, later also PRI appears with about 12 kHz. In a certain range there is a strong interaction between the two instabilities, which leads to an amplitude and frequency modulation of the EICI by PRI (see Fig. 1). The mechanism at the origin of this modulation was already described earlier [3].

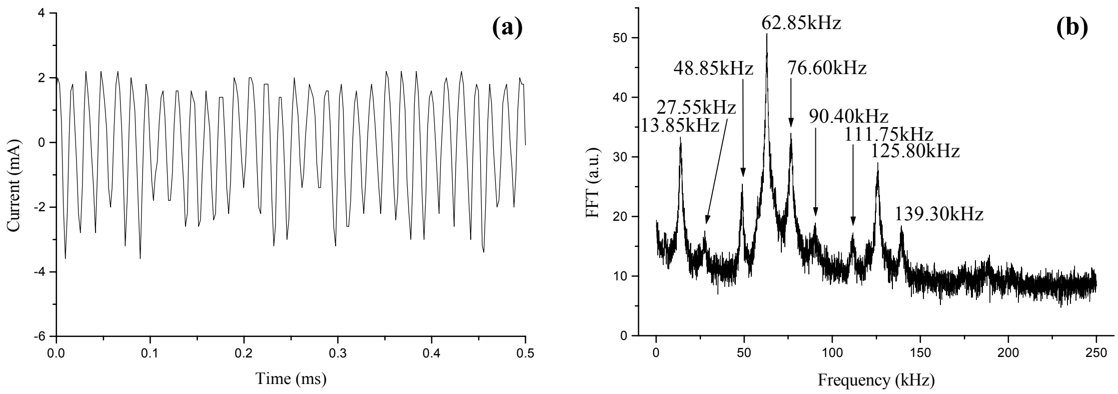

Fig. 1: Time series (a) and FFT (b) of the ac component of the current through collector in the case where both instabilities are excited simultaneously

3. Nonlinear dynamical analysis

The nonlinear dynamical analysis provides us with powerful tools for analysing the evolution of a nonlinear system such as time history, histograms, Fourier spectra, state space plots, Poincaré maps, autocorrelation functions, Lyapunov exponents, dimension calculations, etc. Since the phenomena studied here are obviously strongly nonlinear, we presume that a nonlinear dynamical analysis of the ac components of the collector current can offer an excellent insight into the state space dynamics of our system. For this purpose, we have recorded the time series of the collector current with a sampling rate of 500 kHz delivering 15000 points in 0.03 s, i.e., the sampling time was $\tau_s = 2$ μs.

For analysing the dynamics in the state space of our system, we use the observations of Packard et al. [4], Ruelle [5] and Takens [6], which concluded that we do not need the derivatives to form a coordinate system which describes the structure of orbits in phase space. Instead, we can use directly the time advanced variables $s(t + n\tau)$, where $n = 1,2,\ldots,d$, and

$\tau = k\tau_s$ is an appropriately chosen time delay, and we can define the so-called *delay-coordinate vectors*:

$$y_n = \{s(t_0 + n\tau_s), s(t_0 + n\tau_s + k\tau_s),\ldots, s[t_0 + n\tau_s + k(d-1)\tau_s]\}^T \qquad (1)$$

or

$$y_n = \{s_n, s_{n+k}, s_{n+2k}, \ldots, s_{n+(d-1)k}\}^T, \qquad (2)$$

where $s_n \equiv s(t_0 + n\tau_s)$, $s_{n+k} \equiv s(t_0 + n\tau_s + k\tau_s)$ etc.

The space constructed by using the vectors y_n is called the *reconstructed space*. According to a theory of Takens [6] and Mané [7], the geometric structure of the dynamics of the system, from which the s_n were measured, can be observed in the reconstructed d-dimensional Euclidean space if $d \geq 2d_a + 1$, where d_a is the dimension of the attractor of interest. The parameter τ is called *time delay*, the integer d is called the *embedding dimension*, the constructed coordinates are called *delay coordinates*, and this method of constructing coordinates is called the *method of delays*.

There are many systematic approaches for choosing the time delay and the embedding dimension. In our analysis we have used two methods to determine the time delay and one method to determine the embedding dimension. The two methods to determine the time delay, which provide us with the same results, are the autocorrelation function (Fig. 2) and the average mutual information. To determine the embedding dimension we have chosen the method of *false nearest neighbours*, proposed by Kennel, Brown and Abarbanel [8]. Having the time delay and embedding dimension for each of our signals we can plot the reconstructed space, using the method of delays. Fig. 3 shows the 3D Poincaré maps through the

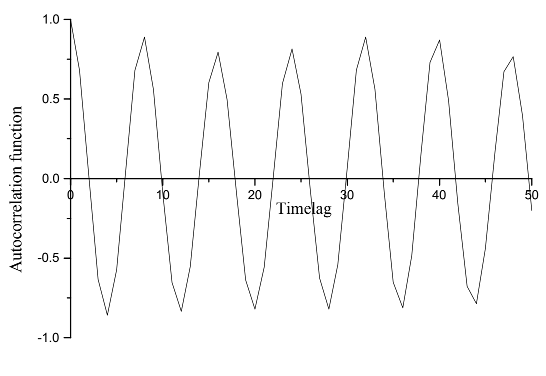

Fig. 2: Autocorrelation function of the time series of Fig. 1(a)

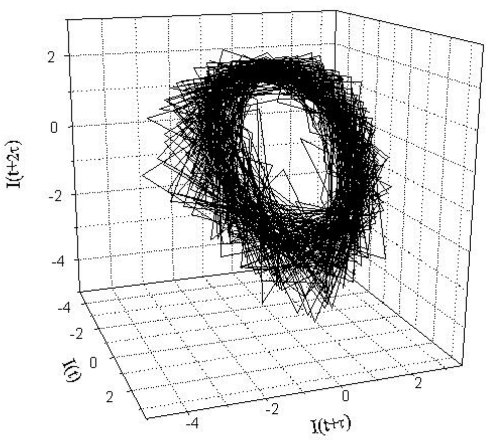

Fig. 3: 3D Poincaré map of the reconstructed space for the case of Fig. 1

reconstructed space corresponding to the signal from Fig. 1. We point out that the trajectories in the phase space, constructed with the derivatives, have almost the same shapes as in Fig. 3. We remark the torus-shaped map for the case where both instabilities appear simultaneously with similar amplitude. The torus is the usual shape in the case of two coupled oscillators like we have [9].

4. Conclusions

We performed a nonlinear dynamical analysis of the obtained times series in the case of the simultaneously excitation of two low-frequency ion-instabilities in magnetized plasma. This analysis allows us to reconstruct the state space dynamics of our plasma system.

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