Time-Dependent Closure Relations of the Collisionless Fluid Equations in Fully Ionized Plasmas and Rarefied Neutral Gases

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I. Introduction

The resolution of the kinetic equation is a problem of great interest in the description of plasma and neutral gases. Indeed, the solutions of this kinetic equation are useful to derive the transport coefficients, which include purely kinetic effects. More precisely, an increasing interest is devoted to the transport effects in the collisionless limit, where Landau resonance can occur. In this work, we present a theoretical analysis of the kinetic equation in the collision free limit in fully ionized plasmas[1] and neutral gases[2]. The time-dependent kinetic equation has been solved and the transport coefficients for arbitrary normalized phase velocities $\xi = \omega / \sqrt{2}k v_t$ have been deduced. These transport coefficients are used to close the low-moment fluid equations.

II. Basic equations

A collisionless system constituted of a great number of particles can be described by the following kinetic equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \cdot \frac{\partial f}{\partial \vec{v}} = 0 \,, \tag{1}$$

where $f(\vec{r}, \vec{v}, t)$ is the distribution function, m is the particle mass and $\vec{F}$ is a force which accounts for both external and internal contributions.

In this work, we study small perturbations which can be described by the linearized kinetic equation written in the Fourier space and in the frame of the fluid,

$$-i\omega \, \delta f + ik v_x \, \delta f + \frac{F}{m} \, \frac{\partial f_M}{\partial v_x} = - \lim_{v \to 0} v(\delta f - \delta f_M) + i \, \omega \, \frac{v_x}{v_t^2} \, \delta V f_M \; - \; i \, k \, \frac{v_x^2}{v_t^2} \, \delta V f_M \,, \tag{2}$$

where, $f_M(v, n_0, T_0)$ is the global Maxwellian and $\delta f_M(v, \delta n, \delta T)$ is the perturbed Maxwellian. We have added in the right hand side of Eq. (2), a collision operator in the limit of vanishing collision frequency $(v \to 0)$. To ensure the conservative properties of this collision operator, we use the projection operators defined in Ref. 1. Thus,

expanding Eq. (2) on the Legendre polynomial basis, we obtain the following set of equations

$$(-i\omega + v)\delta f_0 + (ikv_t \frac{\sqrt{2}}{\sqrt{3}})y^{1/2}\delta f_1 = (-i\omega + v)\delta f_M + \frac{\exp(-y)}{\Gamma(3/2)}(ikv_t \frac{\sqrt{2}}{\sqrt{3}})\left[\frac{5}{2}\delta M^1(\delta f_1) - \delta M^2(\delta f_1)\right]$$

$$+\frac{y\exp(-y)}{\Gamma(5/2)}(ikv_t \frac{\sqrt{2}}{\sqrt{3}})\left[-\frac{3}{2}\delta M^1(\delta f_1) + \delta M^2(\delta f_1)\right], \tag{3}$$

$$\delta f_1 = F_1\left[-(ikv_t \frac{\sqrt{2}}{\sqrt{3}})y^{1/2}\delta f_0 + \frac{F}{m}\frac{1}{v_t}\frac{\sqrt{2}}{\sqrt{3}}y^{1/2}\mu_0 \exp(-y) + i\omega \frac{\delta V}{v_t}\frac{\sqrt{2}}{\sqrt{3}}y^{1/2}\mu_0 \exp(-y)\right]$$

$$+\frac{8}{15}(ikv_t \frac{\sqrt{2}}{\sqrt{3}})(ik\delta V)y^{3/2}F_1 F_2 \mu_0 \exp(-y), \tag{4}$$

$$\delta f_2 = -\frac{2\sqrt{2}}{\sqrt{15}}ik\,v_t\,y^{1/2}F_1 F_2\left[-(ikv_t \frac{\sqrt{2}}{\sqrt{3}})y^{1/2}\delta f_0 + \frac{F}{m}\frac{1}{v_t}\frac{\sqrt{2}}{\sqrt{3}}y^{1/2}\mu_0 \exp(-y)\right.$$

$$\left.+i\omega \frac{\delta V}{v_t}\frac{\sqrt{2}}{\sqrt{3}}y^{1/2}\mu_0 \exp(-y)\right] - i\omega F_1 F_2\left[-ik\delta V \frac{4}{3\sqrt{5}}y\mu_0 \exp(-y)\right], \tag{5}$$

where the continued fractions F_1 and F_2 are defined by the recurrence formula

$$F_n = \left[-i\omega + v + 2k^2 v_t^2 y \frac{(n+1)^2}{4(n+1)^2 - 1}F_{n+1}\right]^{-1}. \tag{6}$$

Equations (3)-(5) constitute a linear set of equations for the unknown functions δf_0, δf_1 and δf_2. By solving these equations, we have computed the explicit expressions of δf_1 and δf_2 which define the transport coefficients. From the standard definitions of the heat flux $\delta \vec{q}$ and the stress tensor $\delta \overline{\overline{\Pi}}$, we have deduced the non vanishing components

$$\delta q_x = -K_T(\xi)\frac{ik}{|k|}n_0 v_t \delta T - \alpha(\xi)n_0 T_0 \delta V, \tag{7}$$

$$\delta \Pi_{xx} = -\eta(\xi)\frac{ik}{|k|}mn_0 v_t \delta V - \alpha(\xi)n_0 \delta T, \tag{8}$$

where, the thermal conductivity K_T, the anisotropic temperature coefficient α (which corresponds also to the convective heat flux coefficient) and the viscosity coefficient η, are functions of the normalized phase velocity ξ.

III. Numerical results

The collisionless transport coefficients in (7) and (8), have been used as closure relations of the low-moment fluid equations[1]. By eliminating the temperature δT with the use of the energy equation, we write only the two first fluid equations, namely the equation of continuity and the equation of motion.

$$- i\omega\delta n + ikn_0\delta V = 0 \tag{9}$$

$$- i\omega\delta V + ik v_t^2 \Gamma\frac{\delta n}{n_0} + \tilde{v}\delta V = \frac{F}{m}, \tag{10}$$

where,

$$\Gamma(\xi) = 1 + \frac{\xi}{[(\frac{3}{2}\xi - \frac{K_{TI}}{\sqrt{2}})^2 + \frac{K_{TR}^2}{2}]} \times [(1 + \alpha_R^2 - \alpha_I^2 - 2\alpha_R)(\frac{3}{2}\xi - \frac{K_{TI}}{\sqrt{2}}) + \sqrt{2}K_{TR}\alpha_I(\alpha_R - 1)] + \sqrt{2}\xi\eta_I, \tag{11}$$

corresponds to the generalized polytropic coefficient[1] and

$$\tilde{v}(\xi) = -\frac{kv_t}{\sqrt{2}[(\frac{3}{2}\xi - \frac{K_{TI}}{\sqrt{2}})^2 + \frac{K_{TR}^2}{2}]}[2\alpha_I(\alpha_R - 1)(\frac{3}{2}\xi - \frac{K_{TI}}{\sqrt{2}}) - \frac{K_{TR}}{\sqrt{2}}(1 + \alpha_R^2 - \alpha_I^2 - 2\alpha_R)] + \eta_R, \tag{12}$$

to the fluid damping rate[1], where the subscripts R and I stand for real and imaginary parts of the transport coefficients. We present in Figs. 1 and 2, these two physical quantities as functions of the normalized phase velocity ξ.
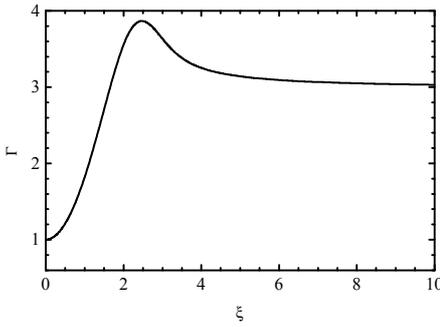

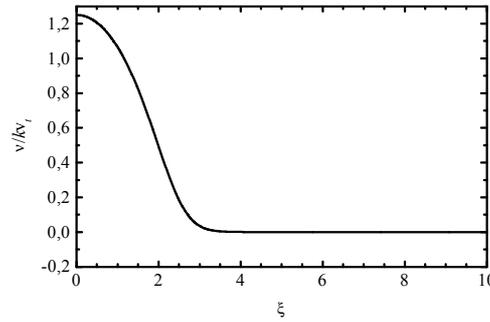Fig.1 The rate of specific heats Γ vs ξ .

Fig. 2 The normalized fluid damping rate $(\tilde{v}/kv_t)$ vs ξ .

Using The Poisson equation and Eqs. (9) and (10), we can deduce from the fluid theory, the dispersion relation of the Langmuir waves in fully ionized plasmas

$$\left(\frac{\omega_r}{\omega_p}\right)^2 = 1 + \Gamma(\xi)(k\lambda_D)^2, \tag{13}$$

$$\frac{\gamma}{\omega_p} = \frac{(k\lambda_D)}{2}\frac{\tilde{v}}{kv_t}(\xi). \tag{14}$$

On the other hand, by dropping the force $\vec{F}$ in Eq. (10) and using Eq. (9), we obtain the dispersion relation of sound waves in rarefied neutral gases,

$$\omega_r^2 = \left[\Gamma(k_R^2 - k_I^2) + 2\omega_r(\frac{\tilde{v}}{k_R^2 v_t^2})k_R k_I\right]v_t^2, \tag{15}$$

$$2\Gamma k_R k_I = \omega_r(k_R^2 - k_I^2)(\frac{\tilde{v}}{k_R^2 v_t^2}). \tag{16}$$

In Eqs. (13) and (14), ω_r is the frequency of the plasma modes, γ is the temporal damping rate ($\gamma \geq 0$) and λ_D is the electron Debye length. In Eqs. (15) and (16), we have used the notation $k = k_R + ik_I$, in order to compare the spatial damping rate (k_I) with the experimental result. We present in Fig. 3, the variation of the normalized damping rate γ / ω_p, for electrostatic waves in plasmas, with respect to the parameter $(k\lambda_D)^2$. We have compared our results to the standard results derived from the Vlasov equation[3]. We note that the two results are in very good agreement

The normalized damping rate k_I / k_0 (where $k_0 = \sqrt{3/5}\,\omega_r / v_t$), for sound waves in neutral gases, is displayed in Fig. 4. We can see that in the weakly collisional limit, our result agrees well with the experimental results presented in Ref. 4.
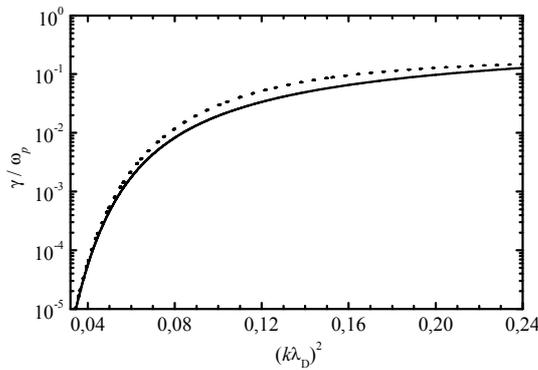

Fig. 3. The normalized damping rate γ / ω_p versus the parameter $(k\lambda_D)^2$. The solid curve corresponds to the theoretical results and the dotted curve of the standard results.
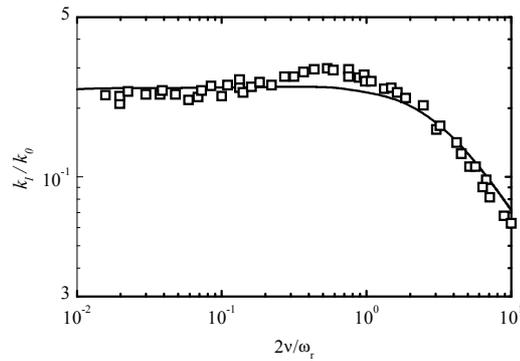
Fig. 4. The normalized damping rate k_I / k_0 versus the parameter $2\nu / \omega_r$. The solid curve corresponds to the theoretical results and the white squares to the experimental results of Ref. 4.

IV. References

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Acknowledgments