

The Effect of Shear Flow on Inverse Landau Damping of Drift Modes in Tokamak Plasma

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In this paper we investigate the effects of inverse Landau damping on two-dimensional drift modes in a rotating tokamak plasma. The model includes how the peaking of the diamagnetic frequency, the flow velocity shear and the magnetic shear govern the structure of drift modes.

Long term plasma confinement has illuded scientists for decades. As the larger MHD disruptions was brought under control, a smaller scale turbulence revealed itself. It is now thought that this microturbulence stems from phase decorrelated drift waves driven by density and temperature gradients. The gradients harbour local free energy and release it in an attempt to reach equilibrium, by exciting low-frequency oscillations. The oscillations, known as drift waves then create the turbulence detrimental to plasma confinement. As a fusion plasma inherently contains gradients, a better knowledge of the excitation mechanisms, the stability and structure of drift waves is needed to ensure the future of a successful fusion reactor.

Inverse Landau damping is one of the instability processes which always occurs in plasmas. We focus on how drift mode suppression by shear flow affects the stability of the Landau resonance. It should be noted that a sheared flow is nowadays thought to be one of the main stabilising mechanisms of plasma turbulence hazardous to plasma confinement.

In high confinement plasmas, the cause and effect of the observed radial electric field and poloidal plasma flow have not yet been experimentally clarified. Theoretical models relating these two quantities explains the flow as a consequence of the electric field in conjunction with the confining magnetic field. The flow can thus been viewed as an $\mathbf{E} \times \mathbf{B}$ -flow. The shear of this flow emerges from the steep radial gradient of the radial electric field. Hence, the flow may have a considerable shear in the radial direction, which can tear turbulent eddies apart.

Previous studies[1], without plasma flow, showed that a prominent radial peaking of the electron diamagnetic frequency can localise drift modes. The peaking creates a potential well which trap the wave energy both radially and along the magnetic field lines. The diamagnetic frequency's peaking arises from its dependence of the electron density and temperature. The peaking makes the mode structure strictly two-dimensional, without possibility to reduce it.

A more recent investigation [2] pointed out that the plasma rotation shear plays a significant role when we explore the influence of the diamagnetic peaking on the drift mode structure. A sufficiently strong velocity shear can entirely suppress the mode localisation imposed by the frequency peaking. If so, magnetic shear damping determines the mode structure.

The key properties of drift waves excited in an axisymmetric tokamak plasma, with circular concentric magnetic surfaces, can be described by a set of fluid equations for the electrons and the ions supplemented by the Maxwell equations for the electromagnetic field. In the limit of low- β (plasma pressure/ magnetic field pressure) the magnetic field perturbations are negligible and the drift waves become collisionless electrostatic waves of low frequency. The application of the usual drift mode ordering to these equations reveals two coupled equations which describe the ions' motion parallel and perpendicular to the magnetic field.

To reduce the two coupled differential equations to a single differential equation for the electrostatic potential, consider drift wave solutions localised on a rational surface $r = r_0$. A rational surface require that $m_0 - nq(r_0) = 0$, where m_0 and n are the poloidal and toroidal mode numbers respectively and $q(r_0)$ is the safety factor, all variables evaluated at the surface. Moreover, suppose that the modes variate sinusoidally, $\exp(-i\omega t - in\phi)$ with a rapid $\exp(im_0\theta)$ but also with a slow, $\partial/\partial\theta \ll m_0$ θ -dependence and that $m_0 \gg 1$. The above manipulations result in the following differential equation for the perturbed part of the electrostatic potential:

$$\left[1 + k^2 - i\delta - \frac{k}{\Omega} \left(1 - \frac{\rho^2 x^2}{r_d^2} \right) \left\{ 1 - \frac{\rho V_0}{r_0 c_s} L_V \right\} + \frac{k \varepsilon_n \rho V_0}{\Omega \varepsilon_0 r_0 c_s} \{ 1 - L_V - L_V L_V' \} \right] F - \frac{\partial^2 F}{\partial x^2} + \frac{k}{\Omega} \left[2\varepsilon_n - \varepsilon_0 \left(1 + \frac{r_n V_0}{\rho c_s} \right) \right] \cos \theta F - \frac{2i\varepsilon_n}{\Omega} \sin \theta \frac{\partial F}{\partial x} + \gamma^2 \left\{ \frac{\partial}{\partial \theta} - ik_{sx} \right\}^2 F = 0. \quad (1)$$

The dimensionless variables are $k = k_\theta \rho$, $\Omega = \Omega r_n / c_s$, $\varepsilon_n = r_n / R$, $\varepsilon_0 = r_0 / R$, $L_V = r_0 V_0' / V_0$, $L_V' = r_0 V_0'' / V_0'$, $\gamma = \varepsilon_n / q \Omega$, where the Larmor radius $\rho = c_s / \omega_{ci}$, the ion sound velocity $c_s = (T_e / m_i)^{1/2}$, the electron temperature T_e , the Doppler shifted frequency, $\Omega = \omega - k_\theta V_0$, the poloidal wave number $k_\theta = m_0 / r_0$, the density inhomogeneity length scale $r_n^{-1} = -(\text{dln } n_0 / \text{dr})|_{r=r_0}$, the magnetic shear $s = r_0 q' / q$, the rotation velocity $V_0 = c / B_T (\text{d}\phi_0 / \text{dr})|_{r=r_0}$, its shear $V_0' = c / B_T (\text{d}^2 \phi_0 / \text{dr}^2)|_{r=r_0}$ and curvature $V_0'' = c / B_T (\text{d}^3 \phi_0 / \text{dr}^3)|_{r=r_0}$. The peaking of the diamagnetic frequency enters the equation as the factor $1 - \rho^2 x^2 / r_d^2$, with r_d as the radial peaking and $x = (r - r_0) / \rho$ as the normalised displacement from a magnetic surface.

To solve Eq. (1), suppose as Horton *et al.* [1] did, that the spatial envelope of a wave has a two-dimensional Gaussian shape, modulated to consider the characteristics of drift waves, *i.e.*

$$F(x, \theta) = \exp \left(-\frac{1}{2} \sigma_\perp x^2 + ibx\theta - \frac{1}{2} \sigma_\parallel \theta^2 \right). \quad (2)$$

The addition of the cross-term $ibx\theta$ ensures the long wavelength along the magnetic field property of drift waves as it makes the phase-front rotate around a magnetic field line.

We assume that the spread in the poloidal direction to be so small that we can approximate $\sin \theta \approx \theta$ and $\cos \theta \approx 1 - \theta^2 / 2$. Applying this approximation and Eq. (2) on Eq. (1) yield an equation of terms multiplied by 1, x^2 , θ^2 and $x\theta$. Since all these variables are independent of

each other and the total equation equals zero, each and every one of these terms has to equal zero. This reveals four separate equations, of which one is the dispersion relation,

$$1 + k^2 - i\delta \left(1 - \frac{\varepsilon_0 k r_n V_0}{\Omega \rho c_s}\right) + \frac{k \varepsilon_n \rho V_0}{\Omega \varepsilon_0 r_0 c_s} \{1 - L_V - L_V L'_V\} - \frac{k}{\Omega} \left\{1 - \frac{\rho V_0}{r_0 c_s} L_V\right\} + \frac{k}{\Omega} \left\{2\varepsilon_n - \varepsilon_0 \left(1 + \frac{r_n V_0}{\rho c_s}\right)\right\} + \sigma_\perp - \gamma^2 \sigma_\parallel = 0 \quad (3)$$

and the remaining three determines the variables σ_\perp , σ_\parallel and b . We can see from Eq. (3) that the phaseshift $\sigma_\perp - \gamma^2 \sigma_\parallel$ plays an important roll when determining the structure of the drift mode. A real phaseshift implies a localised mode whilst an imaginary phaseshift exerts damping on the mode [2]. The stability of a drift mode depends on whether the magnetic shear damping or the non-adiabatic electron response (δ in Eq. (3)) exerts the most influence on the mode.

The magnetic shear damping affects the mode through the phaseshift $\sigma_\perp - \gamma^2 \sigma_\parallel$. In the case when the magnetic shear damping completely deteriorates the mode localisation by the diamagnetic peaking, the phaseshift becomes imaginary [1,3],

$$\sigma_\perp - \gamma^2 \sigma_\parallel = i\gamma \left\{ \left(\frac{\varepsilon_n}{\Omega} + ks \right)^2 - \left[\left\{ \frac{\varepsilon_n}{\Omega} \left(\frac{\varepsilon_n}{\Omega} + k \right) - \frac{\varepsilon_0 k}{2\Omega} \left[1 + \frac{r_n V_0}{\rho c_s} (1 - i\delta) \right] \right\}^{1/2} - \frac{\alpha}{\gamma} \left(1 - \frac{\rho V_0}{r_0 c_s} L_V \right)^{1/2} \right]^2 \right\}^{1/2}, \quad (4)$$

where the diamagnetic peaking enters as $\alpha^2 = \rho^2 k / r_d^2 \Omega$. We can see from Eq. (4) that a strong velocity shear (L_V) may suppress the influence of the diamagnetic peaking. A thorough analysis of the phase shift can be found in [2-3].

The phaseshift (4) stabilises the mode, but the non-adiabatic electron response δ may counteract it. Hence, the sign of δ is of the utmost importance when to determine the stability of a drift mode. From the dispersion relation (3), we see that this sign do not solely emerge from the appropriate expression of δ for the destabilising mechanism we want to consider. In addition the sign of δ depends on the sign of the term multiplied by it. This term includes the effects of the velocity of the plasma and the normalised phase velocity, $\Omega/k = f(L_V, V_0)$ of the wave.

We can see from the dispersion relation (3) that $\delta > 0$ invokes instability. In the particular case of Landau damping [3],

$$\delta = \left(\frac{\pi}{2}\right)^{1/2} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{k}{|k_\parallel| r_n} \left(1 - \frac{\Omega}{k}\right) \quad (5)$$

infers that instability occurs whenever $\Omega/k < 1$. To estimate this ratio from Eq. (3), regard δ and $\sigma_\perp - \gamma^2 \sigma_\parallel$ as minor contributions to the total dispersion relation. Then,

$$\frac{\Omega}{k} \simeq \frac{1 - \frac{\rho V_0}{r_0 c_s} L_V \left[1 - \varepsilon_n \left\{ \frac{1}{\varepsilon_0} + \frac{1}{L_V} \left(\frac{r_0^2}{\rho^2} - \frac{1}{\varepsilon_0} \right) \right\} \right]}{1 + k^2} \approx \frac{1 - \frac{\rho V_0}{r_0 c_s} L_V}{1 + k^2}, \quad (6)$$

for normal edge plasma parameters: $\varepsilon_0 > 2\varepsilon_n$ and $L_V \gg 1$. For $V_0 > 0$, we can conclude from the structure of Eq. (6), that $\Omega/k < 1$ always. Moreover, Ω/k decreases with increasing velocity shear. Returning to Eq. (3), we find the sign of the instability not to be exclusively determined by Ω/k , but also by the sign of

$$1 - \varepsilon_0 \frac{k r_n V_0}{\Omega \rho c_s} \approx 1 - \varepsilon_0 \frac{1 + k^2}{1 - \frac{\rho V_0}{r_0 c_s} L_V} \frac{r_n V_0}{\rho c_s}, \quad (7)$$

where we have replaced Ω/k by Eq. (6). Hence, as the velocity shear, L_V increases, the sign of this factor becomes negative and the instability is reversed. This implies a distinct suppression of drift waves by a strong velocity shear. A threshold in velocity shear exists, $L_V < r_0 c_s / \rho V_0$, as to keep the negative sign in Eq. (7) and hence the restraining of the instability successful.

As we can see from Eq. (7) this mechanism is only effective for positive rotation velocities. But the negative rotation modes are not inherently unstable. Assume that the magnitude of the velocity always increases with radius, disregarding the direction of rotation so $L_V > 0$ holds. Rewrite Eq. (6) for $V_0 < 0$ and apply it to $\Omega/k < 1$ to find the instability criterion

$$k^2 > \frac{\rho |V_0|}{r_0 c_s} L_V \quad \Rightarrow \quad m_0^2 > \frac{r_0 |V_0|}{\rho c_s} L_V. \quad (8)$$

Definition of $k = m_0 \rho / r_0$, makes the last step possible. For large velocity shear and near sonic rotation velocities, the above number is large. Thus, only modes with large poloidal mode numbers are unstable. These high m_0 -modes have very small spatial structures compared to the velocity inhomogeneity length scale. Hence, the velocity shear can not tear the modes apart.

The modes (8) can nevertheless become stable if the magnetic shear damping dominates the mode localisation due to the diamagnetic peaking. The latter was shown [2] to be most likely at strong velocity shears. If so, the energy of the unstable modes then convects out from the magnetic surfaces because of the magnetic shear damping and the mode regains stability.

In conclusion, we have shown that the velocity shear has a suppressing or even reversing effect, on inverse Landau damping. For positive velocities, the expression for the Landau resonance seems to imply that drift modes are always unstable, but due to an external, velocity induced effect the modes are stabilised for large velocity shear. This mechanism does not affect modes with negative velocity, but in this case only modes with high poloidal mode numbers are unstable. The stability threshold for negative rotation is proportional to the velocity shear. Hence, a high velocity shear prevents unstable modes from developing. In either case, magnetic shear damping can drain energy out of the instabilities and suppress them.

References

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