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Active Control of Resistive Wall Modes in Tokamaks

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1. Introduction

The advanced tokamak is a promising candidate for an economically viable fusion reactor, because of its good transport properties. However, the advanced tokamak has rather low beta limits and is attractive only if at least $n = 1$ modes can be wall stabilized. Finite wall conductivity then gives rise to resistive wall modes (RWM), which can be stabilized either by plasma rotation or active feedback. Here we discuss feedback control and show that robust control of $n = 1$ RWM is possible with simple coil systems and controllers. We also show recent results on more complicated multiple-input-multiple-output (MIMO) feedback systems.

The key step in studying the RWM feedback control is to construct an accurate plasma response model. In [1, 2, 3], we constructed this model from toroidal computations with the MHD stability code MARS-F, and low-order rational functions (Padé approximations). Two frequency-dependent transfer functions, $P_1(s)$ and $P_2(s)$, characterize the response of the plasma wall system to the applied feedback currents. $P_1(s)$ is the transfer function from the feedback current I_f to the magnetic flux Ψ_s through the sensor loop:

$$P_1 = \Psi_s / M_{sf} I_f,$$

and M_{sf} is the mutual inductance between the active and sensor coils, in the absence of the plasma and wall. For *current* control $I_f = -K\Psi_s/M_{sf}$, where K is the feedback gain, P_1 is sufficient to determine the plasma response. For *voltage* control, where the input signal is a voltage V_f, we introduce an additional function

$$P_2(s) = \Psi_f / L_f^{(0)} I_f,$$

where Ψ_f is the total flux through the active coil, and $L_f^{(0)}$ is the self-inductance of the active coil in free-space.

The transfer functions $P_1(s)$ and $P_2(s)$ can be constructed analytically for a cylindrical plasma [4]. This showed that the cylindrical transfer function has a significant number of poles, nevertheless it can be approximated by a low order rational function. An analogous technique can be applied to construct toroidal transfer functions, in a rigorous way, from the results of numerical computations [5].

Based on the plasma response model, controllers can be designed, which meet the prescribed performance criteria, and minimize either the activity in the control signal [6], or more physical quantities such as the maximum voltage of the amplifier [7].

2. Feedback with SISO system

A simple configuration for RWM control is to put one set of feedback coils along the poloidal angle of the torus. This is a single-input-single-output (SISO) control system. For this configuration, we studied the robustness of the feedback control to variation of the plasma current. We considered sensors for the poloidal field perturbations just inside the wall, which are superior to radial sensors [2, 6, 3, 4, 8]. Several advanced equilibria of JET shape, that are unstable to the $n = 1$ RWM, are examined. The pressure has been raised to the no-wall beta limit for $n = 2$ or $n = 3$. The total plasma current varies by almost a factor of 2. The solid curves in Fig. 1 show the critical gain (minimum gain for stabilization) for proportional current control versus the the fraction of the poloidal circumference subtended by the feedback coil array, λ, when $r_w = r_s = 1.2a$ and $r_f = 1.5a$. The optimal coil width decreases with increasing safety factor q. For a broad range of coil widths all equilibria can be well controlled. To ensure a stability margin a bound can be imposed for the *sensitivity* $S \equiv 1/(1 + KP_1)$. We choose to constrain $||S||_\infty \equiv \max_{\omega\in R} |S(j\omega)| \leq 2.0$ for *all* the equilibria and minimize the control activity $||KS||_\infty$. The dashed line shows the maximum control activity $||KS||_\infty$ over *all* the equilibria after optimization of a single PD controller $K(s) = K_p(1+T_d s)/(1+T_d s/\xi)$. As an example, for $\lambda = 0.25$ an optimized controller with $K_p = 1.21,\ T_d = 0.12,\ \xi = 1.3$ gives $\|S\|_\infty \leq 1.74,\ \|KS\|_\infty \leq 1.57$, for all the equilibria, which implies good and robust control. The robustness against variations in q can be understood by comparing the unstabilized RWM structures. These are quite similar, despite the difference in q values, because the modes balloon strongly on the outboard side.

The feedback system can also be made robust with respect to toroidal plasma flow. We studied a JET-shaped equilibrium with β_N above the no-wall limit by 40% and uniform toroidal plasma rotation, with rotation frequencies $\omega_0/\omega_A = 0, 0.02, 0.04, 0.05$. For all of these rotation frequencies, a single proportional controller with $K_p = 0.63$ gives stabilization and good performance $\|S\|_\infty < 1.5$, $\|KS\|_\infty < 0.9$. Figure 2 shows how the transfer functions P_1 are modified by the plasma flow. Controller optimization for each separate rotation frequency shows that the feedback works best, if a

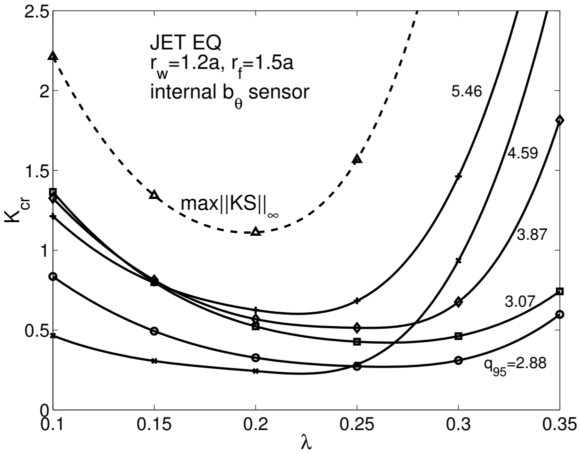

Figure 1: *The minimum gain for stability with proportional current control (solid) versus poloidal width (λ) of the active coils. The dashed line gives the maximum control activity for a single controller optimized for all the equilibria.*

toroidal phase lead of the feedback current is introduced with respect to the sensor signal, such that the feedback system pushes the plasma rotation in the same direction as the toroidal flow. The optimal phase angles of the resulting complex gains are 0^o, -20^o, -31^o, -51^o, for the rotation frequencies $\omega_0/\omega_A = 0,\ 0.02,\ 0.04,\ 0.05$, respectively.

We also studied an advanced equilibrium from ITER 9MA steady state Scenario 4 with weak negative magnetic shear. Only the up-down symmetrized equilibrium with double conformal wall was considered. The radial position of the walls are $r_1 = 1.375a$, $r_2 = 1.725a$, respectively. The wall time is assumed to be 0.15s for each wall. The active coil is placed at $r_f = 3.0a$. The plasma equilibrium has β_N 15% over the no-wall limit.

The controller is optimized for the transfer functions from MARS-F calculations for $\lambda = 0.125$ and 0.1, by minimizing the maximum voltage in the time response. In these calculations we set $P_2 = 1$ because the feedback coil is very far from the wall. The time responses of the amplifier voltage V_f, feed-

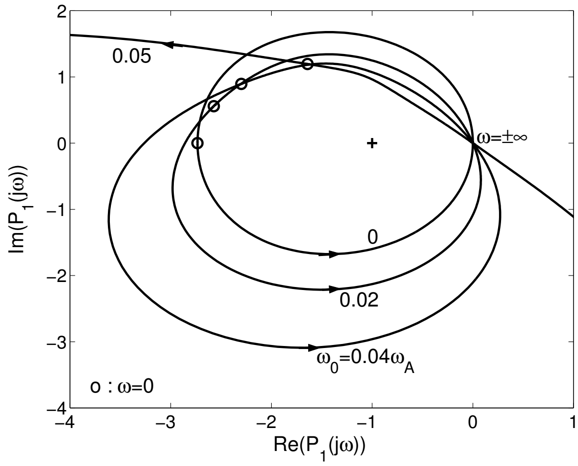

Figure 2: *Nyquist plots of the plasma response transfer functions with different toroidal rotation frequencies of the plasma.*

back current I_f, and the detected poloidal field perturbation B_t, are plotted in Fig. 3. We assume that the feedback system is turned on when the detected signal exceeds 1mT, and that the amplifier voltage saturates at 40V/turn. Figure 3 shows that the RWM can be stabilized with the present ITER design parameters, although a slightly smaller active coil, with poloidal width $\lambda = 0.1$, gives a better time response.

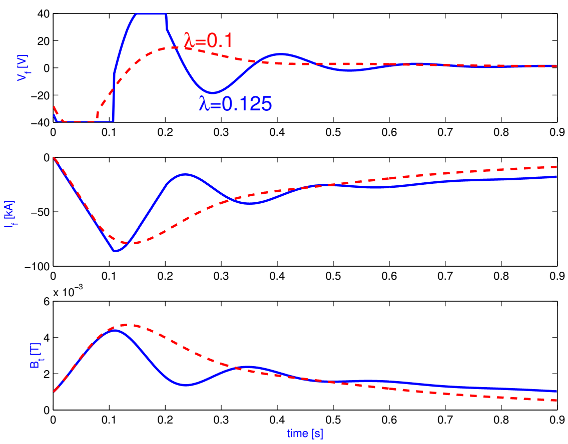

Figure 3: *The time responses of the amplifier voltage V_f, feedback current I_f, and the detected poloidal field perturbation B_t for ITER design, with voltage saturation at 40V/turn.*

3. Feedback with MIMO system

Although the simple feedback system with a single coil poloidally works well for poloidal sensors, there is interest in studying multiple coil arrays in the poloidal direction. This leads to a MIMO control system. The transfer function can be constructed analytically in the cylindrical limit. The plasma response is now described by a transfer function matrix (TFM) $\mathbf{P}(s) = [P_{jk}(s)], j,k = 1,...,N_c$, where N_c is the total number of coils in the poloidal direction. The transfer function from k-th feedback current to the j-th sensor signal is

$$P_{jk}^{r,p\pm}(s) = \frac{r_1^2 + r_f^2 - 2r_1 r_f \cos\theta_c}{2r_f^2 \sin\theta_c} \sum_m \frac{M_m^{r,p\pm} m \sin(m\theta_c)}{\mu} e^{im(\theta_j - \theta_k)},$$

where $M_m^{r,p\pm}$ is defined the same as in the SISO case (ref. [4]), $\mu = |m|$, θ_j is the poloidal position of j-th active coil. The sum is taken over all the Fourier harmonics.

Using cylindrical theory, we studied the effect of feedback coil geometry on the RWM stabilization. With fixed poloidal width $\lambda = 0.2$ for each coil and the number of coils $N_c = 3$, we changed the poloidal distance Λ between two neighboring coils. For almost all the geometrical configurations with multiple coils, there is a second instability, when the (proportional) gain is large. Therefore, the closed loop system is stable only in a window for the gain, as shown by Fig. 4. For poloidal sensors, the upper limit of the gain disappears as Λ approaches 0, i.e., when the three coils coalesce. A large stable window is achieved when the active coils overlap only slightly, or are separated from each other. For radial sensors, stabilization is possible only when the coils are separated. Separation reduces the coupling between the coils, and the driving source for the second instability. MIMO transfer function matrices can also be constructed from toroidal computations, although this is more

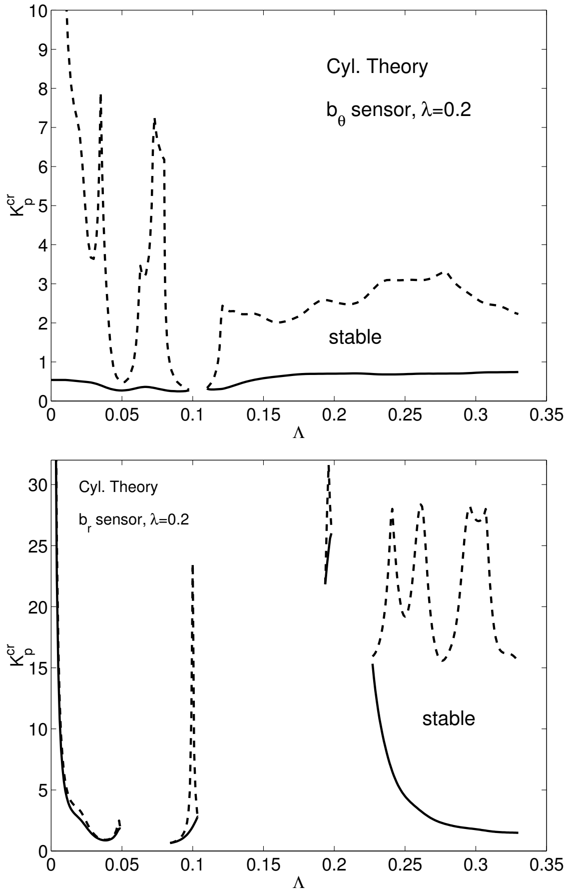

Figure 4: *Lower (solid lines) and upper (dashed) limits of the proportional feedback gain versus the poloidal distance* Λ *between two neighboring coils. A cylindrical plasma with (a) - poloidal sensors, and (b) - radial sensors, is considered.*

difficult than in the SISO case, because the stability property of the closed loop is very sensitive to the accuracy of the MIMO model. However, the stability margin (in terms of critical gains) can easily be calculated by MARS-F, and shows similar behavior as in Fig. 4.

4. Conclusion

With single set of coils in the poloidal direction, $n = 1$ RWM control can be made robust with respect to changes in current, pressure (up to a maximum value) and plasma rotation. Acceptable voltages are found for the ITER design.

For poloidal sensor, a three coil configuration does not improve the stabilization significantly, compared with a single coil system. In most cases, a second instability is induced by the coils coupling in the MIMO system. For the radial sensor, MIMO system can improve the RWM stabilization, but a single array with poloidal sensors is still better.

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